# Modeling and Control of an Active Magnetic Bearing System

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Abstract — Active Magnetic Bearings (AMB) support a body by magnetic pulling forces, without any mechanical contact. The main advantages of such bearings compared with the traditional solutions are: absence of mechanical friction and wear, lubricant-free operation and therefore suitability for severe environments and applications, active vibration control and unbalance compensation. This paper deals with modeling and simulation of radial active magnetic bearing system in a high-speed drive. The system model from previous works [1], [5] has been extended with model of the power amplifier with PWM control. Hence, dynamic behavior of currents in the bearing windings can also be investigated with this model. Two different controllers are used in the work, i.e. PID and state-space controller. Both will be compared to each other for two cases: a) lift-off of the rotor, and b) unbalance in the rotor which causes periodical disturbance force, which has also to be compensated by the magnetic bearing, additionally to the gravitation force.

Index terms — Active magnetic bearings mathematical model, PID controller, STATE SPACE method.

#### I. Introduction

Magnetic bearings have been a research topic for several decades. During this time, magnetic bearings have evolved into an industry product that - due to its advantages over conventional bearing technology - is used in many practical applications as: turbo molecular vacuum pumps, gas pipeline centrifugal compressors, sealed pumps, and electric utilities power plant equipment. This progress was possible due to progress made in power electronics, microprocessors and digital control. Today, there are two trends in AMB technology: on the one hand the high speed drives gain more importance especially in the machining and vacuum technology, and on the other hand, analogue control is abandoned in favor of digital control, which offers much more flexibility for taking full advantage of the AMB technology. For the operation of the AMB system, position control is necessary, since the magnetic force system is inherently unstable. The design of such control is a challenging task since it must compensate for the instability inherent to the magnetic bearing and at the same time avoid destabilization of any flexible

modes the rotor may exhibit, especially at high speed, which is aimed in our case at 40000 min<sup>-1</sup> and for a rotor with a mass of 14 kg. At the moment this high speed drive is supported by AMB with analog control [8], which shall be replaced by digital control in the future. The purpose of this paper is to contribute to digital controller design for AMB rotor system. This paper follows two directions:

- 1. *Identification of AMB rotor system model*, with detailed modeling of PWM of current controller.
- Digital controller design, searching for a well-suited controller that fits for all necessities. Two types of controllers were distinguished and will be discussed:
   a) PID controllers and b) State-space controllers based on LQR method (Linear Quadratic Regulator).

Since in large-signal model the unstable AMB is non-linear, the most useful approaches are: (i) to use differential windings magnetic bearing to reduce the influence of non-linearity and (ii) to linearize the model in a small region around equilibrium points in order to use linear control techniques. However, the performance of a single operating point linear controller can be quite good only near the equilibrium conditions. As the variation of the rotor position is small due to the small air gap of AC motors, this method of linearization applies well.

Comparisons between PID and state-space controller for magnetic bearings are available in several publications in the past, for example [6] and [7]. For a current research on high-speed machine with 40000 min<sup>-1</sup> at *Darmstadt University of Technology*, the comparison here shall be done with a demonstrator (Fig.1), so previous simulation of the control software is necessary, which is included here.

# II. MODELS AND FORMULATIONS

The core of AMB system is an electromagnet, which creates a magnetic field that provides a magnetic pull force that supports the body without mechanical contact, if a current is passed through electromagnet coil.

The AMB exerts an attractive force on the body that opposes the gravitational force, which pulls the body downward



Figure 1. Magnetic bearing demonstrator with two radial bearings (max. force 230 N)

If the current is kept constant, and the body is moving downward, the magnetic force on the body is decreasing as the distance between electromagnet and body is increasing. This leads to the body falling down. If the body is moving upwards, the attractive magnetic force is increasing, and the body is accelerated towards the electromagnet.

To avoid this unstable behavior, the electric current must be permanently adjusted. A position sensor (e.g. capacitive or eddy-current sensor) measures the deviation from reference position. Based on this measurement a control system (controller/microprocessor) computes the value of current that must be applied to electromagnet coil and sends this information to power amplifier. The power amplifier generates this electric current, which excites the electromagnet coil. With an appropriately designed controller the body can be held at its reference position, and system dynamics can be adjusted in a wide range.

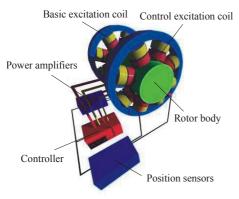


Figure 2. AMB system model

In technical applications, differential winding bearings are used. So, for positioning the rotor in one axis, a second magnet identical to first one, but exerting pulling force in opposite direction, is used. The non-linear force-distance characteristic tends to be more linear. It improves the dynamics, since now the forces on the rotor body can be exerted in both directions of axes. The AMB system (Fig. 2) has two degrees of freedom (x-and y-positions). The two opposing electromagnets are operating in differential winding mode (Fig. 3). A con-

stant bias current  $i_0$  is exciting basic excitation coils for generating the magnetic field, and the control current  $i_c$  is exciting an additional control field which exerts pull forces in positive direction, so adding to the field of one magnet, and subtracting to the opposite magnet.

## A. Magnetic force model

In order to obtain the equation that is describing the AMB behavior the following assumptions are used:

- The permeability of iron part is infinitely high (= ideally unsaturated iron).
- The magnetic flux density *B* is homogeneously distributed in the iron core and the air gap.
- The magnetic pole cross-sectional areas are constant along the magnetic circuit.

In air gap, flux density B is proportional to magnetic field strength H, so the radial force can be expressed as a function of coil current i and air gap d.

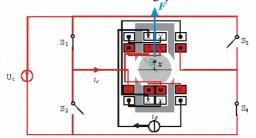


Figure 3. Magnetic bearing with differential windings, connected at constant bias current *i*<sub>0</sub> and switched power amplifier injecting the control current *i*<sub>c</sub>

With control and bias current  $i_c$ ,  $i_0$  and

 $x = d_0 \cdot \cos(\alpha) - d \cdot \cos(\alpha)$ ,

where  $i_0$ ,  $d_0$  are current and air gap, when rotor is placed in symmetric axis and  $\alpha$  is the angle under which the magnetic force attracts the rotor [1], equation (1) is obtained:

$$F = k \cdot \left[ \frac{(i_0 + i_c)^2}{(d_0 - x)^2} - \frac{(i_0 - i_c)^2}{(d_0 + x)^2} \right] \cdot \cos^3 \alpha$$
 (1a)

with 
$$k = \mu_0 \cdot N^2 \cdot A/4$$
 (1b)

 $\mu_0 = 4 \cdot \pi \cdot 10^{-7}$  H/m stands for the vacuum magnetic permeability, N is coil number of turns and A is iron cross-sectional area. Equation (1b) is valid, if number of turns of basic and control coils  $N_0$  and  $N_c$  are identical:  $N_0 = N_c = N$ , otherwise  $i_0$ ' =  $(N_0/N_c)$ .  $i_0$  has to be used. Equation (1a) and Fig. 4 show the quadratic dependence of force with *control current*  $i_c$  and inversely quadratic dependence with *air gap d*, which - due to differential winding mode - gives a wide range of linear dynamic operation as equation (2)

$$F \approx k_{\rm i} \cdot i_{\rm c} + k_{\rm x} \cdot x \tag{2}$$

TABLE 1 SIMULATION: AMB PARAMETERS

Maximum force $F_{\text{max}}$	230	N	
Rotor mass, m	14	kg	
Air gap, d	0.6	mm	
Bias current, $i_0$	6	A	
Control current, ic	15	A	
No. of turns for basic excitation	46		
$N_0$	18		
No. of turns for control excitation			
$N_{c}$			

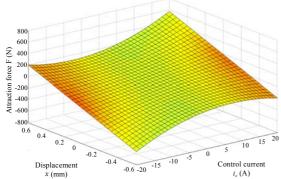


Figure 4. Calculated attraction force  $F(i_c,x)$  of differential winding mode, showing more linear behavior (one radial bearing of demonstrator Fig.1).

## B. Rigid body rotor model

The mathematical model does not take into account axial movement, this being justified by negligible coupling effect between the radial and axial dynamics [2]. So, the mathematical model refers only to a rigid rotor body, suspended in two identical radial active magnetic bearings. The rotor dynamics in the x-z-plane can be described using displacement x of rotor's center of gravity O and rotation angle  $\varphi$  between rotor and horizontal z-axis through O (Fig.5). Usually, even for high-speed operation rotational speed n of rotor is kept well below 70 % of first bending natural frequency  $f_e$ , so the rotor may be treated as rigid body. The rather low dynamic elasticity of AMB due to the low value of  $k_x$ gives vibration of the rigid body in the bearings, called common mode and differential mode vibration.

From Fig. 5 with force and torque in x-z-plane and y-zplane, the mechanical equation for evaluating the AMB system dynamic behavior, considering rotating rigid rotor vibration, can be written as:

$$f = M \cdot \ddot{z} + G \cdot \dot{z} \tag{3}$$

In (3),  $\mathbf{f} = (f_x \quad p_y \quad f_y \quad -p_x)^T$  is force component vector, M is mass matrix (see Equation (6)), G is gyroscopic matrix (see Equation (7)) and  $z = (x \ \beta \ y \ -\alpha)^{T}$  is coordinate component vector, using the global coordinate system x, y, z.

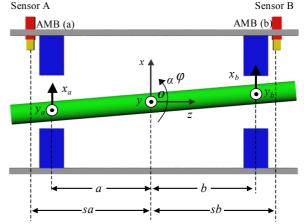


Figure 5. Magnetically suspended rigid rotor shaft

In order to give a proper command, (3) must be translated in AMB coordinate system using a transformation matrix T, with the axial distance a, b of the radial bearings from O [2].

$$T = \frac{1}{a+b} \cdot \begin{bmatrix} b & a & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & b & a \\ 0 & 0 & -1 & 1 \end{bmatrix}$$
 (4)

With  $f_c = T \cdot f$ ,  $z_c = T \cdot z$ , we get:

$$f_{c} = M_{c} \cdot \ddot{Z}_{c} + G_{c} \cdot \dot{Z}_{c} \tag{5}$$

In Eq. (5) the subscript c denotes the control coordinate system.  $\mathbf{M}_{c}$ ,  $\mathbf{G}_{c}$  and  $\mathbf{z}_{c}$  have following expressions, where m is rotor mass,  $I_x$ ,  $I_y$ ,  $I_z$  are rotor inertia moments around x, y, z-axis and  $\Omega = 2\pi n$  is angular speed around z-axis. With the use of inverse T' of matrix T we get

$$\boldsymbol{M}_{c} = \boldsymbol{T}' \cdot \boldsymbol{M} \cdot \boldsymbol{T} = \boldsymbol{T}' \cdot \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_{x} & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & I_{y} \end{bmatrix} \cdot \boldsymbol{T}$$
(6)

$$\mathbf{M}_{c} = \mathbf{T}' \cdot \mathbf{M} \cdot \mathbf{T} = \mathbf{T}' \cdot \begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & I_{x} & 0 & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & I_{y} \end{bmatrix} \cdot \mathbf{T}$$

$$\mathbf{G}_{c} = \mathbf{T}' \cdot \mathbf{G} \cdot \mathbf{T} = \mathbf{T}' \cdot \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \cdot I_{z} \cdot \Omega$$

$$\mathbf{T} = \mathbf{T}' \cdot \mathbf{G} \cdot \mathbf{T} = \mathbf{T}' \cdot \mathbf{G} \cdot \mathbf{G} \cdot \mathbf{G} \cdot \mathbf{G} = \mathbf{G} \cdot \mathbf{G} \cdot \mathbf{G} \cdot \mathbf{G} \cdot \mathbf{G} = \mathbf{G} \cdot \mathbf{G} = \mathbf{G} \cdot \mathbf{G} \cdot \mathbf{G} = \mathbf{G} \cdot \mathbf{G} \cdot \mathbf{G} = \mathbf{G} \cdot \mathbf$$

$$\mathbf{z}_{c} = \begin{bmatrix} x_{a} \\ x_{b} \\ y_{a} \\ y_{b} \end{bmatrix} \tag{8}$$

Equation (2) describes a scalar equation, which is valid for Fig. 3. For describing figure 5, it will be written as vector equation

$$f_{c} = K_{x} \cdot z_{c} + K_{i} \cdot i_{c} \tag{9}$$

Force-displacement factors matrix  $\mathbf{K}_x$  and force-current factors matrix  $K_i$  represent the elements that link the mechanical equations with electric ones for x- and ydirection of bearings (a) and (b). The variable  $i_c$ represents control

$$K_{x} = \begin{bmatrix} k_{x_{-}xa} & 0 & 0 & 0 \\ 0 & k_{x_{-}xb} & 0 & 0 \\ 0 & 0 & k_{x_{-}ya} & 0 \\ 0 & 0 & 0 & k_{x_{-}yb} \end{bmatrix}$$

$$K_{i} = \begin{bmatrix} k_{i_{-}xa} & 0 & 0 & 0 \\ 0 & k_{i_{-}xb} & 0 & 0 \\ 0 & 0 & k_{i_{-}ya} & 0 \\ 0 & 0 & 0 & k_{x_{-}yb} \end{bmatrix}$$

$$(10)$$

Considering gravitation and disturbance force vectors  $f_g$ and  $f_d$ , equation (5) will be extended to

$$\mathbf{f}_{c} + \mathbf{f}_{g} + \mathbf{f}_{d} = \mathbf{M}_{c} \cdot \ddot{\mathbf{z}}_{c} + \mathbf{G}_{c} \cdot \dot{\mathbf{z}}_{c} \tag{11}$$

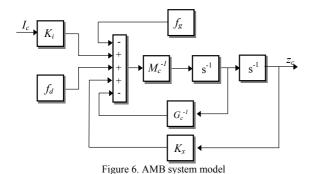
 $f_c + f_g + f_d = M_c \cdot \ddot{z}_c + G_c \cdot \dot{z}_c$  (11) Based on (9), (10) and (11), Fig. 6 shows the AMB model.

# C. PWM inverter model

One of the extensions to the previous works, [1] and [5], is the modeling of power amplifiers. Four switch mode MOSFET full-bridges inject control current ic into the bearing windings with a switching frequency of 50 kHz. The bearing windings are modeled by the transfer

function 
$$\frac{I(s)}{U(s)} = \frac{1}{(sL+R)}$$
 with a current slope of

maximum 50 A/ms due to the inductivity of 500 µH and resistance of 500 m $\Omega$  of the windings.



Given a reference current by the position controller, the current controller calculates appropriate duty cycles for the MOSFETs of the full-bridges. As a two-level PWM hardware, the full-bridges apply either +24V or -24V on

the bearings windings, with a duration depending on the mentioned duty cycle. The simulated current behavior of the PWM inverter model is depicted in Figure 7. The current ripple shows conformity to real current behavior of the power amplifiers.

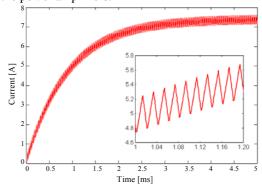


Figure 7. Calculated control current of the PWM inverter model for yaxis of radial bearing with 7.5 A set-point value

#### III. CONTROL SYSTEM

Extending previous works in [1] and [5], following improvements are done:

- 1. Design of PID controller in digital method.
- 2. Design of decentralized state-space controller for each bearing axis with reduced observer. Simulation is performed in fixed-step calculations.
- 3. Modeling of power amplifier injecting control current  $i_c$  into bearing windings.

Based on the AMB model for both radial bearings (a) and (b), two control methods are presented.

# A. PID control

PID control method was already used at the early days of AMB [2]. It requires a small computing power and provides good robustness and stability, if the operating point is inside of linear performance range of the AMB. PID control procedure associated with design of decentralized controllers is able to control AMB systems with collocation of force direction and sensed direction of rotor movement at the position sensor's location, and gives stability for discrete time control.

For position control loop, we consider the AMB as Single Input Single Output (SISO) system. Hence, it is sufficient to investigate only one bearing axis, and we gain a transfer function of the AMB in z-domain of ztransform, using sampling time  $t_s$ :

$$H_{P}(z) = \frac{b_{1} \cdot z + b_{0}}{z^{2} + a_{1} \cdot z + a_{0}}$$

$$k_{Y} \qquad -\sqrt{a_{1}}c_{x} \qquad \sqrt{a_{2}}c_{x}$$
(12)

with 
$$a = \frac{k_x}{m}$$
:  $a_0 = 1$ ;  $a_1 = -e^{-\sqrt{a} \cdot t_s} - e^{\sqrt{a} \cdot t_s}$ ;  
 $b_0 = b_1 = \frac{k_1}{2 \cdot \sqrt{k_x \cdot m}} \cdot \left( e^{\sqrt{a} \cdot t_s} - e^{-\sqrt{a} \cdot t_s} \right)$ 

As the closed loop transfer function is described as a fourth order system, for PID controller implementation an extended parallel model with four parameters  $K_P$ ,  $K_I$ ,  $K_D$ , r was chosen.

$$H_R(z) = K_P + \frac{K_1 \cdot z}{z - 1} + K_D \cdot \frac{z - 1}{z - r}$$
 (13)

In (13)  $K_P$  is proportional gain,  $K_I$  is integrative gain,  $K_D$  is derivative gain and the fourth parameter r is the coefficient of derivative filter, used in order to limit step variations of derivative term as a consequence of fast changes.

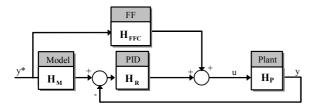


Figure 8. AMB control structure with PID controller

# TABLE 2 PID CONTROLLER PARAMETERS

Proportional gain, $K_P$	1.5
Integrative gain, $K_{\rm I}$	0.0025
Saturation limit for integral part, $I_{LIM}$	300 bits
Time after that the integration is started, $I_{\text{TIME}}$	1 ms
Derivative gain, $K_D$	10
Derivative filter gain, r	-0.0033
Sampling time, $t_{\rm s}$	0.1 ms

To assure a good system response at set-point reference variations even at high frequencies, a feed-forward controller is added into the controller model. In this case the feed-forward controller transfer function is the inverse of discrete plant transfer function in z-domain (12). In Fig. 8,  $H_R$  and  $H_P$  represent the controller respectively plant transfer function,  $H_{FFC}$  is the feed-forward controller transfer function and  $H_M$  is the reference model taken into consideration.

$$H_{FFC}(z) = \frac{b_1 \cdot z + b_0}{z^2 \cdot (b_1 + b_0)} \cdot \frac{1}{H_R(z)} \quad . \tag{14}$$

The corresponding reference model is given by

$$H_{FFC}(z) = \frac{b_1 \cdot z + b_0}{z^2 \cdot (b_1 + b_0)}$$
 (15)

In Table 2, the PID main data obtained via polynomial method for poles allocation are presented.

#### B. State-space control

State-Space controller is perfectly suited for numerical computations. In the last years DSP (Digital Signal Processor) computation power has increased. A large variety of state space controllers can be implemented. The algorithm for controller parameters (K-vector as feed-back controller) is more complex than in PID

controller's case. The method is based on matrices operations that describe the system model, and for this reason, the inner state of the process can be accessed, not only inputs and outputs.

The first step, when dealing with a multivariable system design, is to perform the decoupling of MIMO (Multiple Input Multiple Output) system and to find an approximate model consisting of two or more SISO systems.

A continuous, linear, constant-coefficient system of differential equations can always be expressed as a set of first-order matrix differential equations [3]:

$$\mathbf{x}(k+1) = \mathbf{\Phi} \cdot \mathbf{x}(k) + \mathbf{\Gamma} \cdot \mathbf{u}(k) 
\mathbf{y}(k) = \mathbf{H} \cdot \mathbf{x}(k) ; \text{ with } \mathbf{H} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$
(16)

where x is the inner states vector, u is the control input vector, y represents the outputs vector and

$$\boldsymbol{\Phi} = \mathbf{e}^{\mathbf{A} \cdot t} \; ; \; \boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ \frac{k_x}{m} & 0 \end{bmatrix}$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} 0 \\ \frac{k_i}{m} \end{bmatrix} \cdot \int_0^T \boldsymbol{\Phi} \cdot dt$$
(17)

One of the most attractive features of state-space controller design method is that the procedure consists of two independent steps. First step assumes that all states are known. This assumption allows proceeding with first design step, namely, *control law*. Second step is to design an estimator, in order to estimate entire inner state vector. Finally, control algorithm will consist of a combination of control law and estimator, with control law calculations based on estimated state.

A control law that has considerable convenience is simply the feedback of linear combination of all state elements

$$\boldsymbol{u} = -\boldsymbol{K} \cdot \boldsymbol{x} = -\begin{bmatrix} k_1 & k_2 & \dots \end{bmatrix} \cdot \begin{bmatrix} x_1 & x_2 & \dots \end{bmatrix}^{\mathrm{T}}$$
(18)

Control law design consists in finding elements of *K*-vector, so that roots of *characteristic equation* (19) are in desired location.

$$\det |\mathbf{z} \cdot \mathbf{I} - \mathbf{\Phi} + \mathbf{\Gamma} \cdot \mathbf{K}| = 0 \tag{19}$$

In this paper control **K**-vector will be determined using LQR solution. The LQR solution may be found using two methods [4]:

- 1. First method is to compute K-vector. From the beginning, in constant steps of the problem, the task is to compute S (solution of vector Riccati equation) backward in time until it reaches steady value  $S_{\infty}$  and then to use theory for  $time\ varying\ optimal\ control$ .
- 2. Second method is to look for steady-state solution of *Riccati* equation. In steady state S(k) becomes equal with S(k + 1) and in this case *algebraic Riccati*

equation can be used. This method is going to be used for determining K-vector.

As some of the states are directly measured, a reducedorder estimator will be used [4]. In order to pursue an estimator for unmeasured part, we partition the state vector  $\mathbf{x}$  into two parts:  $\mathbf{x}_a$  represent part directly measured (rotor position) or calculated respectively (rotor movement speed), and  $\mathbf{x}_b$  is the remaining portion to be estimated (i.e. gravitation and disturbance force), as shown in (20).

$$\begin{bmatrix}
\mathbf{x}_{a}(k+1) \\
\mathbf{x}_{b}(k+1)
\end{bmatrix} = \begin{bmatrix}
\mathbf{\Phi}_{aa} & \mathbf{\Phi}_{ab} \\
\mathbf{\Phi}_{ba} & \mathbf{\Phi}_{bb}
\end{bmatrix} \cdot \begin{bmatrix}
\mathbf{x}_{a}(k) \\
\mathbf{x}_{b}(k)
\end{bmatrix} + \begin{bmatrix}
\mathbf{\Gamma}_{a} \\
\mathbf{\Gamma}_{b}
\end{bmatrix} \cdot \mathbf{u}(k)$$

$$\mathbf{y}(k) = \begin{bmatrix}
\mathbf{H}_{a} & \mathbf{H}_{b}
\end{bmatrix} \cdot \begin{bmatrix}
\mathbf{x}_{a}(k) \\
\mathbf{x}_{b}(k)
\end{bmatrix}$$
(20)

From (20), we get the equations that describe unknown states vector part [4], as shown in (21).

$$\mathbf{x}_{b}(k+1) = \mathbf{\Phi}_{bb} \cdot \mathbf{x}_{b}(k) + \mathbf{v}(k) ;$$
with  $\mathbf{v}(k) = \mathbf{\Phi}_{ba} \cdot \mathbf{x}_{a}(k) + \mathbf{\Gamma}_{b} \cdot \mathbf{u}(k)$ 

$$\mathbf{y}_{b}(k) = \mathbf{\Phi}_{ab} \cdot \mathbf{x}_{b}(k)$$
(21)

For this artificial system (21) with well-known entrance vector  $\mathbf{v}$ , *Luenberger* estimator can be sketched (Fig. 9).

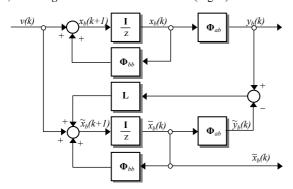


Figure 9. Luenberger estimator block diagram

In Table 3 the state-space main data obtained via LQR method are presented.

# TABLE 3 STATE-SPACE PARAMETERS

Control vector	$K = [5.49 \text{ A/m}; 0.03 \text{ A/(m/s)}] \cdot 10^4$
Luenberger vector	$L = [0.00006 \text{ N/m}; 1.39 \text{ N/(m/s)}] \cdot 10^3$

## IV. SIMULATION RESULTS

The models of AMB and digital control software had been implemented in *MATLAB/Simulink* to get simulation results, before the control software was transferred to the *Texas Instruments* DSP TMS320F2812, which is used in a test-bench (Fig. 10).

In Fig. 11, 12, 13 and 14, the calculated position and current variations during rotor lifting are presented.



Figure 10. Magnetic bearing test-bench at Darmstadt University of Technology

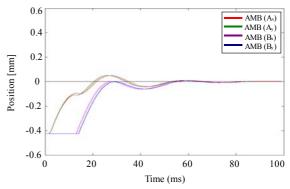


Figure 11. Calculated rotor positions, using PID control of radial AMB of all four axes x, y for A and B-bearing

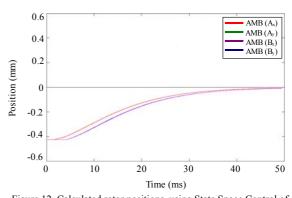


Figure 12. Calculated rotor positions, using State Space Control of radial AMB of all four axes x, y for A and B-bearing

The main difference between Fig. 11 and 12 is the absence of overshoot in the rotor movement with state-space controller, compared to the PID controller, with almost identical lift-off time needed to reach the zero reference position. The oscillations in the rotor movement with the PID controller are also reflected by the behavior of its control current in Fig. 13.

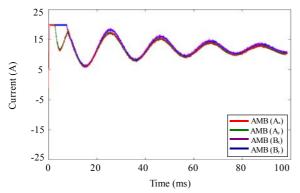


Figure 13. Calculated currents  $i_c$ , required for rotor lifting with PID control

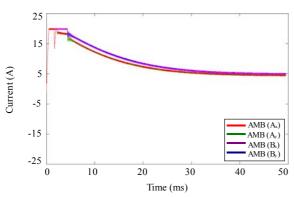


Figure 14. Calculated currents  $i_c$ , required for rotor lifting with state-space control

Although both controllers manage to bring the system in steady state, it is very important to maintain the rotor in levitation, when a disturbance acts upon the system. In order to study the behavior of both control methods, two types of disturbances will be considered:

- Static disturbance simulated as a force step (step function model) is applied to rotor at 17 s after system is started. The static disturbance force fd equals 100 N. The force is located at the axial outer side of the rotor near bearing B with a distance of 231 mm from the gravity center O. Angle between the direction of gravitation force and disturbance force is -30°.
- 2. Imbalance effect due to rotor center of gravity dislocation from rotational axis by a certain displacement  $e_s$ . Dislocation  $e_s$  of gravity center O from rotational axis (that is centered at rotor geometrical center) is given by an additional mass  $\Delta m_s$  on the rotor surface at each axial position of magnetic bearings.

In Fig. 15 and 16, the simulation results are presented, which were obtained for static disturbance force that appears when rotor is rotating. We can see, that the state-space controller can maintain the rotor position better than the PID controller (displacement of the rotor up to 200  $\mu$ m with the PID controller, against the rotor

displacement up to 50  $\mu m$  with the state-space controller).

In Fig. 17 and 18, the geometric locus of symmetry axis of bearing B with state-space controller are shown, when the rotor imbalance is applied according to ISO 1940 (Q = 2.5 mm/s at 40000 min<sup>-1</sup>). This corresponds with two masses, each  $\Delta m_{\rm s}$  of 94 mg, fixed on the rotor surface. Each mass is placed at each magnetic bearing axial distance from the rotor gravity center. The resulting periodical force due to the imbalance equals 74.2 N for the total rotor.

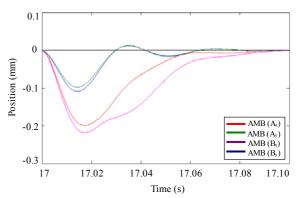


Figure 15. Calculated PID rejection of a static disturbance at speed 40 000 min<sup>-1</sup>

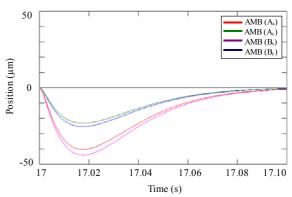


Figure 16. Calculated state-space rejection of a static disturbance at speed 40 000 min<sup>-1</sup>

Fig. 17 shows the rotor movement from lifting-off at standstill and acceleration up to 40000  $\rm min^{-1}$ . The detail movement is shown in Fig. 18, where the rotor is oscillating in a small range of 0.6  $\mu m$  around the zero reference position.

# V. CONCLUSIONS

The advantages of state-space control method in the magnetic bearings over the PID one is presented. The time required for reaching standstill position in state-space control is shorter than in PID case and the rotor moves to the reference position without any oscillations.

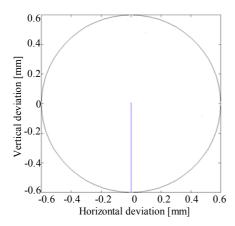


Figure 17. Calculated state-space behavior with static imbalance from lift-off to rotational speed at 40000 min<sup>-1</sup>

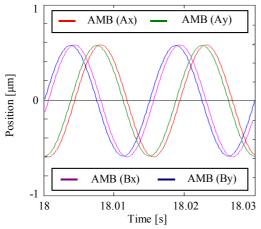


Figure 18. Zoomed calculated position behavior of Fig. 17

The simulations were demonstrated by taking into consideration the disturbances that might appear in system. The state-space controller is capable of dealing with many kinds of disturbances, when the PID might fail to control the levitation process. This advantage is the consequence of the much more complex mathematical model used for the control law and estimator part of state-space model.

Although state-space method is offering more advantages, it must be said that the implementation of the software on a DSP and DSP system requirements are more complex than the PID case. This implementation is done actually on the demonstrator (Fig. 10) and will be reported in the future.

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