DAILY OPTIMAL OPERATING POLICY
OF HYDROPOWER SYSTEMS

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Abstract: - In this paper, we have presented a new model for daily operating policy of hydroelectric power systems, which consists to maximize the potential energy of the whole system. The method used for the solution is based on the discrete maximum principle for determining the optimal daily operating policy of hydroelectric power systems consisting of multi-reservoirs, where the objective is to maximize the potential energy while satisfying all operating constraints over a short-term planning horizon. The major focus of this paper will be also the treatment of the two-sided inequality constraints using the augmented Lagrangian method. The proposed algorithm takes into account spilling and time delays between reservoirs. The proposed algorithm is tested on a large hydroelectric power system consisting of ten reservoirs. The developed algorithm gives a satisfactory solution for the problem and turns out to be very efficient.

Key-words: Daily operating policy, potential energy, discrete maximum principle, augmented Lagrangian method.

1. Introduction

The daily optimal operating policy of hydropower systems is a deterministic problem [1][2], which consists in determining the amount of water to be discharged from each reservoir of the system over the day so that to meet the hourly load demand assigned previously. The prime objective here is to perform the operating policy with the lowest use of water; which is achieved by avoiding spilling and by maximizing the hydroelectric generation, besides satisfying all operating constraints. The maximization of electrical power production is achieved by maximizing the heads. Consequently, this allows maximizing the reservoirs content.

When modeling the problem, and for more accuracy, the following factors which make the problem more complex are taken into consideration; significant water travel time between reservoirs, the multiplicity of the input-output curve of hydroelectric reservoirs that have variable heads, the maximum generation of the hydroelectric plant varies with the hydraulic head i.e. the quantity of water required for a given power output decreases as the hydraulic head increases, the water stored in the upstream reservoir is more valuable than that stored in the downstream reservoir, whether the reservoirs have very different storage capacity and whether the system has quite complex topology with many cascaded reservoirs.

To solve the daily operating policy problem, we use the discrete maximum principle [3-4]. While solving the equations relating to the discrete maximum principle, we use the gradient method [3]. However, to treat equality constraints we use Lagrange's multiplier method. To treat the inequalities constraints we use the augmented Lagrangian method [5]. The present paper is concerned particularly with the treatment of the constraints on the state variables, which are of two-sided inequalities. The augmented Lagrangian method is proposed to deal with this type of inequalities.

The hydroelectric power system considered in this paper consists of ten reservoirs hydraulically

coupled, i.e., the release of an upstream reservoir contributes to the inflow of downstream reservoirs. All reservoirs are located in the same river. The time taken by water to travel from one reservoir to

The decision variables in the optimization problem are the amount of water to be released from each reservoir to their direct downstream reservoirs in a given period. The state variables are the contents of the reservoirs.

2. Problem formulation
The main objective of the daily operating policy of hydroelectric power system is to maximize the reservoir's contents which imply maximizing the value of potential energy stored at the end of the planning horizon, while satisfying demand for electrical energy and all other specified constraints. Thus, the suggested mathematical model for the deterministic short-term operating policy of the hydroelectric power systems is as follows:

2.1 The objective function
The main objective is to maximize the total potential energy of water stored in all the reservoirs. The formulation must take into account the fact that the water stored in one reservoir will be used in all its downstream reservoirs, hence, the water stored in the upstream reservoir is more valuable than that stored in the downstream reservoir, hence:

$$\max \sum_{i=1}^{n} E_P(x_i^{k_f}, h_i^{k_f}) + \sum_{k=k_f-S_{mi}}^{k_f} E_p(u_{mi}^k, v_{mi}^k)$$

Where

$E_p(x_i^{k_f}, h_i^{k_f})$: Potential energy of water stored in reservoir i at the end of the planning horizon k_f. This energy depends on the amount of water stored in the reservoir i, on its effective water head $h_i^{k_f}$, and on the effective water head of the downstream reservoirs.

$x_i^{k_f}$: Content of the reservoir i at the end of period k_f, in Mm^3.

n : Number of reservoirs of the system.

k_f : The last hour of the planning horizon, in hours.

m : The reservoir immediately preceding the reservoir i.

the downstream reservoir [8-10] is taken into account. The natural inflow and the load demand are known beforehand. The scheduling is stretched over one day divided into hours.

u_{mi}^k, v_{mi}^k : Respectively the discharge and the spilled outflows from the upstream reservoir m incoming later to the downstream reservoir i during period k, in Mm^3.

S_{mi} : Time required for the water discharged from reservoir m to reach its direct downstream reservoir i, in hours.

$\sum E_p^k(u_{mi}^k, v_{mi}^k)$: Total potential energy of the outflow from reservoir m, which will reach later the downstream reservoir i after the last hour of the planning horizon k_f.

2.2 Operational constraints
The optimization is performed incorporating the following constraints [1-2] [8-15]:

- Hydraulic continuity constraint:
The flow balance equation of each reservoir i of the system, for every period k is represented by the following hydraulic continuity equation:

$$x_i^k = x_i^{k-1} + q_i^k - u_i^k - v_i^k$$

Where:

u_i^k : Discharge from reservoir i during period k, in Mm^3.

v_i^k : Spillage from reservoir i during period k, in Mm^3.

q_i^k : Total inflow to the reservoir i during period k, in Mm^3.

$$q_i^k = \begin{cases} y_i^k & \text{if } i \le e. \\ y_i^k + \sum_m (u_{mi}^{k-s_{mi}} + v_{mi}^{k-s_{mi}}) & \text{otherwise.} \end{cases}$$

e : The extreme upstream reservoirs.

y_i^k : Natural inflow to the reservoir i during period k, in Mm^3.

$u_{mi}^{k-s_{mi}}, v_{mi}^{k-s_{mi}}$: The discharge and the spilled outflows, respectively, from the upstream reservoir m incoming later to the downstream reservoir i during period k, in Mm^3.

- Limits on storage capacity of each reservoir i:

$$\underline{x}_i \leq x_i^k \leq \overline{x_i}$$

$\underline{x}_i, \overline{x}_i$: Lower and upper bounds on reservoir storage capacity, respectively, for reservoir i , in Mm^3.

- Limits on discharged outflow of hydro plant i :

$$\underline{u}_i \leq u_i^k \leq \overline{u}_i$$

$\underline{u}_i, \overline{u}_i$: Minimum and maximum bound on water discharge, respectively, of hydro power plant i, in Mm^3.

- Load constraints:
The total power generated by all the hydroelectric plants must satisfy the system load demand at each period of the planning horizon. In mathematical terms, it has the following form:

$$\sum_{i=1}^{n} P_i^k = D^k$$

Where:

D^k : System load demand at each period k , in Mw.

P_i^k : Electric power generated by hydro plant i at period k , in Mw. The generation is a function of the water discharge u_i^k and of the effective water head h_i^k .

2.3 Mathematical model formulation:
The suitable mathematical model proposed for the short-term scheduling problem of a hydroelectric plant system is as follows:

$$\max \sum_{i=1}^{n} E_p(x_i^{k_f}, h_i^{k_f}) + \sum_{k=k_f - S_{mi}}^{k_f} E_p(u_{mi}^k) \quad (1)$$

Subject to the following constraints:

$$x_i^k = x_i^{k-1} + q_i^k - u_i^k \quad (2)$$

$$\sum_{i=1}^{n} P_i^k = D^k \quad (3)$$

$$0 \leq u_i^k \leq \overline{u}_i \quad (4)$$

$$\underline{x}_i \leq x_i^k \leq \overline{x}_i \quad (5)$$

To ovoid the spillage, we force v_i^k to vanish.

3. Solution methodology

The problem (1)-(3) is solved by using the discrete maximum principle as follows [3-7]:
Associate the constraint (2) to the criterion (1) with the dual variable $\boldsymbol{l}_i^k$. Furthermore, to satisfy the balance between electric power demand and generation, we associate the constraint (3) to the criterion (1) with the Lagrange multiplier $\boldsymbol{b}^k$, and then we define the function H^k called the Hamiltonian function, which has the following form:

$$H^k = \sum_{i=1}^{n}[\boldsymbol{l}_i^k(x_i^k + q_i^k - u_i^k)] + \boldsymbol{b}^k(\sum P_i^k - D^k)$$

(6)

Where u_i^k and x_i^k represent respectively the control and state variables.

To take into account the possible violation of constraint (5) we proceed as follows:
The two-sided inequality constraint (5) can be broken into two inequalities constraints and rewritten, following the substitution of equation (2) for x_i^k :

$$(x_i^{k-1} + q_i^k - u_i^k) - \overline{x}_i \leq 0 \quad (7)$$

$$(x_i^{k-1} + q_i^k - u_i^k) - \underline{x}_i \geq 0 \quad (8)$$

To treat these inequalities constraints we use the augmented Lagrangian method [6-7], which consists of adding the functions R_i^k and Q_i^k to the Hamiltonian H^k that penalizes respectively the violations of the inequalities constraints (7) and (8), i.e., the violation of lower and upper limits of the original constraint (5). Then the Hamiltonian H^k becomes as follows:

$$H^k = \sum_{i=1}^{n}[\boldsymbol{l}_i^k(x_i^k + y_i - u_i^k) + R_i^k + Q_i^k]$$
$$+ \boldsymbol{b}^k(\sum P_i^k - D^k)$$

(9)

The penalty function R_i^k is defined as follows [6-7]:

$$R_i^k = \boldsymbol{r}_i^k \Psi_i^k + r(\Psi_i^k)^2 \quad (10)$$

Where:

r : Penalty weight.

r_i^k : Lagrange multipliers being updated as follows:

$$r_i^k = r_i^k + 2r \max(x_i^k - \overline{x}_i, -\frac{r_i^k}{2r}) \quad (11)$$

The function Ψ_i^k is determined as follows:

$$\Psi_i^k = \max(x_i^k - \overline{x}_i, -\frac{r_i^k}{2r}) \quad (12)$$

The penalty function Q_i^k is calculated in the same manner as R_i^k.

The problem (1)-(5) becomes:

$$\max H^k \quad (13)$$

The necessary conditions for the optimum are:

$$\frac{\partial H^k}{\partial u_i^k} = 0 \quad (14)$$

To find the optimal water discharge trajectory u_i^k from equation (14), we must solve the difference's equations (2) and the following ones called the adjoint equation [6]:

$$\lambda_i^{k-1} = \frac{\partial H^k}{\partial x_i^{k-1}} \quad (15)$$

The boundary conditions for equations (2) and (15) are:

- The first one is the initial state, which is specified, i.e., the initial content of all reservoirs is known, thus:

$$x_i^0 = b_i \quad (16)$$

- The second one is the terminal condition for the adjoint equation:

$$\lambda_i^{k_f} = \frac{\partial E_p(x_i^{k_f})}{\partial x_i^{k_f}} \quad (17)$$

The necessary conditions for the optimality constitute a two-point boundary value problem, whose solution determines the optimal state and control variables. This problem is solved iteratively by using the gradient method [3].

4. Application

In order to testify the efficiency of the proposed algorithm, it was applied to the system composed of ten reservoirs located on the same river as shown in Fig. 1:
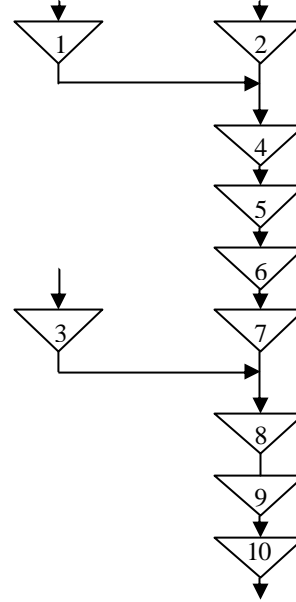

Fig.1: The reservoir network.

The characteristics of the reservoirs and water time travel are shown in table 1.

Table 1: Characteristics of the installations.

i	$\overline{x}_i$ (M.m³)	$\overline{u}_i$ (M.m³/h)	h_i (m)	S_{mi} (h)
1	8777,2	1,1232	0,00	55
2	986,4	0,5272	0,00	70
3	998,0	0,5054	0,00	42
4	504,9	2,5531	66,61	5
5	8,5	2,4181	114,1	7
6	4,2	2,5650	92,41	2
7	4,8	2,5240	83,28	22
8	26,9	2,7648	55,72	3
9	4,54	3,0476	107,66	2
10	3,4	3,4686	40,81	0

Where:

$\overline{x}_i$: Maximum storage capacity of reservoir i.

$\overline{u}_i$: Maximum water discharge of hydro-power plant i.

$h_i^k(x_i^k)$: Effective water head of hydropower plant i at period k.

S_{mi}: Time required for the water discharge from reservoir m to reach its direct downstream reservoir i.

The natural inflows are assumed constant throughout the week in all reservoirs. Their values are depicted in table 2 as well as the initial contents of each reservoir.

Table 2: Natural inflows and initial contents of reservoirs.

i	x_i^0 $_{(M.m^3)}$	y_i^k $_{(M.m^3/h)}$
1	4386,6	0,1476
2	986,4	0,5272
3	998,0	0,5054
4	504,9	0,5531
5	8,5	0,4181
6	4,2	0,5650
7	4,8	0,5240
8	26,9	0,7648
9	4,5	0,0476
10	3,4	0,4686

Where:

y_i^k: Natural inflow to the reservoir of hydropower plant i during each hour of the planning horizon k_f.

x_i^0: Initial content of reservoir i.

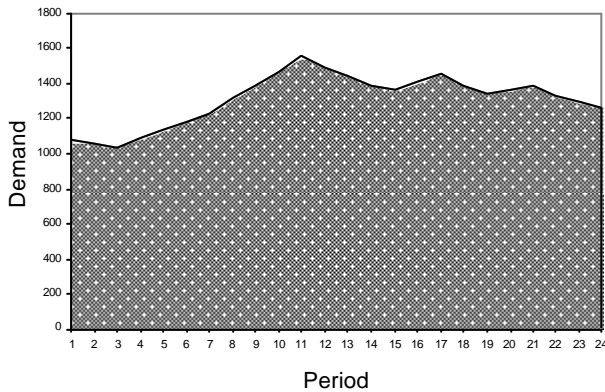The hourly load demand D^k during the day is shown in Fig. 2.

Fig. 2: Hourly demand profile during one day.

The electrical power produced in MW at the hydroelectric plant i during a period k is given by the following expression:

$$P_i(h_i^k, u_i^k) = h_i^k(x_i^k) \cdot u_i^k \qquad (18)$$

Where:

$h_i^k(x_i^k)$: Effective water head of hydropower plant i at period k.

5. Implementation results

In this section, the results, obtained from the implementation of the proposed algorithm are presented. The algorithm is implemented in FORTRAN.

The solution is obtained after a very moderate number of iterations with all constraints being satisfied.

The daily optimal scheduling, i.e., optimal water discharges from each hydropower plant obtained are depicted in Fig. 3.
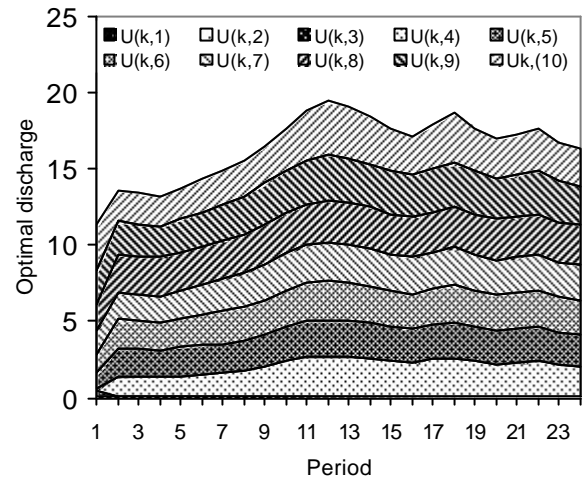

Fig. 3: Optimal discharge trajectories.

It was seen that the discharge of the hydroelectric plants follows the demand because it is proportional to the production on one hand. On the other hand, this production must be equal to the demand. Furthermore, the discharge from the downstream reservoir is greater than in upstream one as shown in Fig. 3, for the reason that the water stored in the upstream reservoir is more valuable than that stored in the upstream one, i.e., the water of the upstream reservoir will be used again in all the downstream ones. Consequently, in economic terms, water in the upstream reservoirs should be preserved as shown in Fig. 4. Thus, the consequences of the optimal scheduling of the water discharge are the filling of the upstream reservoirs as against the downstream ones.
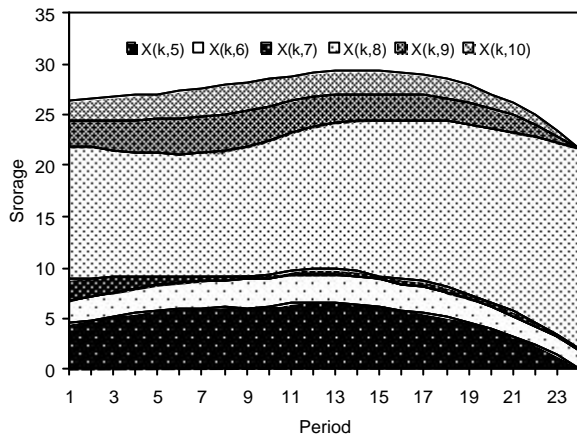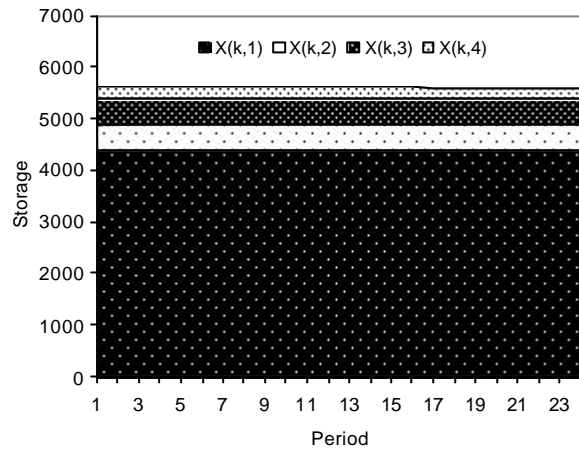
Top: reservoirs 1-4.   Bottom: reservoirs 5-10

Fig. 4: Optimal evolution storage trajectories.

The behaviour of the objective function during the optimization process is illustrated in Fig. 5 The convergence was achieved in about ten iterations with few oscillations. It is also noted that the search for the optimum becomes slower when the optimal solution is approached; this is due to the gradient method itself.
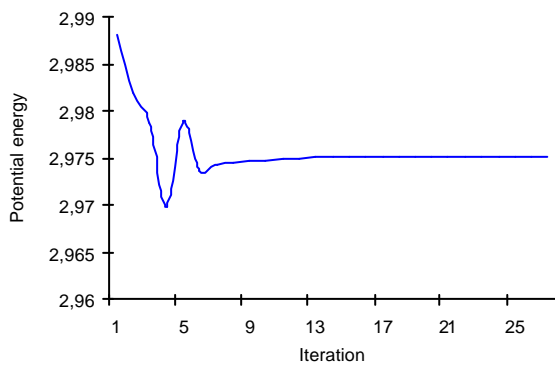

Fig. 5: Behavior of the search trajectory

6. Conclusions

In this paper, a new model is presented for the daily operating policy of hydroelectric power system, which consists to maximize the potential energy of the whole system. With the discrete maximum principle, the optimal solution is obtained by solving simultaneous equations representing the optimality conditions. The principle turned out to be very efficient.

To deal with the two-sided inequality constraints, the augmented Lagrangian method was introduced. The results confirm the promising properties of the augmented Lagrangian.

The proposed algorithm based on those methods requires moderate time and storage for its execution, thus allowing the solution of large-scale scheduling problems.

Some improvements can be made to the proposed algorithm in order to increase the convergence speed of the algorithm and its execution time by using an optimal step size a rather than a fixed one.

The proposed algorithm can take into account the time of water travel between upstream and downstream reservoirs.

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