A Simple Active Loss Allocation Algorithm for an Unbalanced Radial Distribution System

Sivkumar Mishra and Debapriya Das

Abstract-- **This paper presents a simple algorithm to allocate active losses to the various consumers in an unbalanced radial distribution system. An active loss allocation formula for a balanced radial distribution system is first developed .and the loss allocation scheme is then proposed based on this formula. The scheme is then extended to allocate loss for an unbalanced radial distribution system. The proposed loss allocation method utilizes the load flow results of the multiphase unbalanced radial distribution system to allocate the active losses to the various consumers of the system connected at the different buses. To obtain the load flow results, the forward and backward sweep method has been used in which the system loads are modeled as composite loads. The feeder power losses for various phases are calculated first from the load flow. Using the loss allocation formula developed earlier, the real power losses are allocated to the various consumers. The loss allocation scheme has been successfully implemented with the three practical test distribution systems.**

Index Terms-- **Active Loss Allocation, Unbalanced Radial Distribution Systems.**

I. INTRODUCTION

ELECTRICITY supply industries worldwide are undergoing major structural changes with the objective of introducing competition and choice in electricity supply. These changes are motivated primarily by the belief that competition will bring better service, at a lower price to electricity consumers. The vertically integrated systems have been restructured and unbundled to one or more generation companies, transmission companies and a number of distribution companies. An essential condition for competition to develop is open access, on a nondiscriminatory basis, to transmission and distribution networks. The central issue in the concept of open access is setting an adequate price for transmission and distribution services. This is because price affects the future sitting of generators and loads, and network operating costs as well as strongly influencing further development of the network. Under such a scenario, there is

Sivkumar Mishra is an Assistant Professor with the Department of Electrical Engineering ,Ghanashyam Hemalata Institute of Technology & Management, Puri, Orissa, India, Pin -752002 (e-mail: sivmishra@yahoo.com).

Debapriya Das is a Professor with the Electrical Engineering Department, Indian Institute of Technology, Kharagpur 721302, India(e-mail: ddas@ee.iitkgp.ernet.in)

ever-growing pressure for all components of costs to be clearly identified and assigned equitably to all parties taking care to avoid or minimize any temporal or spatial cross-subsidies. The cost of transmission and distribution activities needs to be allocated to the users of these networks. Allocation can be done through network use tariffs, with a focus on the true impact they have on these costs. Among others, distribution power losses are one of the costs to be allocated. The main difficulty faced in allocating losses is the nonlinearity between the losses and delivered power complicates the impact of each user on network losses [1]. It is impossible to calculate the exact amount of losses in advance, without running a power flow. At the same time, even after computing the power flow solution, there is a strong interdependence among all the users, expressed by the presence of cross terms due to the fact that losses are a nearly quadratic function of the power flows. Hence, allocating the losses to the market participants cannot be carried out in a straight forward way. Different techniques have been published in the literature for allocation of losses, most of them dedicated to transmission networks and can be classified into three broad categories – pro rata procedures, marginal procedures and proportional sharing procedures [2-8]. Costa and Matos [9] have addressed the allocation of losses in distribution networks with embedded generation by considering quadratic loss allocation technique. In general, a first distinction can be made between loss allocation methods dedicated to transmission and to distribution systems. The difference between these two classes of methods basically lies in the role of given to the slack node. In transmission systems, the generator located in the slack node compensates for all the losses and is explicitly considered in the mechanism of loss allocation. In radial distribution systems, the location of the slack node at the root node of the distribution tree is naturally unique, and the slack node usually represents the connection to the higher voltage network. In this paper, a novel loss allocation scheme for radial distribution system is proposed, which is based on the loss formula derived. In this loss formula the cross coupling terms are carefully avoided resulting in the simplification of the loss allocation problem. This formula can be easily extended to multiphase unbalanced distribution networks.

II. PROPOSED LOSS ALLOCATION SCHEME

Different techniques have been published in the literature

for allocation of losses, most of them dedicated to transmission networks. Though some of them can be applied to the radial distribution system, but are not fully suitable for such networks. Recently, Savier and Das **[10]** have dealt the issue of impact of network reconfiguration on loss allocation of radial distribution systems using the quadratic way. However, no method really considers the unbalanced nature of the distribution networks for active loss allocation. In the light of such developments, a simple loss allocation scheme is proposed in this chapter for balanced radial distribution systems which is then extended to three phase unbalanced radial networks.

A. Methodology

Consider a sample distribution network as shown in Fig.1. In Fig.1, branch numbers are shown in (.). The branch number, sending end and receiving end nodes are given in Table 1. The load current at any node p, is given as:

$$IL(p) = \frac{PL_p - jQL_p}{V_p^*}, \qquad p = 2,3,...,NB \qquad (1)$$

Consider branch-5 of Fig. 1, number of node beyond branch-5 is one and this is node 6. Therefore, current through branch-5 is:

$$I(5) = IL(6) \qquad (2)$$

Now consider branch-4. The total number of nodes beyond branch-4 is two and these nodes are 5 and 6, respectively. Therefore, current through branch-4 is:

$$I(4) = IL(5) + IL(6) \qquad (3)$$

Similarly consider branch-3. Total number of nodes beyond branch-3 is five and these nodes are 4, 5, 6, 10 and 11. Therefore, current through branch-3 is:

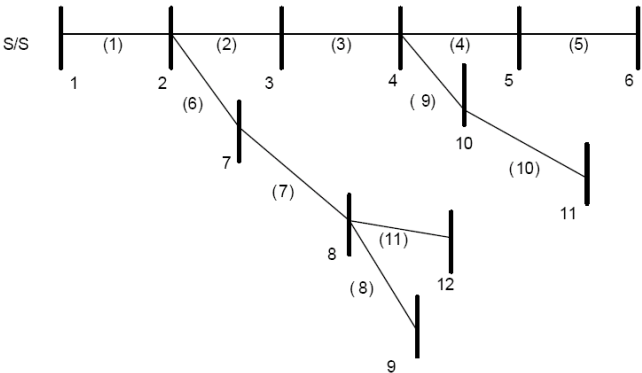$$I(3) = IL(4) + IL(5) + IL(6) + IL(10) + IL(11) \qquad (4)$$

Fig. 1   Simple distribution network with 12 buses

General expression of branch current through branch-j is given by:

$$I(jj) = \sum_{k=1}^{N(jj)} IL\{ie(jj,k)\} \qquad (5)$$

In Table-1, total number of nodes (consumers) beyond branch-jj (N(jj)) and nodes (consumers) beyond branch jj (ie(jj,i) ; i= 1, 2, …, N(jj)) are given for Fig. 1 for the purpose of explanation. In (5), the load currents can be replaced by the following relation:

$$IL(i) = \frac{PL(i) - j\ QL(i)}{V(i)^*} \qquad (6)$$

Thus, (5) modifies to

$$I(jj) = \sum_{k=1}^{N(jj)} \frac{PL\{ie(jj,k)\} - jQL\{ie(jj,k)\}}{V\{ie(jj,k)\}^*} \qquad (7)$$

TABLE1

Branch No. (jj)	Sending end node IS(jj)	Receiving end node IR(jj)	Total no. of nodes beyond branch-jj N(jj)	Nodes beyond branch-jj ie(jj,i) i=1,2….N(jj)
1	1	2	11	2,3,4,5,6,7,8,9,10,11,12
2	2	3	6	3,4,5,6,10,11
3	3	4	5	4,5,6,10,11
4	4	5	2	5,6
5	5	6	1	6
6	2	7	4	7,8,9,12
7	7	8	3	8,9,12
8	8	9	1	9
9	4	10	2	10,11
10	10	11	1	11
11	8	12	1	12

Real power loss of branch-jj with sending end and receiving end voltages V_i and V_j is given by:

$$PLOSS(jj) = \mathrm{Re}\,al(V_i - V_j)^* . I(jj) \qquad (8)$$

$$\Rightarrow PLOSS(jj) =$$
$$\mathrm{Re}\,al\{(V_i - V_j)^* \sum_{k=1}^{N(jj)} (\frac{PL\{ie(jj,k)\} - j\ QL\{ie(jj,k)\}}{V\{ie(jj,k)\}^*})\} \qquad (9)$$

$$\Rightarrow PLOSS(jj) =$$
$$\mathrm{Re}\,al\left\{ \sum_{k=1}^{N(jj)} \left(\frac{V_i - V_j}{V\{ie(jj,k)\}} \right)^* (PL\{ie(jj,k)\} - j\ QL\{ie(jj,k)\}) \right\} \qquad (10)$$

Let $\left(\dfrac{V_i - V_j}{V\{ie(jj,k)\}} \right)^* = A\{ie(jj,k)\} + j\ B\{ie(jj,k)\}$ $\qquad (11)$

$$\therefore PLOSS(jj) =$$
$$\mathrm{Re}\,al\left\{ \sum_{k=1}^{N(jj)} \begin{matrix} (A\{ie(jj,k)\} + j\ \mathrm{B}\{ie(jj,k)\}). \\ (PL\{ie(jj,k)\} - jQL\{ie(jj,k)\}) \end{matrix} \right\} \qquad (12)$$

Hence,

$$PLOSS(jj) = \sum_{i=1}^{N(jj)} (A\{ie(jj,k)\}.PL\{ie(jj,k)\} + \quad (13)$$
$$B\{ie(jj,k)\}.QL\{ie(jj,k)\})$$

Using (13) active power loss in branch-jj can be allocated to consumers beyond branch-jj. Active power loss of branch-jj allocated to a consumer connected to node $\{ie(jj,k)\}$ is given by:

$$ploss\{jj,ie(jj,k)\} = A\{ie(jj,k)\}.PL\{ie(jj,k)\}$$
$$+ B\{ie(jj,k)\}.QL\{ie(jj,k)\}$$
$$for\ jj = 1,2,...........NB-1 \quad (14)$$
$$and\ k=1,2..............N(jj)$$

The global value of losses to be supported by consumer connected to node ℓ results from the sum of the losses allocated to it in each branch-jj of the network, which is given by:

$$Tploss(\ell) = \sum_{jj=1}^{NB-1} ploss(jj,\ell) \quad for\ \ell = 2,3,...,NB \quad (15)$$

B. Extension to Three Phase Unbalanced Networks

The loss formula in (13) derived for a balanced single phase radial distribution network can be extended to write the loss formula for individual phases of a three phase unbalanced radial distribution network. Thus, rewriting (13) for individual phases:

For phase-a,

$$PLOSS_a(jj) = \sum_{i=1}^{N(jj)_a} (A_a\{ie(jj,k)\}.PL_a\{ie(jj,k)\}$$
$$+ B_a\{ie(jj,k)\}.QL_a\{ie(jj,k)\}) \quad (16)$$

For phase-b,

$$PLOSS_b(jj) = \sum_{i=1}^{N(jj)_b} (A_b\{ie(jj,k)\}.PL_b\{ie(jj,k)\}$$
$$+ B_b\{ie(jj,k)\}.QL_b\{ie(jj,k)\}) \quad (17)$$

For phase-c,

$$PLOSS_c(jj) = \sum_{i=1}^{N(jj)_c} (A_c\{ie(jj,k)\}.PL_c\{ie(jj,k)\}$$
$$+ B_c\{ie(jj,k)\}.QL_c\{ie(jj,k)\}) \quad (18)$$

Here, $N(jj)_a$, $N(jj)_b$, $N(jj)_c$ refers to the no. of subsequent nodes of the branch-jj of phase-a, phase-b and phase-c respectively. Similarly, PL, QL, A and B terms refer to the corresponding phases in (16-18). Based on the loss formulae (16-18) loss allocation methods can also be extended to the three phase unbalanced distribution network:

$$ploss_a\{jj,ie(jj,k)\} =$$
$$A_a\{ie(jj,k)\}.PL_a\{ie(jj,k)\} + B_a\{ie(jj,k)\}.QL_a\{ie(jj,k)\}$$
$$for\ jj = 1,2,...........NB-1 \quad (19)$$
$$and\ k=1,2..............N(jj)_a$$

$$ploss_b\{jj,ie(jj,k)\} =$$
$$A_b\{ie(jj,k)\}.PL_b\{ie(jj,k)\} + B_b\{ie(jj,k)\}.QL_b\{ie(jj,k)\}$$
$$for\ jj = 1,2,...........NB-1 \quad (20)$$
$$and\ k=1,2..............N(jj)_b$$

$$ploss_c\{jj,ie(jj,k)\} =$$
$$A_c\{ie(jj,k)\}.PL_c\{ie(jj,k)\} + B_c\{ie(jj,k)\}.QL_c\{ie(jj,k)\}$$
$$for\ jj = 1,2,...........NB-1 \quad (21)$$
$$and\ k=1,2..............N(jj)_c$$

The total loss for each of the individual phases is:

$$Tploss_a(\ell) = \sum_{jj=1}^{NB-1} ploss_a(jj,\ell) \quad for\ \ell = 2,3,...,NB \quad (22)$$

$$Tploss_b(\ell) = \sum_{jj=1}^{NB-1} ploss_b(jj,\ell) \quad for\ \ell = 2,3,...,NB \quad (23)$$

$$Tploss_c(\ell) = \sum_{jj=1}^{NB-1} ploss_c(jj,\ell) \quad for\ \ell = 2,3,...,NB \quad (24)$$

C. Implementation and Flow chart of the proposed method

In order to allocate the losses to the consumers of a three phase unbalanced radial distribution network, the load flow of the system for a particular load pattern has to be performed first. The load flow solution gives the converged voltages at various nodes, which are used latter on for allocating losses to the various customers. For load flow solution, the forward backward sweep based method **[11]** is used. In order to implement the proposed loss allocation method, four arrays are introduced to store the data regarding the subsequent buses to all the branches of the distribution network. The vector **sb[ ]** would store the subsequent buses of each of the buses of the radial distribution network and the vector **nsb[ ]** would store the number of subsequent buses of each of the buses of the network. Two other vectors **mfs [ ]** and **mts[ ]** are introduced, which act as pointers to the **sb[ ]** vector. These vectors in turn govern the reservation allocation of memory location for each bus, where **mfs[i]** and **mts[i]** hold the data of starting memory location and end memory location of bus-i in the **sb[ ]** vector for **i = 1,2……… nb.** All the buses are numbered in the increasing order down stream with the substation node numbered as 1. A branch preceding a bus will be numbered one less than the bus number. In Fig.2, the storage and pointer operation of the four arrays are clearly explained. These vectors, along with the other vectors introduced earlier are used to implement the proposed loss allocation scheme in the unbalanced radial distribution network.
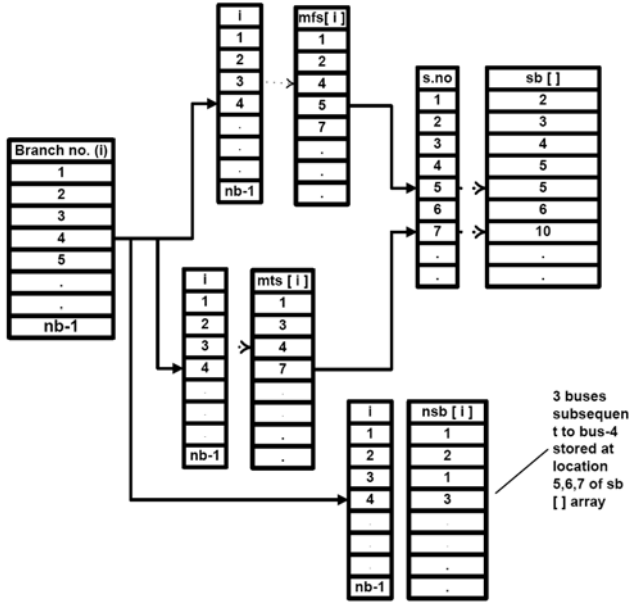
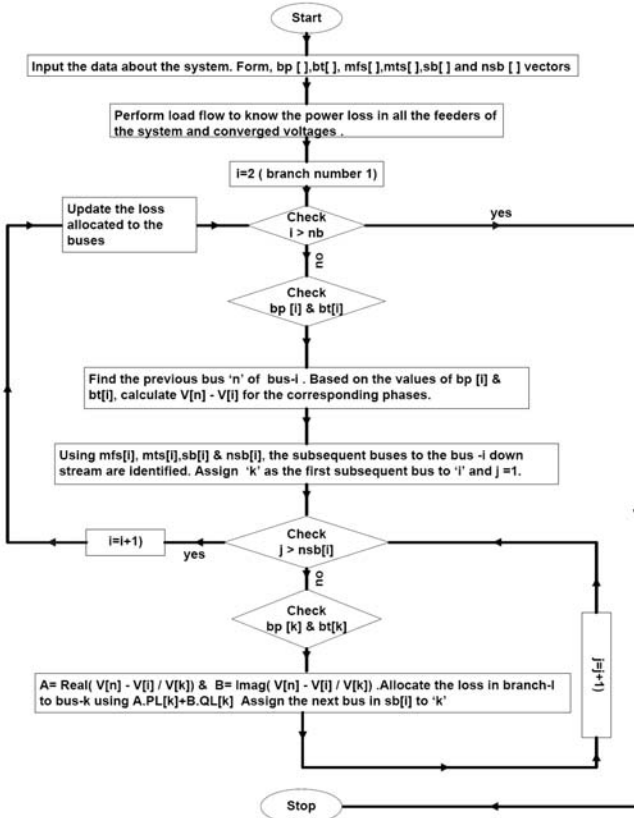**Fig. 2 Storage and Pointer Operation of the proposed arrays**

Fig. 3 Flowchart for the proposed Loss Allocation Scheme

The complete algorithm of the proposed loss allocation scheme for three phase unbalanced network has been presented in fig. 3.In the proposed method the loads are modeled as composite loads:

$$P = P_0(a_0 + a_1V + a_2V^2) \qquad (25)$$

$$Q = Q_0(b_0 + b_1V + b_2V^2) \qquad (26)$$

$$a_0 + a_1 + a_2 = b_0 + b_1 + b_2 = 1 \qquad (27)$$

where, V is the p.u value of the bus voltage magnitude ;
   P_0, Q_0 are the real and reactive power consumed
   at the specific node under the reference voltage;
   a_0,b_0 are the parameters for constant power
   (constant P and Q) load component ;
   a_1, b_1 are the parameter for constant
   current (constant I) load component ;
   a_2, b_2 are the parameters for constant impedance
   (constant -Z) load component.

D. Test Systems

In order to verify the proposed allocation method three test systems have been chosen. Test system-1 is an eight bus system (equivalent 13 node system), which includes the three phase, double phase, and single phase line sections and buses as shown in fig. 4 . The other details are shown in Table-2, and Table-3**.** Power Factor is assumed to be 0.8. Base kV and base kVA are chosen to be: 4.16 kV and 300 kVA respectively. The three phase loads are all balanced. However, the system is having unbalanced loading with respect to the substation.
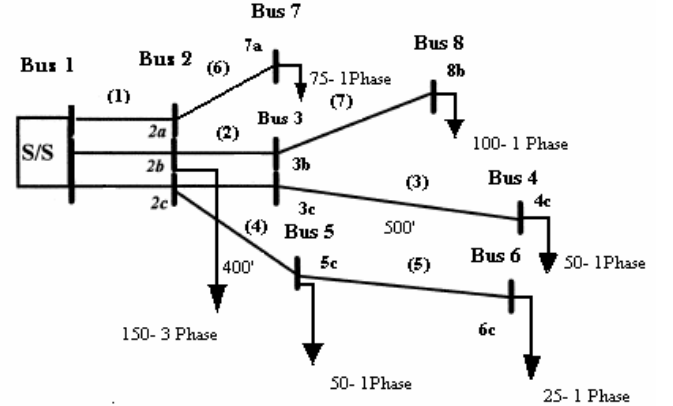
Fig. 4 Test System-1 (8 bus unbalanced multiphase distribution system)

The test system-2 is a 24-bus practical distribution system **[12]** and test system-3 is an IEEE 34-node system **[13-14].**

TABLE 2

Sending end Node	Sending end node phase	Receiving end Node	Receiving end node Phase	Br. no.	Conf. ID.	Branch Length (ft.)
1	3	2	3	1	1	1000
2	3	3	2	2	7	400
3	2	4	1	3	5	500
2	3	5	1	4	5	400
5	1	6	1	5	5	400
2	3	7	1	6	3	500
3	2	8	1	7	4	500

E. Results and Discussion

The forward backward sweep method of load flow program for the three test systems have been implemented,

where the composite load models have been implemented. In (25 – 27), the values of a_0, a1, a2, b0, b1 and b_2 have been considered as $a_0= b_0= 0.3$, $a_1= b_1= 0.3$ and $a_2= b_2= 0.4$ for the composite nature of the load. The value of P_0 and Q_0 are taken as the values of given constant P and Q values of the load. If the constant P and Q models are to be considered, then $a_0= b_0= 1.0$, $a_1= b_1= 0.0$ and $a_2= b_2= 0.0$ are set. The allocated active power losses to the various buses using the loss allocation scheme proposed are presented in the Table-4, 5,6,7,8 and 9 for the three test systems considering two different load models. In the first case, the loads are modeled as constant power loads (Table-4, 6&8) whereas in the latter case the composite loads have been considered with load composition as described (Table-5, 7&9).

TABLE 3

Conf. ID	Phasing	Phase Cond.	Neutral Cond.	Spacing ID	Type
1	ABCN	336,400	336,400	500	3-Φ 4wire
2	ABCN	1/0	1/0	500	3-Φ 4wire
3	AN	1/0	1/0	510	1-Φ 2wire
4	BN	1/0	1/0	510	1-Φ 2wire
5	CN	1/0	1/0	510	1-Φ 2wire
6	ABN	1/0	1/0	505	2-Φ 3wire
7	BCN	1/0	1/0	505	2-Φ 3wire
8	CAN	1/0	1/0	505	2-Φ 3wire

TABLE 4

Node No.	Loss Allocated(kW) Phase-a	Loss Allocated(kW) Phase-b	Loss Allocated(kW) Phase-c
2	0.076694	0.113514	0.078759
3	0.000000	0.000000	-
4	-	-	0.183477
5	-	-	0.174681
6	-	-	0.103481
7	0.294929	-	-
8	-	0.687582	-
Total Loss	0.371623	0.801096	0.540398

TABLE 5

Node No.	Loss Allocated(kW) Phase-a	Loss Allocated(kW) Phase-b	Loss Allocated(kW) Phase-c
2	0.076164	0.112438	0.078279
3	0.000000	0.000000	-
4	-	-	0.181645
5	-	-	0.173064
6	-	-	0.102430
7	0.292067	-	-
8	-	0.676141	-
Total Loss	0.368231	0.788579	0.535418
Total Loss	113.332683	75.734907	100.578099

III. CONCLUSIONS

A loss allocation scheme for unbalanced distribution system has been proposed. This loss allocation scheme successfully allocates the active losses to various buses of the three test systems considered. Composite load modeling has been implemented in the load flow program and allocations have been made accordingly.

Recent technology improvements in micro turbines, fuel cells and energy storage devices have provided the opportunity for dispersed generation at the distribution level. With the possibility of significant penetration of distributed generation, more studies are needed on dynamic analysis of distribution systems. Research is under progress for applying the loss allocation scheme to distribution networks with distributed generations.

TABLE 6

Node No.	Loss Allocated(kW) Phase-a	Loss Allocated(kW) Phase-b	Loss Allocated(kW) Phase-c
2	0.000000	0.000000	0.000000
3	0.000000	0.000000	0.000000
4	0.000000	0.000000	0.000000
5	0.613903	0.359618	0.421616
6	0.000000	0.000000	0.000000
7	0.000000	0.000000	0.000000
8	0.973274	0.532386	0.763904
9	2.304830	0.000000	0.000000
10	0.000000	0.696534	0.000000
11	1.823625	0.000000	-
12	2.790733	-	-
13	-	0.720862	-
14	0.000000	0.000000	1.931400
15	4.992988	3.063675	4.617106
16	-	-	1.323284
17	0.000000	0.000000	0.000000
18	0.579770	0.424480	0.535988
19	0.662366	0.000000	0.000000
20	-	0.828019	-
21	-	-	0.650379
22	-	-	1.079012
23	0.959916	0.000000	0.000000
24	-	0.000000	0.409703
25	-	1.154647	-
Total Loss	15.701406	7.780221	11.732390

TABLE 7

Node No.	Loss Allocated(kW) Phase-a	Loss Allocated(kW) Phase-b	Loss Allocated(kW) Phase-c
2	0.000000	0.000000	0.000000
3	0.000000	0.000000	0.000000
4	0.000000	0.000000	0.000000
5	0.588274	0.353573	0.400213
6	0.000000	0.000000	0.000000
7	0.000000	0.000000	0.000000
8	0.919648	0.521097	0.720333
9	2.143575	0.000000	0.000000
10	0.000000	0.679870	0.000000

11	1.684640	0.000000	-
12	2.575396	-	-
13	-	0.703192	-
14	0.000000	0.000000	1.798606
15	4.650280	2.958997	4.290127
16	-	-	1.231436
17	0.000000	0.000000	0.000000
18	0.556187	0.416022	0.508790
19	0.634906	0.000000	0.000000
20	-	0.806745	
21	-	-	0.616897
22	-	-	1.022610
23		0.000000	0.000000
24	-	0.000000	0.388895
25	-	1.122101	-
Total Loss	14.672452	7.561592	10.977906

TABLE 8

Node No.	Loss Allocated(kW) Phase-a	Loss Allocated(kW) Phase-b	Loss Allocated(kW) Phase-c
2	0.275314	0.184005	0.236047
3	0.457451	0.304756	0.392168
4	1.703049	1.124449	1.459000
5	-	1.778958	-
6	1.554142	0.994392	1.350722
7	1.852089	1.180259	1.625020
8	2.034059	1.287129	1.791679
9	4.129336	2.609815	3.639610
10	6.676705	-	-
11	10.016303	-	-
12	23.169165	-	-
13	2.275569	1.585686	2.089888
14		2.977639	3.848247
15	4.025753	2.847382	3.724544
16	4.433426	3.444826	4.309089
17	4.732981	3.836091	4.710027
18	-	4.920301	-
19	2.869447	2.340483	2.873195
20	4.615476	3.880234	4.761832
21	-	-	9.263170
22	-	-	13.054650
23	2.465390	4.149677	5.066936
24	6.240955	-	-
25	2.195604	1.876686	2.239107
26	2.643415	5.091733	6.067129
27	19.628426	17.019880	20.017008
28	2.995952	2.622701	3.063573
29	1.398387	4.322931	5.026576
30	3.520374	3.008379	3.588588
31	3.111413	2.655956	3.165386
32	8.737969	-	-
33	-	3.712270	4.418522
34	-	3.520074	4.144557
Total Loss	127.758139	83.276689	115.926281

TABLE 9

Node No.	Loss Allocated(kW) Phase-a	Loss Allocated(kW) Phase-b	Loss Allocated(kW) Phase-c
2	0.259704	0.176893	0.218953
3	0.429568	0.292044	0.362267
4	1.575582	1.066347	1.329808
5	-	1.687752	-
6	1.440413	0.939254	1.221553
7	1.686282	1.107350	1.460637
8	1.844385	1.204373	1.604564
9	3.741694	2.440907	3.257494
10	6.029620	-	-

11	8.840397	-	-
12	20.246409	-	-
13	2.053279	1.471988	1.857161
14	-	2.762009	3.418980
15	3.628999	2.639243	3.304742
16	3.968627	3.159088	3.780596
17	4.221095	3.498570	4.107389
18	-	4.475436	-
19	2.557445	2.132668	2.502761
20	4.100870	3.520727	4.125134
21	-	-	7.999561
22	-	-	11.216855
23	2.187196	3.759510	4.382842
24	5.532885	-	-
25	1.939428	1.691430	1.927360
26	2.334534	4.587859	5.221209
27	17.283041	15.281864	17.172305
28	2.637240	2.352645	2.626250
29	1.230832	3.875372	4.307244
30	3.109156	2.711069	3.088537
31	2.746043	2.392081	2.722596
32	7.707969	-	-
33	-	3.342904	3.800041
34	-	3.165525	3.561265

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V. BIOGRAPHIES

Sivkumar Mishra received the B. E degree from Malaviya National Institute of Technology, Jaipur(formerly known as MREC,Jaipur) and M.Tech degree from Indian Institute of Technology, Kharagpur, India . He is currently an Assistant Professor in Ghanashyam Hemalata Institute of Technology and Management, Puri, Orissa. His research interests include electric power distribution system analysis.

Debapriya Das received the B. E degree from Calcutta University, Calcutta in 1982, the M. Tech degree from Indian Institute of Technology, Kharagpur, India in 1984, and the Ph.D. degree from Indian Institute of Technology, Delhi, India. Currently, he is a Professor at Indian Institute of Technology, Kharagpur, India. His research interests are electric power distribution system and power system operation and control.