THE PERMANENT MAGNET SYNCHRONOUS MOTOR ROBUST NONLINEAR FEEDBACK CONTROL BASED ON LYAPUNOV METHOD -EXPERIMENTAL EVALUATION-

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Abstract: *This paper deals with a robust control intended for permanent magnet synchronous motor. Two approaches are used: the first one a nonlinear input output state feedback linearizing control and second one is a proposed nonlinear feedback control based on Lyapunov method. This second solution shows good robustness with respect to parameter variations, measurement errors and noises. Finally, simulation and experimental results are given to demonstrate the effectiveness and the good performance of the proposed control.*

Key words: *Lyapunov function, permanents magnet synchronous motors, robust nonlinear control.*

1. Introduction.

The PMSM is becoming more and more popular in servo systems because of its high power density, large torque to inertia ratio and high efficiency [1], [2].Also the PMSM model is nonlinear coupled and subjected to parameter variation. The PMSM model is described by a fifth-order nonlinear differential equation, where a part of states are not easily measurable, and often perturbed by an unknown load torque. Classical PI controller is a simple method used to control PMSM drives. However the main drawbacks of PI controller are the sensitivity of performances to the system parameter variations and inadequate rejection of external disturbances and load change.
Recently, several nonlinear control methods have been applied to control PMSM considering the nonlinear PMSM dynamics, such as feedback linearization [3], back-stepping [4].

This paper deals with a development of a nonlinear control of current based on Lyapunov Method. Introducing sliding mode control conducts to efficient robustness against parameters variations, measurements errors and noises. The asymptotic stability of the overall system is theoretically proven.

2. Model machine

Its dynamic model expressed in the rotor reference frame is given by voltage equations:

$$\begin{aligned} v_d &= R_s I_d + \frac{d\Phi_d}{dt} + p\Omega\Phi_q \\ v_q &= R_s I_q + \frac{d\Phi_q}{dt} + p\Omega\Phi_d \end{aligned} \tag{1}$$

Where the flux expression are given by

$$\begin{aligned} \Phi_d &= L_d I_d + \Phi_f \\ \Phi_q &= L_q I_q \end{aligned}$$

Considering I_d and I_q are stats variables (1) can be written as

$$\begin{aligned} \frac{dI_d}{dt} &= -\frac{R_s}{L_d} I_d + \frac{L_q}{L_d} p\Omega + \frac{v_d}{L_d} \\ \frac{dI_q}{dt} &= -\frac{R_s}{L_q} I_q - \frac{L_d}{L_q} p\Omega I_d - \frac{\Phi_f}{L_q} p\Omega + \frac{v_q}{L_q} \end{aligned} \tag{2}$$

The electromagnetic torque is given by

$$T_e = \frac{3}{2} p\left[\left(L_d - L_q\right) I_d I_q + \Phi_f I_q\right] \tag{3}$$

And the associated equation of motion is

$$J_m \frac{d\Omega}{dt} = T_e - T_L - f_m \Omega \tag{4}$$

Form (2), (3) and (4), the state model is rewritten as:

$$\begin{aligned} \dot{x} &= f_r(x) + g_d(x) v_d + g_q(x) v_q \\ y &= h(x) \end{aligned} \tag{5}$$

Where

$$f_r(x) = \begin{bmatrix} a_{11}x_1 + a_{12}x_1x_2 \\ a_{21}x_2 + a_{22}x_1x_3 + a_{23}x_3 \\ a_{31}x_1x_3 + a_{32}x_2 + a_{33}x_3 + a_{34}T_r \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \tag{6}$$

and

$$g_d(x)=\begin{pmatrix}\lambda_d\\0\\0\end{pmatrix},\quad g_q(x)=\begin{pmatrix}0\\\lambda_q\\0\end{pmatrix}$$

$$v_s=\begin{bmatrix}v_d & v_q\end{bmatrix}^T \qquad \begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}=\begin{bmatrix}I_d & I_q & \Omega\end{bmatrix}^T$$

$$a_{11}=-\frac{R_s}{L_d},\ a_{12}=\frac{L_q}{L_d}p,\ a_{21}=-\frac{R_s}{L_q},\ a_{22}=-\frac{L_d}{L_q}p,$$

$$a_{23}=-\frac{\Phi_f}{L_q}p,\ a_{31}=\frac{3p}{2J}\left(L_d-L_q\right),\ a_{32}=\frac{3p}{2J}\Phi_f,$$

$$a_{33}=-\frac{f}{J},\ a_{34}=-\frac{1}{J},\ \lambda_d=1/L_d,\ \lambda_q=1/L_q$$

3. Input output feedback linearization control.

The outputs to be controlled are the motor speed Ω and the stator current I_d, i.e. the function $h(x)$ in (5) is defined as

$$h(x)=\begin{bmatrix}I_d\\\Omega\end{bmatrix} \tag{7}$$

The derivative of (7) is given by

$$\begin{bmatrix}\dot{y}_1\\\ddot{y}_2\end{bmatrix}=\begin{bmatrix}L_f h_1(x)\\L_f^2 h_2(x)\end{bmatrix}+D(x)\begin{bmatrix}v_d\\v_q\end{bmatrix} \tag{8}$$

The systems has relative degree 1 for I_d and 2 for Ω, $D(x)$ is the decoupling matrix defined by

$$D(x)=\begin{bmatrix}\lambda_d & 0\\\lambda_d a_{31}x_2 & \lambda_q\left(a_{32}+a_{31}x_1\right)\end{bmatrix} \tag{9}$$

and

$$L_f h_1(x)=a_{11}x_1+a_{12}x_2x_3 \tag{10}$$

$$L_f^2 h_2(x)=a_{31}x_2 f_1(x)+\left(a_{32}+a_{31}x_1\right)f_2(x)+a_{33}f_3(x)+a_{34}\dot{T}_L+a_{33}a_{34}T_L \tag{11}$$

Since $\left|D(x)\right|=\lambda_d\lambda_q\left(a_{32}+a_{31}x_1\right)\neq 0$, then $D(x)$ is not singular (machine with permanents magnets) and the MIMO system is input-output linearizable since the state feedback control given by reduces Input-Output map to:

$$\begin{bmatrix}v_d\\v_q\end{bmatrix}=D^{-1}(x)\left[\begin{pmatrix}-L_f h_1(x)\\-L_f^2 h_2(x)\end{pmatrix}\quad\begin{pmatrix}v_1\\v_2\end{pmatrix}\right] \tag{12}$$

Where $v=\begin{bmatrix}v_1 & v_2\end{bmatrix}^T$ is the new input vector.

$$\begin{bmatrix}\dot{y}_1\\\ddot{y}_2\end{bmatrix}=\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}k_{11}\left(I_d-I_{d\_ref}\right)\\k_{21}\left(\Omega-\Omega_{ref}\right)+k_{22}\left(\dot{\Omega}-\dot{\Omega}_{ref}\right)\end{bmatrix} \tag{13}$$

A bloc diagram of control nonlinear input-output is shown in fig.1

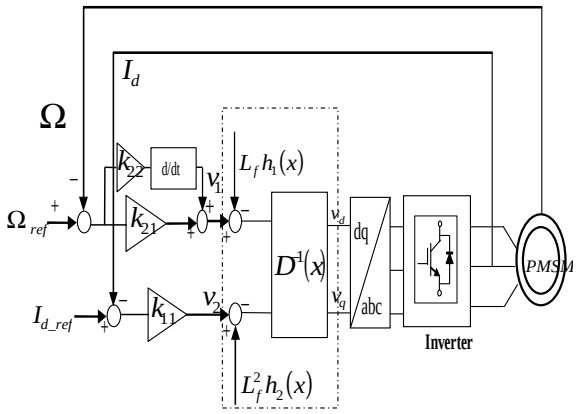

Fig. 1. Bloc diagram of PMSM control (FLIOC) scheme

The drawback of (12) is that it requires exact knowledge of the motor parameters and any variation in the parameters or the load torque will deteriorate the controller performances. In order to overcome this problem feedback nonlinear control based on Lyapunov theory is proposed.

4. Robust nonlinear feedback control based on Lyapunov method

The suggested PMSM control scheme is shown in Fig. 2. We can see that only one PI speed controller is used and the currents are feedback-controlled in association with a sliding mode controller. We can also note the placement of the estimator block which evaluates the feedback function f_1 and f_2 given by (6).

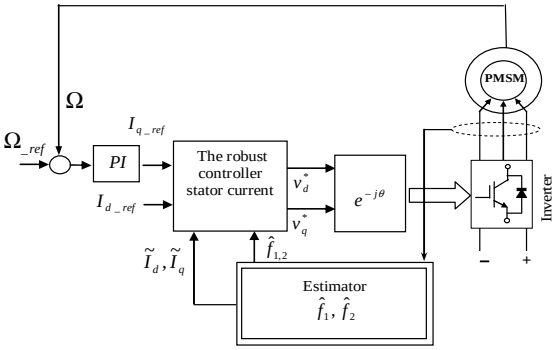

Fig. 2. Bloc diagram of nonlinear feedback control based on Lyapunov method system.

To determine the control feedback, starting with the equation (2) represented as follow

$$\frac{dI_d}{dt} = \lambda_d v_d + f_1$$
$$\frac{dI_q}{dt} = \lambda_q v_q + f_2 \quad (14)$$

Where

$$f_1 = a_{11}x_1 + a_{12}x_1x_2$$
$$f_2 = a_{21}x_2 + a_{22}x_1x_2 + a_{23}x_3$$

To prove the stability of system considering the candidate Lyapunov function [2]:

$$V = 0.5\left(I_d - I_{d\_ref}\right)^2 + 0.5\left(I_q - I_{q\_ref}\right)^2 \quad (15)$$

The time derivative is given by

$$\dot{V} = \left(I_d - I_{d\_ref}\right)\left(\dot{I}_d - \dot{I}_{d\_ref}\right) + \left(I_q - I_{q\_ref}\right)\left(\dot{I}_q - \dot{I}_{q\_ref}\right) \quad (16)$$

Taking in to account of (14) in (16) yields we obtain

$$\dot{V} = \left(I_d - I_{d\_ref}\right)\left(\lambda_d v_d + f_1 - \dot{I}_{d\_ref}\right) + \left(I_q - I_{q\_ref}\right)\left(\lambda_q v_q + f_2 - \dot{I}_{q\_ref}\right) \quad (17)$$

Selecting the control law as

$$v_d = \frac{1}{\lambda_d}\left(-f_1 + \dot{I}_{d\_ref} - K_1\left(I_d - I_{d\_ref}\right)\right)$$
$$v_q = \frac{1}{\lambda_q}\left(-f_2 + \dot{I}_{q\_ref} - K_2\left(I_q - I_{q\_ref}\right)\right) \quad (18)$$

Where K_1, K_2 are positive gains and inserting them (17), we found:

$$\dot{V} = -K_1\left(I_d - I_{d\_ref}\right)^2 - K_2\left(I_q - I_{q\_ref}\right)^2 < 0 \quad (19)$$

If K_1, K_2 are chosen, the time negativity of derivative of Lyapunov function $\dot{V}$ is satisfied, and control system will be stabilized.

We concluded that

$$\lim_{t\to\infty}\left(I_d - I_{d\_ref}\right) = 0$$
$$\lim_{t\to\infty}\left(I_q - I_{q\_ref}\right) = 0 \quad (20)$$

4.1 Robust Feedback nonlinear control

The nonlinear functions $f_i$involved in the state-space model (14) are strongly affected by the conventional effects of PMS motors, such temperature, saturation, skin effects and the noise measurements. Since the control law developed in the preceding section is based on exact knowledge of these functions f_i, one can expect that in a real situation. Our objective is to determine a new control law robust to parameter variations and measurement noise. Globally, we can write

$$f_i = \hat{f} + \Delta f \quad (21)$$

where $\hat{f}_i$ is the true nonlinear feedback function (NLFF), f_i is the effective NLFF and Δf_i is the NLFF variation around f_i .The Δf_i can be generated from all of the parameters and variables as indicated above. We assume that all of the Δf_i are bounded as follows: $|\Delta f_i| < \beta_i$ where the β_i are known bounds. Knowledge of the β_i is not difficulties obtain, since one can use a sufficiently large number to satisfy the constraint $|\Delta f_i| < \beta_i$.

The Δf_i can be generated from the all of the parameters and variations as indicated above.

Substitution of (21) into (14) yields:

$$\frac{dI_d}{dt} = \lambda_d v_d + \hat{f}_1 + \Delta f_1$$
$$\frac{dI_q}{dt} = \lambda_q v_q + \hat{f}_2 + \Delta f_2 \quad (22)$$

Select the control law as

$$v_d = \frac{1}{\lambda_d}\left(-\hat{f}_1 + \dot{I}_{d\_ref} - K_1\left(I_d - I_{d\_ref}\right) - K_{11}sign\left(I_d - I_{d\_ref}\right)\right)$$
$$v_q = \frac{1}{\lambda_q}\left(-\hat{f}_2 + \dot{I}_{q\_ref} - K_2\left(I_q - I_{q\_ref}\right) - K_{22}sign\left(I_q - I_{q\_ref}\right)\right) \quad (23)$$

Where $K_{ii} \geq \beta_i$, $K_i > 0$ and $i = 1,2$.

The Lyapunov function relativity to (22) is defined by

$$V = 0.5\left(I_d - I_{d\_ref}\right)^2 + 0.5\left(I_q - I_{q\_ref}\right)^2 \quad (24)$$

We have

$$\dot{V}_1 = \left(I_d - I_{d\_ref}\right)\left(\Delta f_1 - K_{11}sign\left(I_d - I_{d\_ref}\right)\right) + \left(I_q - I_{q\_ref}\right)\left(\Delta f_2 - K_{22}sign\left(I_q - I_{q\_ref}\right)\right) + \dot{V} < 0 \quad (25)$$

Where $\dot{V}$ is given by (17). Hence the Δf_i variations can be absorbed if we take

$$K_{111} > |\Delta f_1|$$
$$K_{22} > |\Delta f_2| \qquad (26)$$

These inequalities are satisfied since $K_i > 0$ and $|\Delta f_i| < \beta_i < K_{ii}$. Finally, we can write

$$\dot{V}_1 < \dot{V} < 0 \qquad (27)$$

Insuring the negativity of Lyapunov function. Hence, using Lyapunov theorem [2], we conclude that

$$\lim_{t\to\infty}\left(I_d - I_{d\_ref}\right) = 0$$
$$\lim_{t\to\infty}\left(I_q - I_{q\_ref}\right) = 0 \qquad (28)$$

The design of these robust controllers, resulting from (27), is given in Fig.3.

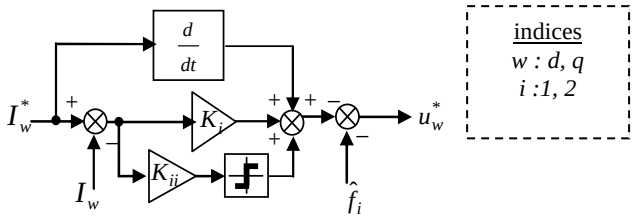

Fig. 3. Design of a robust controller.

5. Simulation results

Two schemes have been implemented: 1) a feedback linearizing input output Control (FLIOC) and 2) the proposed nonlinear feedback control based on Lyapunov Method scheme (NLFC). To the end of more significant comparison between the two schemes, the tests have been conduced to analyze and compare the performance of the PMSM in terms of accuracy, dynamic performance and load disturbance rejection.

The parameters of the PMSM used in the simulation are given as: rated voltage V=511 V, number of poles p=3, armature resistance Rs=1.4Ω, Stator inductances Ld= 0.0066H, Lq=0.0058H, viscous damping fm=0.00038N.m.s/rad, moment of inertia Jm=0.0016 kg.m2, rotor flux Φf =0.1546 wb, rated torque Tn =10 N.m.

Figs. 4 and 5 shows the PMSM response to square-wave speed reference 200 rad/s, using the FLIOC and NLFC. The NLFC PMSM drive speed trajectory is characterized by zero steady-state error and very fast dynamic response. The comparison between the FLIOC and NLFC speed transients, reported in Figs. 6 and 7, highlights the better performance of the proposed control. The considerations indicated above are confirmed by the following test imposing a 5-Hz sinusoidal reference speed of 10rad/s peak value. The comparison between the actuel speed profiles, Fig. 8 shows a better dynamic response of FLL scheme.

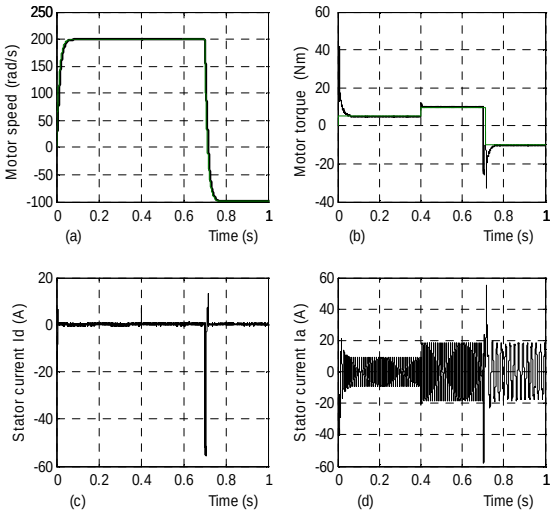

Fig.4. Simulation results of feedback linearizing control : a) motor speed b) motor torque c) stator current Id d) stator current Ia (A)

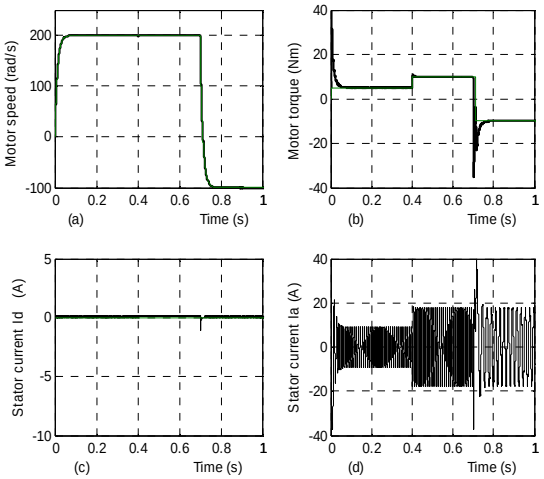

Fig.5. Simulation results of Nonlinear feedback Control Based on Lyapunov Method : a) motor speed b) motor torque c) stator current Id d) stator current Ia (A)

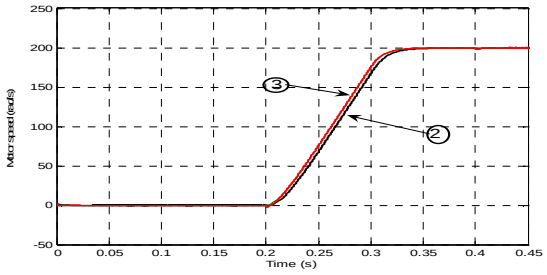

Fig. 6. Comparison between FLIOC (tr.2) and NLFC (tr.3) speed transient evolutions

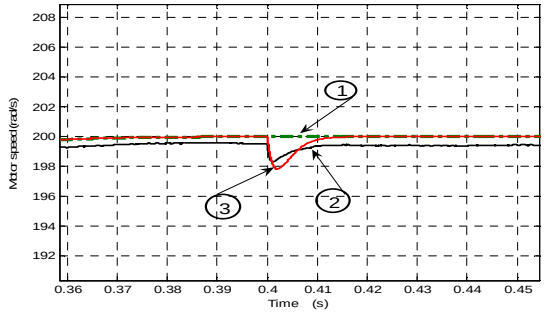

Fig. 7. Zoom of FLIOC (tr.2) and NLFC (tr.3) speed transient. (tr.1 reference speed).

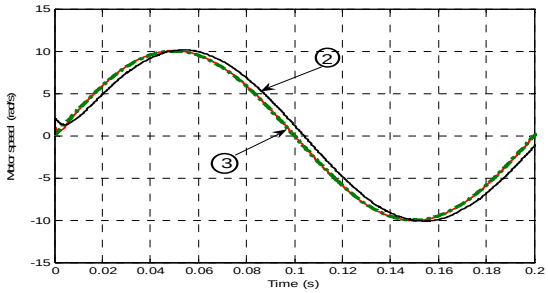

Fig. 8. Comparison between FLIOC (tr.2) and NLFC (tr.3) speed evolutions to the application of the same sinusoidal reference speed. (Crest value 10 rad/s, frequency 5Hz

To test robustness of feedback control. The simulation involves the following operating sequences:
The PMSM started with a constant acceleration after 0.1s, the speed was maintained to10 rad/s, while the motor is loaded with a constant torque of 5 Nm at starting. Then the motor is loaded with a constant torque of 10 Nm at t=0.4s. at t=0.7s, the speed change form 10 rad/s to 0 rad/s with same constant load torque. Maintaining a reference current Id to zero. Tow sets of simulation tests are carried out.
The first set is carried out with stator resistance having a mismatch of 100% at t=0.5s using the control law given by (12) the results of this test set are shown in Fig. 9 (tr.2). It is clear that when considering stator resistance uncertainty, a very large steady state error occurred in motor speed.
Finally the motor having a f_i (NLFF) mismatch of 300% at t=0.5s and in the presence of noise is simulated using the proposed control (23). The results are shown in Fig. 9 (tr.3). The control shows better speed response even in the presence of parameter uncertainty and measurement noises.

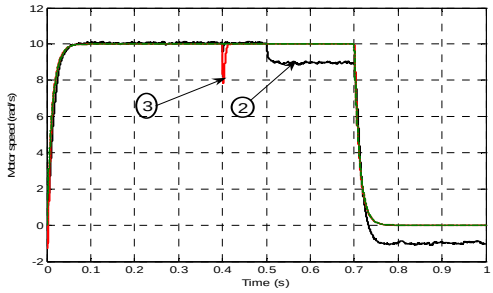

Fig. 9. Comparison between FLIOC (tr.2) and NLFC (tr.3) speed transient evolutions with parameter uncertainty and measurement noises.

6. Experimental the al system

The basic structure of the laboratory setup is depicted in Fig. 10. The DC motor is used as a load. The PMSM stator is fed by a converter controlled directly by the DS1103 board. The dSPACE DS1103 PPC is plugged in the host PC. The encoder is used for the measure mechanical speed. The sensors used for the currents and voltages measure are respectively LA-25NP and LV-25P. The Interface to provide galvanic isolation to all signals connected to the DS1103 PPC controller.

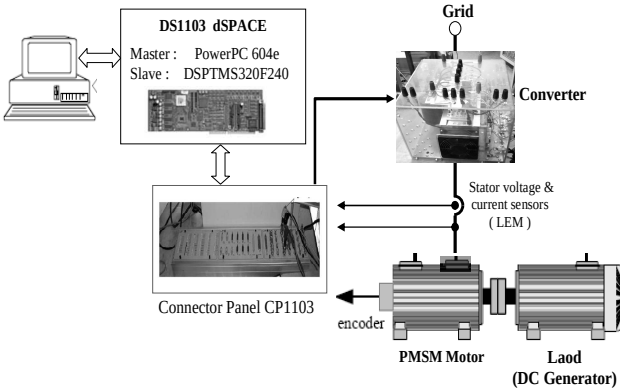

Fig.10 Structure of the laboratory setup

6.1 Reference profiles and machine parameters

To test the speed tracking, we applied two kinds of speed references. In the first, the PMSM is started with a constant acceleration 33.33 (rd/s²), after 3 s, the speed was maintained to 100(rd/s). After that, a reversal speed test was applied to the loaded machine at 21 s, where the speed changed to -100 (rd/s), Fig. 11. The load is 1 mN. In the second, a sinusoidal speed reference of magnitude 100 rad/s with frequency 0.05 Hz. The machine was loaded at 1 mN. (see Figs. 11a and 11b).
The machine parameters are given as follows:
Rs=11Ω; Ld=0.995 H; Lq=0.885H; p=2;J=0.025SI; f=0.0001SI; Φf=0.15wb

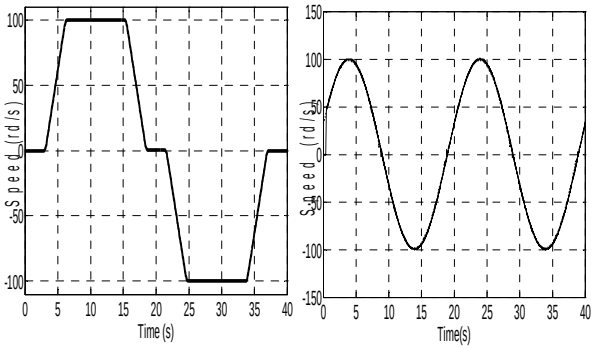

Fig. 11 Speed references versus time

6.2 Experimental results

Fig. 12 and 14 present the speed response versus time according to the profiles described above.

The actual speed converges to the reference speed very quickly with zero steady-state error and almost without any overshoot/undershoot in real-time.

Figs. 13 and 15 show, respectively, the stator phase current and the quadratic current Iq versus time during the test.

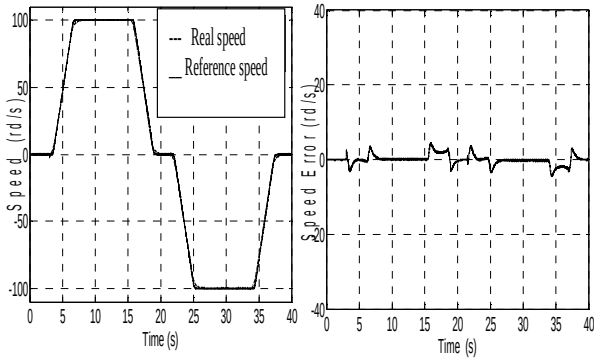

Fig. 12 Speed response versus time and speed error versus time

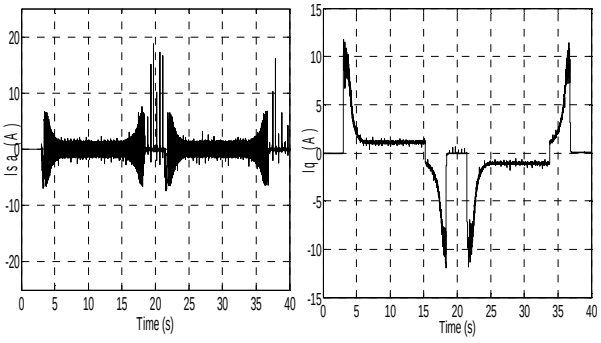

Fig. 13 Stator phase current versus time and Iq current versus time

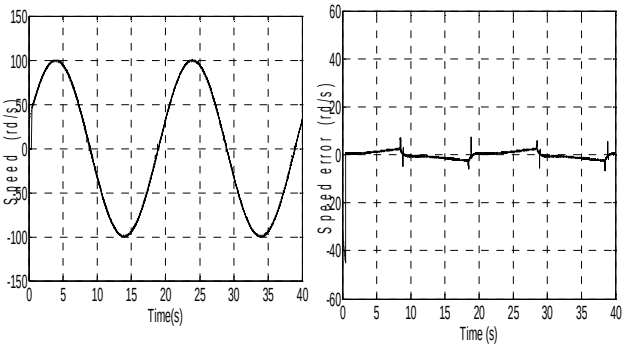

Fig. 14 Speed response versus time and speed error versus time

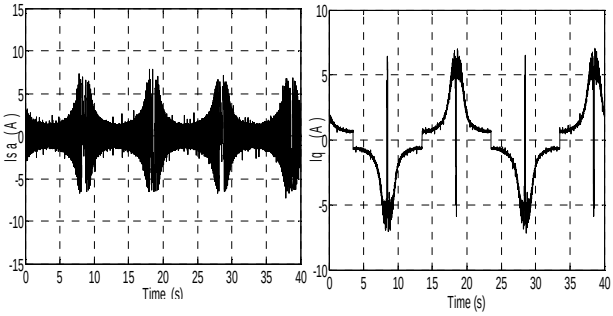

Fig. 15 Stator phase current versus time and Iq current versus time

Figs. 10 and 12 show speed and stator current response versus time; we observe that with the proposed control a good tracking speed end currents were achieved.

7. Conclusion

This paper present tow control schemes for the PMS motor. Firstly an input-output linearizing control is developed. Secondly a feedback nonlinear control based on Lyapunov method is proposed in order to reduce the effects of the parametric variations and measurement noise.

The theoretical study of the proposed control technique (NLFC) has been discussed, and the simulation results of the overall system have been presented to prove the effectiveness of this control strategy. Control robustness is achieved by a sliding-mode controller in order to reduce the effects of parametric variations, uncertainties and measurement noise.

The control stability is verified via Lyapunov stability analysis. From the simulation and experimental results for the proposed scheme, very well performances for both high and low speed are achieved. The proposed control is robust to parameter changes.

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