OPTIMAL REPAIR AND PLACEMENT OF PHASOR MEASUREMENT UNITS ON POWER SYSTEMS

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Abstract *- This paper proposes an algorithm to determine optimal repair rates of components of a phasor measurement unit (PMU) under the budgetary constraint. Based on optimal repair rates of components of a PMU; we determine the reliability of a PMU; the reliability of observability of buses falling within the scope of a PMU and the reliability of observability of a given power system. The repair-rate linked PMU placement problem has been formulated as a non-linear mathematical program. To achieve the targeted reliability of observability of a power system, the optimal numbers of PMUs and their candidate buses have been determined. Two new indices called the bus observability reliability index and the system observability reliability index have been introduced to determine the best solutions. The proposed algorithm and the repair-rate linked optimal placement of PMUs have been illustrated through examples.*

Keywords - *Optimal Repair Rates, Geometric Programming, Zero-One programming, Phasor Measurement Unit, Reliability, Observability, Power System*

1. INTRODUCTION

There has been a great interest in solving the PMU placement problem for the last two decades. The PMU placement problem is about determining the optimal number of PMUs and their candidate buses on a power system in such a manner that the given power system is fully observed. Several engineers from academia, research laboratories and industry have dealt with variants of the PMU placement problem. Comprehensive surveys on PMU placement on power systems are found in Shahraeini and Javidi [1], Cai and Ai [2], Almutaire and Milanovic [3], Reddy, Ramesh, Choudhary and Choudhary [4], Dongjie, Renmu, Peng and Tao [5], Manousakis, Korres and Georgilakis [6] and Yuill, Edwards, Choudhary and Choudhary [7]. What we perceive from existing literature on PMU placement on power systems is that the literature on reliability of a PMU and reliability-linked PMU placement is quite scantier. The authors in Aminifar, Bagheri-Souraki, Fotuhi-Firuzabad and Shahidehpour [8] proposed the descriptive model of a PMU system and the authors in Khiabani, Yadav and Kavesseri [9] dealt with reliability-based placement of phasor measurement units on a power system.

There is no literature on prescriptive behaviour of a PMU and repair-rate-linked PMU placement on power systems. It prompts the author to present the prescriptive model of a PMU and repair-rate linked optimal placement of PMUs on a power system. Higher the repair rates of components of a PMU, higher the reliability of the PMU and, in turn, higher the reliability of observability of connected buses. It means the reliability of observability of a complete power system depends on the optimal repair rates of components of each of the PMUs installed on a power system. This paper presents the prescriptive model of a PMU and optimal-repair-rate-linked optimal placement of PMUs to achieve the targeted reliability of observability of a given power system.

Section 2 presents the descriptive model of a PMU. Section 3 presents the prescriptive model of a PMU to determine optimal repair rates of the components of a PMU, reliability of observability of each bus in a power system and reliability of observability of a power system. Section 4 presents the repair-rate-linked optimal placement of PMUs in a power system. Section 5 gives illustrative examples.

2. DESCRIPTIVE MODEL

A PMU consists of six major components as shown in Figure. 1.

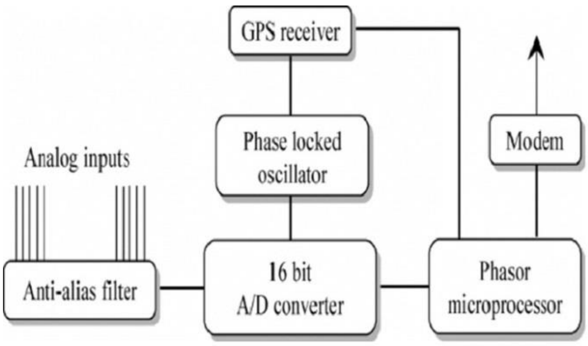


Figure 1: Block Diagram of a PMU

It is assumed that failure of a component means failure of the PMU and that a failed PMU is immediately sent for its repair. The state transition diagram of a PMU is as given in Figure 2 (Aminifar, Bagheri-Souraki, Fotuhi-Firuzabad and Shahidehpour [8]).

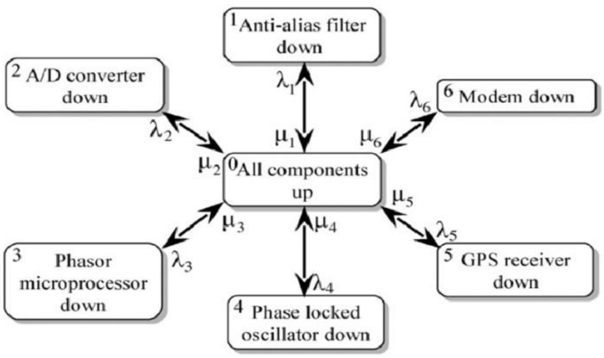


Figure 2: Seven-State Markov Model of PMU

Eqns. (1) and (2) govern the steady-state behaviour of a PMU.

$$p_0\lambda_i = p_i\mu_i \qquad (1\leq i \leq 6) \qquad (1)$$

$$\sum_{i=0}^{6} p_i = 1 \qquad (2)$$

The solution to Eqns. (1)-(2) is as follows.

$$p_i = p_0.\lambda_i / \mu_i \qquad (1\leq i \leq 6) \qquad (3)$$

$$p_0 = \left(1 + \sum_{i=1}^{6} \lambda_i / \mu_i\right)^{-1} \qquad (4)$$

, where

λ_i = Failure rate of the i^{th} component of a PMU,
μ_i = Repair rate of the i^{th} component of a PMU,
p_0 = Probability that all components are up,
p_1 = Probability that the anti-alias filer is down,
p_2 = probability that the A/D converter is down,
p_3 = probability that the microprocessor is down,
p_4 = probability that the phase-locked oscillator is down,
p_5 = probability that GPS receiver is down, and
p_6 = probability that modem is down.

(Aminifar, Bagheri-Souraki, Fotuhi-Firuzabad and Shahidehpour [8])

3. PRESCRIPTIVE MODEL

Let c_i be the cost of repair of the i^{th} component of a PMU ($1\leq i\leq 6$) and let c be the budget available for the purpose of repair of a PMU. The total cost incurred on repair of components of a PMU cannot exceed total budget (c) available for the purpose of repair of a PMU. Therefore, *the budgetary constraint* is as follows.

$$\sum_{i=1}^{6} c_i \mu_i \leq c \qquad (5)$$

It is justified to assume that the repair rate is always positive i.e. $\mu_i > 0$ ($1\leq i\leq 6$).

Our objective is to maximize the reliability of a PMU under budgetary constraint and $\mu_i > 0$ ($1\leq i\leq 6$). Mathematical formulation of the prescriptive model of a PMU is as given by program $\mathbf{P_1}$.

$\mathbf{P_1}$: Min $Z = p_0^{-1}$ s. t. (5), $\mu_i > 0$ $(1\leq i\leq 6)$

Let $\mu_0 > 0$ be an extra variable constrained to satisfy $\mu_0^{-1}.p_0^{-1} \leq 1$. We get following complementary geometric program.

$\mathbf{P_2}$: Min $Z = \mu_0$ s.t. $\mu_0^{-1}\, p_0^{-1} \leq 1$, (5), $\mu_i > 0$ $(1\leq i\leq 6)$

$\mathbf{P_2}$ is a complementary geometric program (Avriel and Williams [10], Beightler and Phillips [11]) whose solution may be obtained by condensing $\mu_0^{-1} p_0^{-1}$ to a monomial function. Given the point of condensation ($\underline{\mu}_0$, $\underline{\mu}_1$, $\underline{\mu}_2$, $\underline{\mu}_3$, $\underline{\mu}_4$, $\underline{\mu}_5$, $\underline{\mu}_6$), the condensation of $\mu_0^{-1} p_0^{-1}$ gives rise to the following geometric program with zero degree-of-difficulty (DoD).

P$_3$: Min Z = μ_0 s.t. $a\mu_0^{-1} \prod_{i=1}^{6} \mu_i^{b_i} \le 1$, (5), $\mu_i > 0$ ($0 \le i \le 6$)

, where

$$a = F \prod_{i=1}^{6} \underline{\mu}_i^{-b_i} \quad (6)$$

$$b_i = -\lambda_i \underline{\mu}_i^{-1}/F \quad (7)$$

with (8)

$$F = 1 + \sum_{i=1}^{6} \lambda_i/\underline{\mu}_i$$

Applying primal-dual relationship ([10], [11]), we get (μ_0^*, μ_1^*, μ_2^*, μ_3^*, μ_4^*, μ_5^*, μ_6^*) = $(a.\prod_{j=1}^{6} [(c_j/c)(\sum_{i=1}^{6} b_i/b_j)]^{-b_j}$, $(c/c_1).(b_1/\sum_{j=1}^{6} b_j)$, $(c/c_2).(b_2/\sum_{j=1}^{6} b_j)$, $(c/c_3).(b_3/\sum_{j=1}^{6} b_j)$, $(c/c_4).(b_4/\sum_{j=1}^{6} b_j)$, $(c/c_5).(b_5/\sum_{j=1}^{6} b_j)$, $(c/c_6).(b_6/\sum_{j=1}^{6} b_j))$ as solution to P$_3$. At this solution to P$_3$, we get $p_0^* = (1 + \sum_{i=1}^{6} \lambda_i/\mu_i^*)^{-1}$. If $|\underline{p} - p^*|$ is less than or equal to an infinitesimally small quantity ε, either (μ_1^*, μ_2^*, μ_3^*, μ_4^*, μ_5^*, μ_6^*) or ($\underline{\mu}_1$, $\underline{\mu}_2$, $\underline{\mu}_3$, $\underline{\mu}_4$, $\underline{\mu}_5$, $\underline{\mu}_6$) may be considered as the optimal solution to P$_2$. If $|\underline{p} - p^*|$ is greater than ε, we perform $\underline{\mu}_i \leftarrow \mu_i^o$ ∀i ($0 \le i \le 6$) and refine solution repeatedly through condensation. For given values of λ_1, λ_2, λ_3, λ_4, λ_5, λ_6, $\underline{\mu}_0$, $\underline{\mu}_1$, $\underline{\mu}_2$, $\underline{\mu}_3$, $\underline{\mu}_4$, $\underline{\mu}_5$, $\underline{\mu}_6$, c_1, c_2, c_3, c_4, c_5, c_6, c and ε, this process gives rise to the following algorithm (Figure 3) to deliver optimal solution (μ_0^o, μ_1^o, μ_2^o, μ_3^o, μ_4^o, μ_5^o, μ_6^o) to **P$_2$**. In the algorithm, $\underline{\mu}_i$, μ_i^*, p_0^o and p_0^*, respectively, are represented by $o\mu_i$, $n\mu_i$, op_0 and np_0. At optimality, we get

$$p_i^o = (\lambda_i/\mu_i^o).p_0^o \quad (1 \le i \le 6) \quad (9)$$

, where

$$p_0^o = (1 + \sum_{i=1}^{6} \lambda_i/\mu_i^o)^{-1} \quad (10)$$

Therefore,

$$q^o = \sum_{i=1}^{6} (\lambda_i/\mu_i^o)/(1 + \sum_{i=1}^{6} \lambda_i/\mu_i^o) \quad (11)$$

A bus in an n-bus system may be observed by a maximum of as many PMUs as the number of incident links to the bus plus one. The reliability of observability of the i[th] bus is given by

$$B_i = 1 - q^{o^{\sum_{j=1}^{n} a(i,j) x_j}} \quad (12)$$

, where a(i,j) = 1 if buses B_i and B_j are connected ,a(i,j) = 1 if i=j, a(i,j) = 0 if B_i and B_j are not connected and x_i ($1 \le i \le n$) have their usual meanings and q^o is given by Eqn. (11).

	Iter ← 0
Label1:	$o\mu_i \leftarrow \underline{\mu}_i$ ($0 \le i \le 6$)
	$op_o \leftarrow 1/(1 + \sum_{i=1}^{6} \lambda_i / o\mu_i)$
	$F \leftarrow 1 + \sum_{i=1}^{6} \lambda_i / o\mu_i$
	$b_i \leftarrow -(\lambda_i / o\mu_i)/F$ ($0 \le i \le 6$)
	$n\mu_i \leftarrow \left(\frac{c}{c_i}\right).b_i/\sum_{j=1}^{6} b_j$ ($1 \le i \le 6$)
	$np_o \leftarrow 1/(1 + \sum_{i=1}^{6} \lambda_i/n\mu_i)$
	If $\|op_0 - np_0\| \le \varepsilon$ then
	{
	$\mu_i^o \leftarrow o\mu_i$ ($1 \le i \le 6$)
	$p_0^o \leftarrow op_0$
	print p_0^o and μ_i^o ($1 \le i \le 6$) and stop
	}
	$\underline{\mu}_i \leftarrow n\mu_i$ ($0 \le i \le 6$)
	Iter++
	Go to Label1

Figure 3: Algorithm to solve P$_2$

The reliability of observability of entire n-bus power system is given by

$$R = \prod_{k=1}^{n} [1.0 - q^{o^{\sum_{j=1}^{n} a(k,j) x_j}}] \quad (13)$$

The values of B_i ($1 \le i \le 6$) and R play important roles in assessing the quality of a solution to the PMU placement problem. We may call B_i the *bus observability reliability index* (BORI) of the i[th] bus and R the *system observability reliability index* (SORI).

An illustrative example of the prescriptive model has been given in Subsection 5.1.

With the failure of a component of a PMU installed on a power system, the reliability of observability of the power system decreases. In situations when the reliability of observability of a power system is not satisfactory, we re-solve the PMU placement problem to achieve a targeted level of reliability of observability of the given power system. For example; we require four PMUs to be installed at buses numbered B2, B6, B7 and B9 to observe the IEEE 14-Bus system (Mohammadi-Ivatloo [12]), given that no PMU ever enters the state of failure. If the probability of failure of a PMU (q) is non-zero, the PMU is only (100-100q) per cent available to observe the buses falling in its scope. It shows (100-100q) percent availability of a PMU and it indicates a significant loss of observability of connected buses. This is an alarming situation as $100\prod_{k=1}^{n}[1.0 - q^{o\{\sum_{j=1}^{n} a(k,j)x_j\}}]$ per cent availability of observability of the n-bus power system indicates significant loss of system-wide data what could be recorded had all the PMUs been fully available. If the reliability of observability of a power system is to be raised to a targeted level σ (say), we have to re-determine the optimal number of PMUs and their candidate buses. In Section 4, we revisit the PMU placement problem from viewpoint of improving reliability of observability of entire n-bus power system.

4. OPTIMAL PLACEMENT OF PHASOR MEASUREMENT UNITS

Assume that all PMUs are identical and let σ be the targeted level of reliability of observability of the n-bus power system. Then, the targeted reliability of observability of a bus in the power system is given by $\rho = \sigma^{1/n}$. The general formulation of the PMU placement problem is as follows.

P_4: $\text{Min } Z = \sum_{i=1}^{n} x_i$ s.t. $1 - q^{o\sum_{j=1}^{n} a(i,j)x_j} \ge \rho\ (1 \le i \le n)$, $x_i = 0/1\ (1 \le i \le n)$

, where q, a(i,j) and ρ have their usual meanings and x_i is the i^{th} binary design variable such that $x_i = 1$ if a PMU is placed on the i^{th} bus and $x_i = 0$ if no PMU is placed on the i^{th} bus.

P_4 is a binary integer non-linear program in n variables x_i ($1 \le i \le n$) and its solution gives us the optimal number of PMUs and their candidate locations in the n-bus power system. For a given power system, P4 may possess multiple solutions which may be ranked on the basis of the *system observability reliability index* (SORI) in order to select the best multiple solutions.

4.1 A Particular Case

The buses connected to power generation units are considered as critical buses. Keeping in mind the importance of observation data, we have to set relatively higher reliability of observability of these buses. If the r^{th} bus is the only critical bus in the power system, the mathematical formulation of repair-linked PMU placement problem is as follows.

P_5: $\text{Min } Z = \sum_{i=1}^{n} x_i$ s.t. $1 - q^{o\sum_{j=1}^{n} a(i,j)x_j} \ge \rho\ (1 \le i \le n, i \ne r)$, $1 - q^{o\sum_{i=1}^{n} a(r,j)x_i} \ge \delta$, $x_i = 0/1$ $(1 \le i \le n)$

, where δ ($>\rho$) is the targeted level of reliability of observability of the r^{th} bus ($0 < \delta, \rho < 1$).

P_5 is also a binary integer non-linear program whose solution can be obtained after converting it into a binary integer linear program. An example of the particular case has been given in Subsection 5.2.

The PMU placement problem can be solved for power systems with zero injection buses by using the bus merging method explained in Mohammadi-Ivatloo [12].

5. ILLUSTRATIVE EXAMPLES

Consider the standard IEEE 14-Bus system **as** shown in Figure 4.

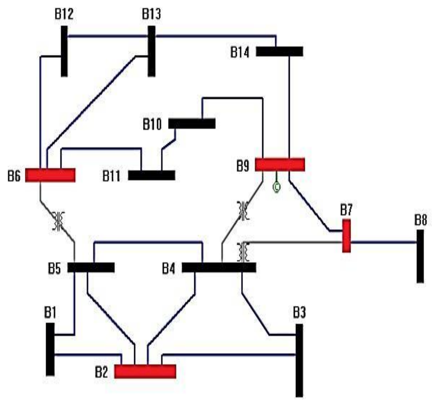


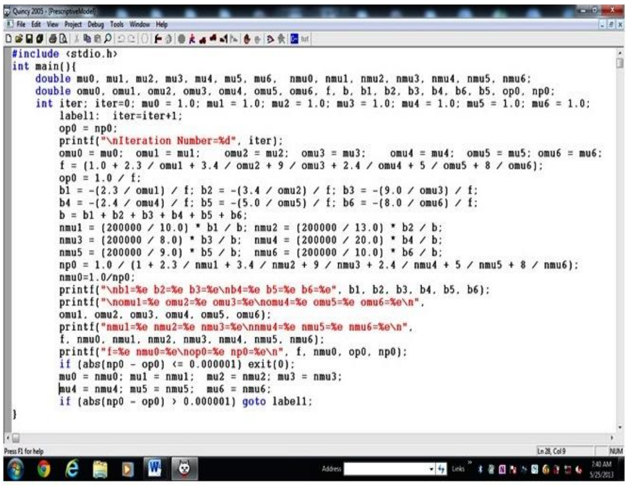
Figure 4: IEEE 14-Bus System

5.1. Prescriptive Model Example

Let $\lambda_1 = 2.3$, $\lambda_2 = 3.4$, $\lambda_3 = 9$, $\lambda_4 = 2.4$, $\lambda_5 = 5.0$, $\lambda_6 = 8.0$, $c_1 = 10.0$, $c_2 = 13.0$, $c_3 = 8.0$, $c_4 = 20.0$, $c_5 = 9.0$, $c_6 = 10.0$ and $c = 200000.0$. Let ($\underline{\mu}_0$, $\underline{\mu}_1$, $\underline{\mu}_2$, $\underline{\mu}_3$, $\underline{\mu}_4$, $\underline{\mu}_5$, $\underline{\mu}_6$) = (1, 1, 1, 1, 1, 1, 1) be the starting point of condensation. The optimal repair rates of six components of a PMU are obtained by solving the geometric program $\mathbf{Q_1}$.

Q_1: Min $Z = \mu_0$ s.t. μ_0^{-1} $(1.0 + 2.3/\mu_1 + 3.4/\mu_2 + 9.0/\mu_3 + 2.4/\mu_4 + 5.0/\mu_5 + 8.0/\mu_6) \leq 1$, $10\mu_1 + 13\mu_2 + 8\mu_3 + 20\mu_4 + 9\mu_5 + 10\mu_6 \leq 200000$, $\mu_i > 0$ $(0 \leq i \leq 6)$

Geometric program $\mathbf{Q_1}$ is solved by the algorithm proposed in Section 3. For this example, the proposed algorithm has been implemented in programming language ′C′ (Figure 5a) in Quincy IDE [13]. In the C program, μ is represented by *mu*.



```
#include <stdio.h>
int main(){
    double mu0, mu1, mu2, mu3, mu4, mu5, mu6,  nmu0, nmu1, nmu2, nmu3, nmu4, nmu5, nmu6;
    double omu0, omu1, omu2, omu3, omu4, omu5, omu6, f, b, b1, b2, b3, b4, b6, b5, op0, np0;
    int iter; iter=0; mu0 = 1.0; mu1 = 1.0; mu2 = 1.0; mu3 = 1.0; mu4 = 1.0; mu5 = 1.0; mu6 = 1.0;
        label1:  iter=iter+1;
        op0 = np0;
        printf("\nIteration Number=%d", iter);
        omu0 = mu0;  omu1 = mu1;   omu2 = mu2;  omu3 = mu3;   omu4 = mu4;  omu5 = mu5; omu6 = mu6;
        f = (1.0 + 2.3 / omu1 + 3.4 / omu2 + 9 / omu3 + 2.4 / omu4 + 5 / omu5 + 8 / omu6);
        op0 = 1.0 / f;
        b1 = -(2.3 / omu1) / f; b2 = -(3.4 / omu2) / f; b3 = -(9.0 / omu3) / f;
        b4 = -(2.4 / omu4) / f; b5 = -(5.0 / omu5) / f; b6 = -(8.0 / omu6) / f;
        b = b1 + b2 + b3 + b4 + b5 + b6;
        nmu1 = (200000 / 10.0) * b1 / b; nmu2 = (200000 / 13.0) * b2 / b;
        nmu3 = (200000 / 8.0) * b3 / b;  nmu4 = (200000 / 20.0) * b4 / b;
        nmu5 = (200000 / 9.0) * b5 / b;  nmu6 = (200000 / 10.0) * b6 / b;
        np0 = 1.0 / (1 + 2.3 / nmu1 + 3.4 / nmu2 + 9 / nmu3 + 2.4 / nmu4 + 5 / nmu5 + 8 / nmu6);
        nmu0=1.0/np0;
        printf("\nb1=%e b2=%e b3=%e\nb4=%e b5=%e b6=%e", b1, b2, b3, b4, b5, b6);
        printf("\nomu1=%e omu2=%e omu3=%e\nomu4=%e omu5=%e omu6=%e\n",
        omu1, omu2, omu3, omu4, omu5, omu6);
        printf("nmu1=%e nmu2=%e nmu3=%e\nnmu4=%e nmu5=%e nmu6=%e\n",
        f, nmu0, nmu1, nmu2, nmu3, nmu4, nmu5, nmu6);
        printf("f=%e nmu0=%e\nop0=%e np0=%e\n", f, nmu0, op0, np0);
        if (abs(np0 - op0) <= 0.000001) exit(0);
        mu0 = nmu0; mu1 = nmu1;  mu2 = nmu2; mu3 = nmu3;
        mu4 = nmu4; mu5 = nmu5;  mu6 = nmu6;
        if (abs(np0 - op0) > 0.000001) goto label1;
}
```

Figure 5a: C Program and Solution to Q_1

Execution of the C program (Figure 5b) delivers following results at successive iterations.

Iter = 0, op_0 = 3.321156e-004, f = 3.011000e+003, b_1 = - 7.395498e-002, b_2 = - 1.093248e-001, b_3 = - 2.893891e-001, b_4 = - 7.717042e-002, b_5 = - 1.607717e-001, b_6 = - 2.572347e-001, $n\mu_6$ = 5.315615e+003, $\underline{\mu}_i$ = 1 $(0 \leq i \leq 6)$, $o\mu_i$ = 1 $(0 \leq i \leq 6)$, np_0 = 9.895748e-001, $n\mu_0$ = 3.110000e+001, $n\mu_1$ = 1.528239e+003, $n\mu_2$ = 1.737797e+003, $n\mu_3$ = 7.475083e+003, $n\mu_4$ = 7.973422e+002, $n\mu_5$ = 3.691399e+003.

As $|op_0 - np_0| > \varepsilon$, we perform $o\mu_i \leftarrow n\mu_i$ $(1 \leq i \leq 6)$ and Iter ← Iter + 1 and go to *Label1*. At Iter = 1, we get $o\mu_0$ = 3.020224e-00, $o\mu_1$ = 1.528239e+003, $o\mu_2$ = 1.737797e+003, $o\mu_3$ = 7.475083e+003, $o\mu_4$ = 7.973422e+002, $o\mu_5$ = 3.691399e+003, $o\mu_6$ = 5.315615e+003, F = 1.010535e+000, op_0 = 9.895748e-001, a = 1.094346e+000, b1 = - 1.489310e-003, b2 = -1.936103e-003, b3 = - 1.191448e-003, b4 = -2.978620e-003, b5 = - 1.340379e-003, b6 = -1.489310e-003, np_0 = 9.895748e-001, $n\mu_1$ = 2.857143e+003, $n\mu_2$ = 2.857143e+003, $n\mu_3$ = 2.857143e+003, $n\mu_0$ = 1.185108e+000, $n\mu_4$ = 2.857143e+003, $n\mu_5$ = 2.857143e+003 and $n\mu_6$ = 2.857143e+003.

Since $|op_0 - np_0| = 0$, therefore, $\mu_i^o \leftarrow o\mu_i$ $(0 \leq i \leq 6)$, $p_0^o \leftarrow op_0$ and stop. Thus, the proposed algorithm delivers μ_0^o =3.020224e - 001, μ_1^o = 1.528239e+003, μ_2^o = 1.737797e+003, μ_3^o = 7.475083e+003, μ_4^o = 7.973422e+002, μ_5^o = 3.691399e+003 and μ_6^o = 5.315615e+003 as the following optimal solution to $\mathbf{Q_1}$.

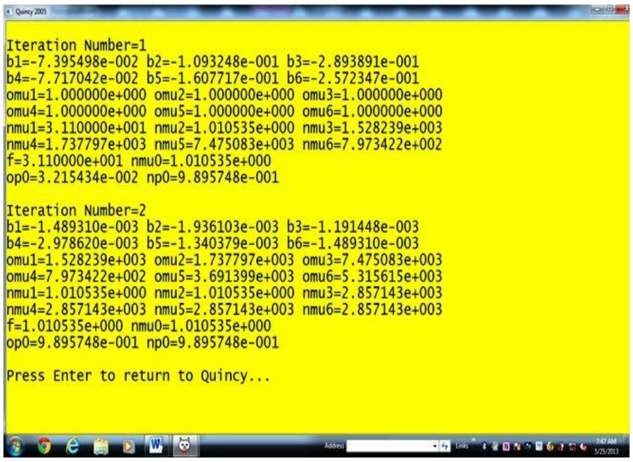


Figure 5b: Solution to Q_1

The state probabilities at $(\mu_0^o, \mu_1^o, \mu_2^o, \mu_3^o, \mu_4^o, \mu_5^o, \mu_6^o)$ are given by $(p_0^o, p_1^o, p_2^o, p_3^o, p_4^o, p_5^o, p_6^o)$ = (9.895748e-001, 7.966077e-004, 1.177594e-003, 3.117161e-003, 8.312429e-004, 1.731756e-003, 2.770810e-003). The state probabilities at optimality imply that the reliability a bus is observed by one PMU, two PMUs, three PMUs and more than three PMUs, respectively, are 9.895748e-001, 9.998913e-001, 9.999989e-001 and 1.

At optimality, we get $BORI_1 = BORI_2 = BORI_3 = BORI_6 = BORI_8 = BORI_{10} = BORI_{11} = BORI_{12} = BORI_{13} = BORI_{14}$ = 0.98957483, $BORI_5 = BORI_7 = BORI_9$ = 0.999891315830471, $BORI_4$ = 0.999998866949056 and SORI = 0.900210971146585.

5.2. PMU Placement Examples

Example1: Given that μ_0^o = 0.989575 and μ_i^o ($1 \le i \le 6$) = 2.857143e+003, it is targeted to raise the reliability of observability of complete system to 0.99 at least. So, $\rho = 0.99^{1/14}$ or 0.999282376486922. Then, the mathematical formulation of the PMU placement problem for the standard IEEE 14-Bus system is as follows.

Q₂: $\text{Min } Z = \sum_{i=1}^{14} x_i$ s.t. $1 - 0.01042517^{f_i} \ge 0.999282376486922$ $(1 \le i \le 14)$, x_i=0/1 ($1 \le i \le 14$)

, where

$f_1 = x_1 + x_2 + x_5$,
$f_2 = x_2 + x_1 + x_3 + x_4 + x_5$,
$f_3 = x_3 + x_2 + x_4$,
$f_4 = x_4 + x_3 + x_2 + x_5 + x_7 + x_9$,
$f_5 = x_5 + x_2 + x_4 + x_1 + x_6$,
$f_6 = x_6 + x_{11} + x_{12} + x_{13} + x_5$,
$f_7 = x_7 + x_4 + x_8 + x_9$,
$f_8 = x_8 + x_7$,
$f_9 = x_9 + x_7 + x_4 + x_{10} + x_{14}$,
$f_{10} = x_{10} + x_9 + x_{11}$,
$f_{11} = x_{11} + x_{10} + x_6$,
$f_{12} = x_{12} + x_6 + x_{13}$,
$f_{13} = x_{13} + x_{12} + x_6 + x_{14}$
and
$f_{14} = x_{14} + x_{13} + x_9$.

Program **Q_2** can be solved by using ILP Solver [14] after converting **Q_2** into a binary integer linear program (Figure 6).

The ILP Solver [14] delivers (x_1^o, x_2^o, x_3^o, x_4^o, x_5^o, x_6^o, x_7^o, x_8^o, x_9^o, x_{10}^o, x_{11}^o, x_{12}^o, x_{13}^o, x_{14}^o, Z^o) =(1, 1, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 0, 9) as solution to **Q_2** (Figure 7).

Add the constraint $x_1 + x_2 + x_4 + x_6 + x_7 + x_8 + x_9 + x_{11} + x_{13} + x_{14} \le 8$ to obtain (x_1^o, x_2^o, x_3^o, x_4^o, x_5^o, x_6^o, x_7^o, x_8^o, x_9^o, x_{10}^o, x_{11}^o, x_{12}^o, x_{13}^o, x_{14}^o, Z^o) = (1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0, 9) as another solution to **Q_2**. Continuing in this fashion, we obtain all the eight solutions to Q_2 given by (x_1^o, x_2^o, x_3^o, x_4^o, x_5^o, x_6^o, x_7^o, x_8^o, x_9^o, x_{10}^o, x_{11}^o, x_{12}^o, x_{13}^o, x_{14}^o, Z^o) ∈ { (1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0, 9), (1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0, 9), (0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1, 0, 9), **(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 0, 9),** (1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 0, 9), (1, 1, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 0, 9), (0, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 9), **(0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 9) }**. The boldfaced solutions correspond to the best SORI value i.e. 0.999128557654907.

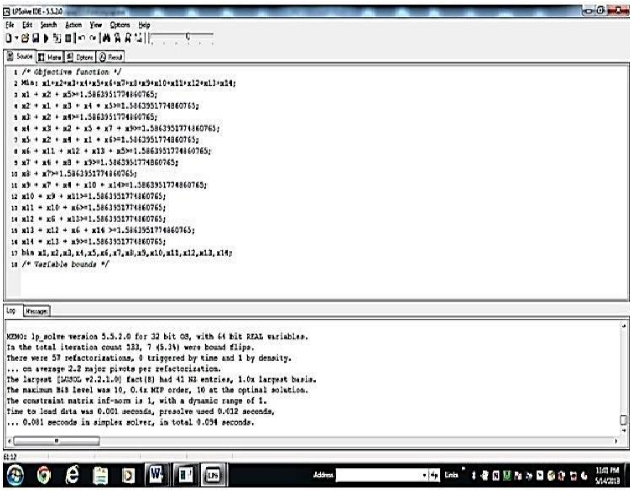


Figure 6: Binary Integer Linear Program from Q_2

At the first boldfaced solution, the values of $BORI_i$ ($1 \le i \le 14$) are $BORI_i$ = 0.999998866949056 (i = 2, 6), $BORI_4$ = 0.999999999876855, $BORI_j$ = 0.999999988187751(j = 5, 7, 9) and $BORI_k$ = 0.999891315830471 (k = 1, 3, 8, 10-14). At the next boldfaced solution, the values of $BORI_i$ ($1 \le i \le 14$) are $BORI_i$ = 0.999998866949056 (i = 2, 9), $BORI_4$ = 0.999999999876855 and $BORI_j$ = 0.999891315830471 (j = 1, 3, 5-8, 10-14).

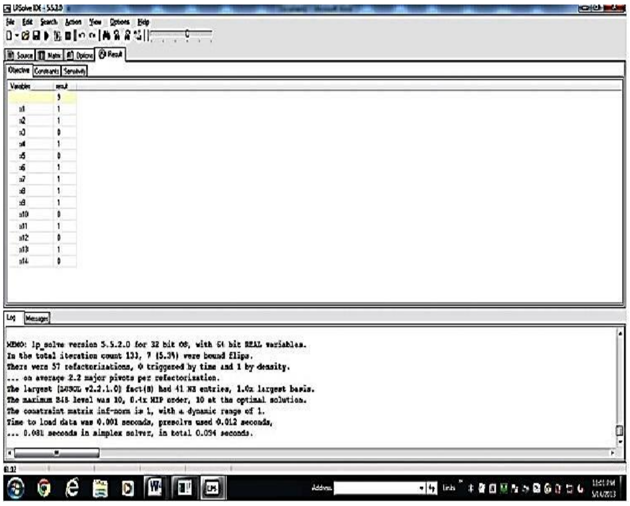


Figure 7: Solution to Q_2

It suggests us to place only *nine* PMUs either on the set of buses numbered 2, 4-10 and 13 or on the set of buses numbered 2, 4-9, 11 and 13, given that the optimal repair rates of PMU components are μ_i^o ($1 \le i \le 6$) = 2.857143e+003 and the targeted reliability of observability of the system is 0.99. **It may be interesting to note that the SORI value has improved from 0.900210971146585 to**

0.999128557654907. This improvement is significant.

Example 2: We now give an example covering critical buses. Buses numbered 4-7 and 9 connected to power generation units in the IEEE 14-Bus system have been considered as critical buses. Suppose that the reliability of observability of every critical bus is required to be 0.999999 at least and that the reliability of every other bus is required to be 0.989575 at least. The mathematical formulation of the PMU placement problem is as follows.

Q_3: $\text{Min } Z = \sum_{i=1}^{n} x_i$ s.t. $1.0 - 0.01042517^{f_i} \geq 0.989575 \quad (1 \leq i \leq 3), 1 - 0.01042517^{f_j} \geq 0.999999 \quad (4 \leq j \leq 7), \quad 1 - 0.01042517^{f_8} \geq 0.989575, 1 - 0.01042517^{f_9} \geq 0.999999, \; 1 - 0.01042517^{f_k} \geq 0.989575 \quad (10 \leq k \leq 14), \; x_i = 0/1 \; (1 \leq i \leq 14)$

, where f_i $(1 \leq i \leq 14)$ have their usual meanings.

Q_3 is converted into a binary integer linear program (Figure 8) to obtain its solution.

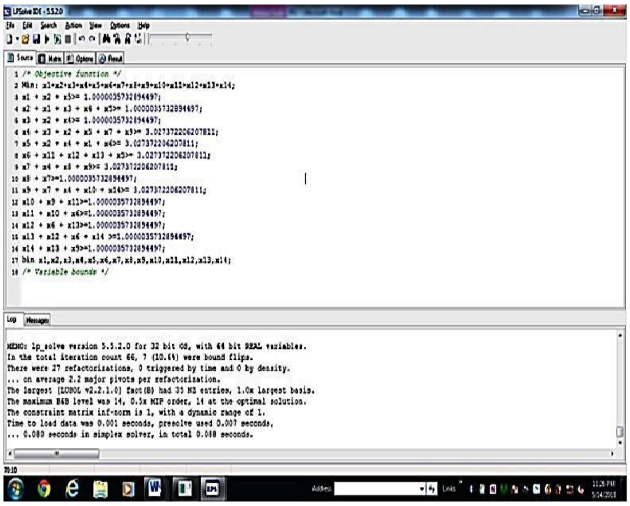


Figure 8: Binary Integer Program from Q_3

The ILP Solver [14] delivers (x_1^o, x_2^o, x_3^o, x_4^o, x_5^o, x_6^o, x_7^o, x_8^o, x_9^o, x_{10}^o, x_{11}^o, x_{12}^o, x_{13}^o, x_{14}^o, Z^o) = (0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 10) (Figure 9).

Remaining solutions to Q_3 have been found in usual way. All the solutions to Q_3 are given by (x_1^o, x_2^o, x_3^o,x_4^o, x_5^o, x_6^o, x_7^o, x_8^o, x_9^o, x_{10}^o, x_{11}^o, x_{12}^o, x_{13}^o, x_{14}^o, Z^o) ∈ { **(0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 10)** , (0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 1, 10), **(0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 10) }.** The boldfaced solutions correspond to the highest SORI value. The reliability of observability of complete power system is 0.999344627866809. At optimality, the values of $BORI_1$, $BORI_2$, $BORI_3$, $BORI_4$, $BORI_5$, $BORI_6$, $BORI_7$, $BORI_8$, $BORI_9$, $BORI_{10}$, $BORI_{11}$, $BORI_{12}$, $BORI_{13}$, $BORI_{14}$, respectively, are 0.999891315830471, 0.999998866949056, 0.999891315830471, 0.999999999876855, 0.999999988187751, 0.999999988187751, 0.999999988187751, 0.999891315830471, 0.999999988187751, 0.999891315830471, 0.999891315830471, 0.999891315830471, 0.999998866949056 and 0.999998866949056.

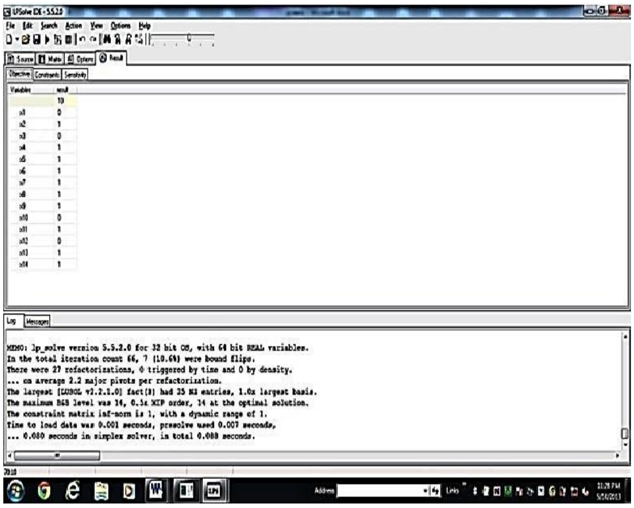


Figure 9: A Solution to Q_3

6. CONCLUSIONS

1. This paper advances the state-of-the-art of optimal placement of PMUs on power systems.

2. The reliability of observability of a power system depends on the reliability of observability of each bus in the power system. Further, the reliability of observability of a bus depends on the reliability of PMUs observing the bus. It incurs a fixed cost to repair a component of PMU and the reliability of observability of a bus depends on repair rates of components of PMUs observing it. It concludes that the reliability of observability of a complete power system depends on repair rates of various components of each of the PMUs installed on a given power system.

3. To the best of the author's knowledge, this paper stands first to have presented the prescriptive model of a PMU and to have dealt with the repair-rate-linked PMU placement. A prescriptive model of a PMU system has been introduced. An algorithm to determine the optimal repair rates of various components of a PMU has been developed.

4. Prescribed optimal repair rates of the components of a PMU may not necessarily give a satisfactory level of reliability of observability of a given power system. This paper determines repair-rate linked optimal number of PMUs and their candidate locations to attain a targeted level of reliability of observability of a complete power system.

5. To find the best of multiple solutions to the PMU placement problem, we have introduced two new indices that are altogether different from those used when failure and repair of components of a PMU are not modelled.

6. The work embodied in this paper has been enriched by giving illustrative examples.

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