A Novel Sample Shifting Technique for Sinusoidal Steady State Solution of R-L-C Circuit

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Abstract: *In this work, an attempt has been made to find out the steady state solution of R-L-C circuit for sinusoidal input using a state-of-the-art sample shifting technique (SST) with lesser computational burden in discrete time domain. The basic integro-differential equations involved in different R-L-C circuits have been solved with the above SST. The samples of the output current waveform are evaluated for each of the circuits from the samples of input voltage and the values of the circuit parameters. The samples of current waveform are also evaluated using conventional technique for the solution of current signal under steady state. A pretty good matching of the samples of output current in MATLAB simulation for both the techniques justifies this proposed SST. Microcontroller-based experimental validation of the method is also discussed.*

Key words: *Sample Shifting Technique, Circuit Analysis, Sampled Analysis.*

1. Introduction

There are lots of age-old conventional techniques to solve the R-L-C circuit with sinusoidal input either in the continuous or in the discrete domain. Several mathematical models and equations are well established [1,2] in order to solve them. More and more techniques are evolved to have a better conceptualization, lesser computational burden within acceptable error limit [3-5]. Moreover approximations of these mathematical solutions are being made in order to implement them in real world situations.

In these methods, several real world based approximation, considering the computational complexity, iteration time as well as error, are being continuously evolved [6]. For example methods like RK, NR, GS, Wavelet, Fuzzy logic are the result of this evolution [4,5,7]. A deep insight for the implementation of these methods reveals different types of processors like DSP, PC, FPGA etc. are suitable to find out the ultimate solutions. But to find out the solution with ordinary processor or microcontroller has yet to be studied.

The authors, in this attempt, have tried to propose and use a new technique to find out the steady state solution with lesser computational burden and this technique is implemented with the help of an ordinary microcontroller. This steady state solution is essential for frequency domain characterization of the system to sinusoidal inputs. As this method deals with the sinusoidal steady state solution of R-L-C circuits whose equations are basically integro-differential in nature, it is required, by the proposed method, to modify them in an all integral form. Now the integration of a sinusoidal function involves the shifting of original function by a quarter cycle, the required integration of the system equations is implemented by our state-of-the-art sample shifting technique (SST). The samples of the output current waveform are evaluated from the samples of input voltage and the values of the circuit parameters. The samples of current waveform are also evaluated using conventional equation for the solution of current signal under steady state. The proposed method is supported by the simulation of a numerical example and compared with the steady-state solution of standard methods. A simple experiment is also performed to show the validity of the proposed method.

2. Materials and Methods

2.1. *Sample Shifting Technique* (SST)

Shifting of any sinusoid by any angle (or equivalent time) only shifts the time of occurrence of instantaneous amplitudes of the sinusoid by that angle. Hence, in a digital acquisition system, a cosine wave data can be generated from the acquired data of sine wave by simply shifting the data set by 90^o. Such conversion of sinusoid by shifting is much simpler than using sine to cosine conversion formula. For example, if the original sine wave is sampled at 1^o interval, i.e., if there are 361 samples over a full cycle, then its cosine wave data can be generated by rearranging the original samples as follows:
This shifting utilizes the fundamental nature of sine and cosine waveforms where a zero value in a sine wave at 0^o corresponds to the zero value at 90^o of a cosine wave (for lagging type). This 90^o difference

continues till 270° of sine wave. But the values from 270° to 360° of the sine wave correspond to the values from 0° to 90° of the cosine wave. This is illustrated in Table 1.

Table 1
Sample Shifting Principle

Original Wave		Converted Wave		
Type	sample numbers	Type	Shifted sample numbers	
Sine	1	Cosine	91	$(N\text{-}1)/4 + 1$
	2		92	$(N\text{-}1)/4 + 2$
	3		93	$(N\text{-}1)/4 + 3$
	….		…	…..
	270		360	$(N\text{-}1)/4 + 270$
	271		361	$(N\text{-}1)/4 + 271$
	271		1	$271\text{-}3(N\text{-}1)/4$
	…..		….	….
	361		91	$361\text{-}3(N\text{-}1)/4$

As shown in the table, the 1st sample i.e. $(0^o)^{th}$ sample is to be placed at 91st $(90^o)^{th}$ position, the 2nd one at 92nd position and so on upto the 271th one at 361th position, while the 271st is to be placed at 1st position, the 272nd one at 2nd position and so on upto the 361th one at 91th position.
Pictorial presentation of this shifting is shown in fig.1 where the portions A′-B′-C′-D′-E′ on the shifted wave are the corresponding portions of A-B-C-D-E of the original wave. That is, the segment D′ to E′ at the starting portion of the shifted waveform is an exact replica of the segment D to E of the original one.

2.2. Integration *Evaluation by Sample Shifting Technique*

In general, let a sinusoidal function $f(t) = a \sin \omega t$ is sampled by $4n$ number of times over a full cycle period $[\,0, 2\pi\,]$ starting from its zero crossing instant. In order to represent the function $f(t)$ in digital domain, these $4n$ samples can be designated, as

$$[f(t)]_{\text{Samples}} = [a_0,\ a_1,\ a_2,\ \dots,\ a_{n-1},\ a_n,\ a_{n+1},\dots,\ a_{2n-1},\ a_{2n},\ a_{2n+1},\dots,a_{3n-1},\ a_{3n},\ a_{3n+1},\dots,\ a_{4n-1}] \quad (1)$$

where n is the number of samples per quarter cycle.

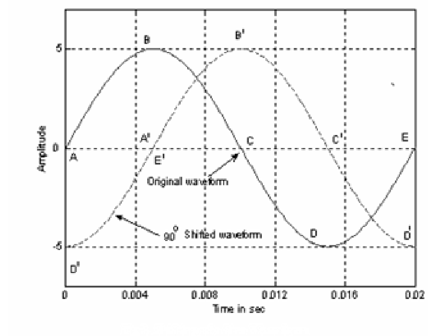


Fig. 1. Illustration of Sample Shifting Technique.

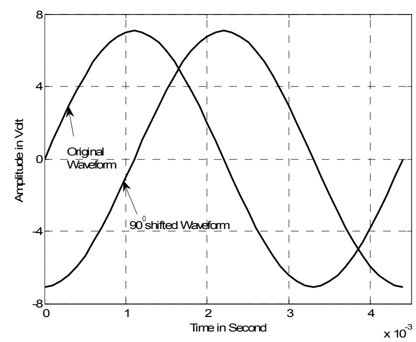


Fig. 2. A sinusoidal function and its first integrated form.

The first integration of the function is

$$\int f(t)\mathrm{d}t = \frac{a}{\omega}\sin(\omega t - \frac{\pi}{2})$$

It is seen from above equation that the integrated function is simply a phase shift of the original function by $\pi/2$ with its modified amplitude $1/\omega$. The graphical representation of the original function and its integrated form are shown in fig. 2. As the point of interest is to be within $[0, 2\pi]$ i.e. a full cycle, a deep insight of fig. 2 reveals that the samples from $3\pi/2$ to 2π of original function, surpass the limit of interest in the integrated function. Hence, to have a feel of the samples of the integrated function within the limit, the samples of surpassed zone be brought back and fit into the zone from 0 to $\pi/2$. This is nothing but the basis of the sample shifting principle which is to be adopted here for its practical realization.

Hence, after rearranging, the samples of the integrated function take the form like,

$$\left[\int f(t)\mathrm{d}t\right] = \frac{1}{\omega}\begin{bmatrix} a_{3n}, a_{3n+1}, ..., a_{4n-1}, a_{4n} (= a_0), a_1, ..., a_{n-1}, a_n, \\ a_{n+1}, ..., a_{2n-1}, a_{2n}, a_{2n+1}, ..., a_{3n-1} \end{bmatrix} \quad (3)$$

Similarly, a double integration of the function $f(t)$ introduces a phase shift by π to the original function with modified magnitudes which is shown in fig. 3. Here the surpassed zone is from π to 2π and this has to be brought back and fit into the segments between 0 to π.

Hence, the samples of the doubly integrated function, with the same argument, are generated as,

$$\left[\iint f(t)\mathrm{d}t\right]_{\text{Samples}} = \frac{1}{\omega^2}\begin{bmatrix} a_{2n}, a_{2n+1}, ..., a_{3n-1}, a_{3n}, a_{3n+1}, \\ ..., a_{4n-1}, ..., a_{4n} (=a_0), a_1, a_2, \\ ..., a_{n-1}, a_n, a_{n+1}, ..., a_{2n-1} \end{bmatrix} \quad (4)$$

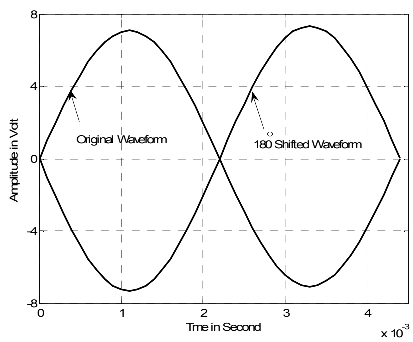


Fig. 3. A sinusoidal function and its second integrated form.

In fact, proceeding in this way, it is possible to derive the samples of any order of integration of a sinusoidal function from the original set of samples.

3. Case Studies

3.1. *R-L-C Series Circuit*

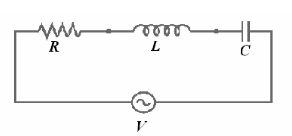


Fig. 4. Typical RLC series circuit fed from sinusoidal source

The basic differential equation describing a R-L-C circuit is given by,

$$v(t) = R\, i(t) + L\,\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{C}\int i(t)\mathrm{d}t$$

$$\text{or, } \int v(t)\,\mathrm{d}t = R\int i(t)\,\mathrm{d}t + L\, i(t) + \frac{1}{C}\iint i(t)\mathrm{d}t \quad (5)$$

Let the voltage and current samples are given by,

$$\left.\begin{array}{l} v(t) = \left[v_0, v_1, ..., v_n, ..., v_{2n}, ..., v_{3n}, ..., v_{4n} (=v_0)\right] \\ \text{and} \\ i(t) = \left[i_0, i_1, ..., i_n, ..., i_{2n}, ..., i_{3n}, ..., i_{4n} (=i_0)\right] \end{array}\right\} \quad (6)$$

Now by employing the proposed sample shifting technique for integration, in the sample domain, equation (5) can be rewritten with the zero initial condition as,

$$\frac{1}{\omega}\mathrm{v}_{\frac{-\pi}{2}}(t) = \frac{R}{\omega}i_{\frac{-\pi}{2}}(t) + Li(t) + \frac{1}{\omega^2 C}i_{-\pi}(t)$$

$$\text{or, } \mathrm{v}_{\frac{-\pi}{2}}(t) = Ri_{\frac{-\pi}{2}}(t) + \omega Li(t) + \frac{1}{\omega C}i_{-\pi}(t) \quad (7)$$

Thus with equation (3), (4), and (6), equation (7) can be written as,

$$\begin{bmatrix} v_{3n} \\ v_{3n+1} \\ \vdots \\ v_{4n-1} \\ v_{4n}(=v_0) \\ v_1 \\ \vdots \\ v_{n-1} \\ v_n \\ v_{n+1} \\ \vdots \\ v_{2n-1} \\ v_{2n} \\ v_{2n+1} \\ \vdots \\ v_{3n-1} \end{bmatrix} = R\begin{bmatrix} i_{3n} \\ i_{3n+1} \\ \vdots \\ i_{4n-1} \\ i_{4n}(=i_0) \\ i_1 \\ \vdots \\ i_{n-1} \\ i_n \\ i_{n+1} \\ \vdots \\ i_{2n-1} \\ i_{2n} \\ i_{2n+1} \\ \vdots \\ i_{3n-1} \end{bmatrix} + \omega L\begin{bmatrix} i_0 \\ i_1 \\ \vdots \\ i_{n-1} \\ i_n \\ i_{n+1} \\ \vdots \\ i_{2n-1} \\ i_{2n} \\ i_{2n+1} \\ \vdots \\ i_{3n-1} \\ i_{3n} \\ i_{3n+1} \\ \vdots \\ i_{4n-1} \end{bmatrix} + \frac{1}{\omega C}\begin{bmatrix} i_{2n} \\ i_{2n+1} \\ \vdots \\ i_{3n-1} \\ i_{3n} \\ i_{3n+1} \\ \vdots \\ i_{4n-1} \\ i_{4n}(=i_0) \\ i_1 \\ \vdots \\ i_{n-1} \\ i_n \\ i_{n+1} \\ \vdots \\ i_{2n-1} \end{bmatrix} \quad (8)$$

The above equation can be simplified and arranged in a matrix form as,

$$\begin{bmatrix} v_0 & v_1 & .. & v_{n-1} \\ v_n & v_{n+1} & .. & v_{2n-1} \\ v_{2n} & v_{2n+1} & .. & v_{3n-1} \\ v_{3n} & v_{3n+1} & .. & v_{4n-1} \end{bmatrix} =$$

$$\begin{bmatrix} R & \omega L & 0 & 1/\omega C \\ 1/\omega C & R & \omega L & 0 \\ 0 & 1/\omega C & R & \omega L \\ \omega L & 0 & 1/\omega C & R \end{bmatrix}\begin{bmatrix} i_0 & i_1 & .. & i_{n-1} \\ i_n & i_{n+1} & .. & i_{2n-1} \\ i_{2n} & i_{2n+1} & .. & i_{3n-1} \\ i_{3n} & i_{3n+1} & .. & i_{4n-1} \end{bmatrix}$$

(9)

$$[I]=[Z]^{-1}_{RLC}\,[V] \qquad (10)$$

where, $[V]$, $[I]$, and $[Z]_{RLC}$ are respectively the voltage current and impedance matrix of equation (9). The characteristic features of arranging the samples are made in such a manner that the voltage and current matrices of equation (9) are always of the order of (4xn), where 4 rows are interpreted as each of the 90 degree segments over a line cycle period and n columns indicate number of samples within each of these segments. The impedance matrix is always of the order of (4x4) and is circulant in nature. Its inverse can apriori be calculated for the known values of impedance parameters. With these values equation (10) can then be used in finding out the samples of the steady state current output signal (with zero initial condition) for various values of voltage inputs.

On the other hand by conventional method, the solution for current (considering only sinusoidal part) is given by

$$i(t) = A_1 \sin(\omega t + \theta_1)$$

where $A_1 = \omega CV\sqrt{(1-\omega^2 CL)^2 + (\omega CR)^2}$ and

$$\theta_1 = \tan^{-1}\{(1-\omega^2 CL)/\omega CR)\}$$

3.2. *R-L-C Series-Parallel Circuit*

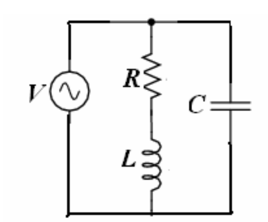


Fig. 5. Typical RLC series-parallel circuit fed from sinusoidal source.

The differential equation describing an R-L-C series-parallel circuit is given by,

$$\iint i(t)\mathrm{d}t + \frac{L}{R}\int i(t)\,\mathrm{d}t = \frac{1}{R}\iint v(t)\mathrm{d}t + C\int v(t)\,\mathrm{d}t + \frac{LC}{R}v(t) \qquad (12)$$

The equation can be written in the sampled domain as,

$$\frac{1}{\omega^2}i_{-\pi}(t) + \frac{L}{\omega R}i_{\frac{-\pi}{2}}(t) = \frac{1}{\omega^2 R}v_{-\pi}(t) + \frac{C}{\omega}v_{\frac{-\pi}{2}}(t) + \frac{LC}{R}v(t)$$

or, $$i_{-\pi}(t) + \frac{\omega L}{R}i_{\frac{-\pi}{2}}(t) = \frac{1}{R}v_{-\pi}(t) + \omega C v_{\frac{-\pi}{2}}(t) + \frac{\omega^2 LC}{R}v(t)$$

(13)

Thus with equation (3), (4), and (6), equation (13) can be written as,

$$\begin{bmatrix} i_{2n} \\ i_{2n+1} \\ \vdots \\ i_{3n-1} \\ i_{3n} \\ i_{3n+1} \\ \vdots \\ i_{4n-1} \\ i_{4n/0} \\ i_1 \\ \vdots \\ i_{n-1} \\ i_n \\ i_{n+1} \\ \vdots \\ i_{2n-1} \end{bmatrix} + \frac{\omega L}{R}\begin{bmatrix} i_{3n} \\ i_{3n+1} \\ \vdots \\ i_{4n-1} \\ i_{4n/0} \\ i_1 \\ \vdots \\ i_{n-1} \\ i_n \\ i_{n+1} \\ \vdots \\ i_{2n-1} \\ i_{2n} \\ i_{2n+1} \\ \vdots \\ i_{3n-1} \end{bmatrix} = \frac{1}{R}\begin{bmatrix} v_{2n} \\ v_{2n+1} \\ \vdots \\ v_{3n-1} \\ v_{3n} \\ v_{3n+1} \\ \vdots \\ v_{4n-1} \\ v_{4n/0} \\ v_1 \\ \vdots \\ v_{n-1} \\ v_n \\ v_{n+1} \\ \vdots \\ v_{2n-1} \end{bmatrix} + \omega C\begin{bmatrix} v_{3n} \\ v_{3n+1} \\ \vdots \\ v_{4n-1} \\ v_{4n/0} \\ v_1 \\ \vdots \\ v_{n-1} \\ v_n \\ v_{n+1} \\ \vdots \\ v_{2n+1} \\ v_{2n} \\ v_{2n+1} \\ \vdots \\ v_{3n-1} \end{bmatrix} + \frac{\omega^2 LC}{R}\begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_{n-1} \\ v_n \\ v_{n+1} \\ \vdots \\ v_{2n-1} \\ v_{2n} \\ v_{2n+1} \\ \vdots \\ v_{3n-1} \\ v_{3n} \\ v_{3n+1} \\ \vdots \\ v_{4n-1} \end{bmatrix}$$

(14)

(11)

This equation can be simplified and arranged in a matrix form as,

$$\begin{bmatrix} i_0 & i_1 & \cdots & i_{n-1} \\ i_n & i_{n+1} & \cdots & i_{2n-1} \\ i_{2n} & i_{2n+1} & \cdots & i_{3n-1} \\ i_{3n} & i_{3n+1} & \cdots & i_{4n-1} \end{bmatrix} = \begin{bmatrix} 1 & \omega L/R & 0 & 0 \\ 0 & 1 & \omega L/R & 0 \\ 0 & 0 & 1 & \omega L/R \\ \omega L/R & 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$\times \begin{bmatrix} 1/R & \omega C & \omega^2 LC/R & 0 \\ 0 & 1/R & \omega C & \omega^2 LC/R \\ \omega^2 LC/R & 0 & 1/R & \omega C \\ \omega C & \omega^2 LC/R & 0 & 1/R \end{bmatrix}\begin{bmatrix} v_0 & v_1 & \cdots & v_{n-1} \\ v_n & v_{n+1} & \cdots & v_{2n-1} \\ v_{2n} & v_{2n+1} & \cdots & v_{3n-1} \\ v_{3n} & v_{3n+1} & \cdots & v_{4n-1} \end{bmatrix} \qquad (15)$$

Again by conventional method, the solution for current (considering only sinusoidal part) is given by,

$$i(t) = A_2 \sin(\omega t + \theta_2)$$

where

$$A_2 = \frac{V\left(\sqrt{\omega^2 C^2 (R^2+\omega^2 L^2) - 2\omega^2 LC(R^2+\omega^2 L^2) + (R^2+\omega^2 L^2)}\right)}{\left(R^2+\omega^2 L^2\right)}$$

and $\theta_2 = \tan^{-1}\left\{\left(\omega C(R^2+\omega^2 L^2) - \omega L\right)/R\right\}$

(16)

3.3. *R-L-C Parallel Circuit*

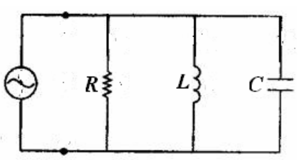


Fig. 6. Typical RL parallel circuit fed from sinusoidal source.

The differential equation describing an R-L-C parallel circuit is given by,

$$\int i(t)\,\mathrm{d}t = \frac{1}{R}\int v(t)\,\mathrm{d}t + \frac{1}{L}\iint v(t)\mathrm{d}t + Cv(t) \tag{17}$$

The equation can be written in the sampled domain as,

$$\frac{1}{\omega} i_{\frac{-\pi}{2}}(t) = \frac{1}{\omega R} v_{\frac{-\pi}{2}}(t) + \frac{1}{\omega^2 L} v_{-\pi}(t) + Cv(t)$$

or,
$$i_{\frac{-\pi}{2}}(t) = \frac{1}{R} v_{\frac{-\pi}{2}}(t) + \frac{1}{\omega L} v_{-\pi}(t) + \omega Cv(t) \tag{18}$$

Thus with equation (3), (4), and (6), equation (18) can be written as,

$$\begin{bmatrix} i_{3n} \\ i_{3n+1} \\ \vdots \\ i_{4n-1} \\ i_{4n}(=i_0) \\ i_1 \\ \vdots \\ i_{n-1} \\ i_n \\ i_{n+1} \\ \vdots \\ i_{2n-1} \\ i_{2n} \\ i_{2n+1} \\ \vdots \\ i_{3n-1} \end{bmatrix} = \frac{1}{R}\begin{bmatrix} v_{3n} \\ v_{3n+1} \\ \vdots \\ v_{4n-1} \\ v_{4n}(=v_0) \\ v_1 \\ \vdots \\ v_{n-1} \\ v_n \\ v_{n+1} \\ \vdots \\ v_{2n+1} \\ v_{2n} \\ v_{2n+1} \\ \vdots \\ v_{3n-1} \end{bmatrix} + \omega C \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_{n-1} \\ v_n \\ v_{n+1} \\ \vdots \\ v_{2n-1} \\ v_{2n} \\ v_{2n+1} \\ \vdots \\ v_{3n-1} \\ v_{3n} \\ v_{3n+1} \\ \vdots \\ v_{4n-1} \end{bmatrix} + \frac{1}{\omega L}\begin{bmatrix} v_{2n} \\ v_{2n+1} \\ \vdots \\ v_{3n-1} \\ v_{3n} \\ v_{3n+1} \\ \vdots \\ v_{4n-1} \\ v_{4n}(=v_0) \\ v_1 \\ \vdots \\ v_{n-1} \\ v_n \\ v_{n+1} \\ \vdots \\ v_{2n-1} \end{bmatrix} \tag{19}$$

This equation can be simplified and arranged in a matrix form as,

$$\begin{bmatrix} i_0 & i_1 & .. & i_{n-1} \\ i_n & i_{n+1} & .. & i_{2n-1} \\ i_{2n} & i_{2n+1} & .. & i_{3n-1} \\ i_{3n} & i_{3n+1} & .. & i_{4n-1} \end{bmatrix} = \tag{20}$$

$$\begin{bmatrix} 1/R & \omega C & 0 & 1/\omega L \\ 1/\omega L & 1/R & \omega C & 0 \\ 0 & 1/\omega L & 1/R & \omega C \\ \omega C & 0 & 1/\omega L & 1/R \end{bmatrix}\begin{bmatrix} v_0 & v_1 & .. & v_{n-1} \\ v_n & v_{n+1} & .. & v_{2n-1} \\ v_{2n} & v_{2n+1} & .. & v_{3n-1} \\ v_{3n} & v_{3n+1} & .. & v_{4n-1} \end{bmatrix}$$

Again by conventional method, the solution for current (considering only sinusoidal part) is given by,

$$i(t) = \frac{\omega V}{LR}\left[A_3 \sin(\omega t + \theta_3)\right]$$

where $A_3 = \left(\sqrt{(RLC\omega^2 - R)^2 + \omega^2 L^2}\right) / \omega^2$ and

$$\theta_3 = \tan^{-1}\left\{\left(\omega^2 RLC - R\right) / \omega L\right\} \tag{21}$$

4. Experimentation

4.1. *Simulation*

Matlab based programs are used to simulate the solution for R-L-C circuits following equations (10), (15) and (20) respectively. The program algorithms given below provide easy understanding of the logics for calculation of current samples, phase difference between input and output as well as output amplitude. These simulated samples using SST are compared with the samples obtained from the simulation using conventional formulae for steady state solution for series R-L-C circuit and for parallel R-L-C circuit respectively by equations (11), (16) and (21) for an input $v(t) = v\sin(\omega t)$.

Algorithm for R-L-C circuit

1. Define amplitude v, R, L, C freq and nsample variables.
2. Generate nsample no. of sample values for the sinusoidal voltage signal following the equation $v(t) = v\sin(\omega t)$ and store them in an array of v.
3. Generate impedance matrix Z of (9) with the above parameters store them in an array of mat.
4. Calculate imat from the inverse of mat.
5. Define no_of_row i.e. nrow as 4.
6. Evaluate no_of_column i.e. ncol from nsamples/nrow .
7. Initialise row_counter with 1.
8. Initialise col_counter with 1.
9. Initialise ((row_counter-1)*ncol + col_counter)th element of current i with 0.

10. Initialise element_counter with 1.
11. Multiply (row_counter, element_counter)th element of imat with ((element_counter-1)*ncol + col_counter)th element of *v***.**
12. Add the multiplication result with ((row_counter-1)*ncol + col_counter)th element of *i*.
13. Store the addition result in the ((row_counter-1)*ncol + col_counter)th element of *i*.
14. Increase element_counter by 1.
15. Repeat from **step 11** for element_counter upto 4.
16. Increase col_counter by 1.
17. Repeat from **step 9** for col_counter upto ncol.

Table 2
Voltage and current samples

Samples of				Samples of			
No.	V	i_{conv}	i_{prop}	No.	V	i_{conv}	i_{prop}
1	0	0.0629	0.0629	11	0	-0.0629	-0.0629
2	30.9017	0.0598	0.0598	12	-30.9017	-0.0598	-0.0598
3	58.7785	0.0509	0.0509	13	-58.7785	-0.0509	-0.0509
4	80.9017	0.0369	0.0369	14	-80.9017	-0.0369	-0.0369
5	95.1057	0.0195	0.0195	15	-95.1057	-0.0195	-0.0195
6	100	0.00003	0.00003	16	-100	-0.00003	-0.00003
7	95.1057	-0.0194	-0.0194	17	-95.1057	0.0194	0.0194
8	80.9017	-0.0369	-0.0369	18	-80.9017	0.0369	0.0369
9	58.7785	-0.0508	-0.0508	19	-58.7785	0.0508	0.0508
10	30.9017	-0.0598	-0.0598	20	-30.9017	0.0598	0.0598

Table 3
Voltage and current samples

Samples of				Samples of			
No.	V	i_{conv}	i_{prop}	No.	V	i_{conv}	i_{prop}
1	0	-44.9849	-44.9849	11	0	44.9849	44.9849
2	30.9017	-20.6279	-20.6279	12	-30.9017	20.6279	20.6279
3	58.7785	5.7481	5.7481	13	-58.7785	-5.7481	-5.7481
4	80.9017	31.5616	31.5616	14	-80.9017	-31.5616	-31.5616
5	95.1057	54.2855	54.2855	15	-95.1057	-54.2855	-54.2855
6	100	71.6957	71.6957	16	-100	-71.6957	-71.6957
7	95.1057	82.0877	82.0877	17	-95.1057	-82.0877	-82.0877
8	80.9017	84.4445	84.4445	18	-80.9017	-84.4445	-84.4445
9	58.7785	78.5352	78.5352	19	-58.7785	-78.5352	-78.5352
10	30.9017	64.9384	64.9384	20	-30.9017	-64.9384	-64.9384

18. Increase row_counter by 1.
19. Repeat from **step 8** for row_counter upto nrow.
20. Evaluate A_1 and θ_1 using above equations.
21. Evaluate samples for y_1 for the nsample element.
22. Plot the samples of y_1 and *i*.

4.2. *Numerical Example*

4.2.1 *For R-L-C series circuit*

Let the voltage be of $v(t) = v \sin \omega t$. For R=22 Ohm, L= 2mH, C=2μF, Frequency = 50 Hz, Amplitude *v* =100 V and nsample = 20. The voltage and current samples are then obtained following equations (10) and (11), as in the Table 2.

The output current samples are obtained by proposed and conventional method following equations (10) and (11) respectively. The peak value of the current wave, by the proposed method, is estimated as the maximum value from the samples of current matrix. The phase angle of the current wave is also estimated from these samples by considering the angle corresponding to the first zero sample value. For the case of non-zero sample, this angle is approximated from the intersecting point of zero line and the line connecting two consecutive samples on either side of the zero line. In this case, as seen from the above table, the phase angle lies between 5th and 6th sample which is 77°. Fig. 6 shows the exact matching of the plots of the samples of the output current waveform in both the proposed and conventional simulation methods.

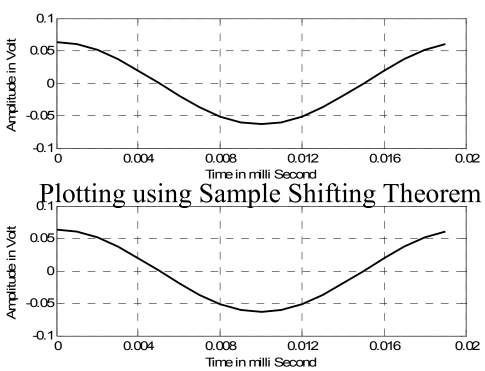
Considering the same applied voltage for above impedance parameters the voltage and current samples, following equations (15) and (16), are given as per the Table-3.

4.2.2. *For R-L-C series-parallel circuit*

Table 4

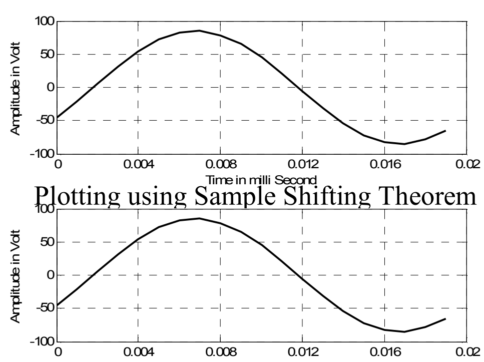
Voltage and current samples

Samples of				Samples of			
No.	V	i_{conv}	i_{prop}	No.	V	i_{conv}	i_{prop}
1	0	-159.0921	-159.0921	11	0	159.0921	159.0921
2	30.9017	-120.4039	-120.4039	12	-30.9017	120.4039	120.4039
3	58.7785	-69.9297	-69.9297	13	-58.7785	69.9297	69.9297
4	80.9017	-12.6103	-12.6103	14	-80.9017	12.6103	12.6103
5	95.1057	45.9435	45.9435	15	-95.1057	-45.9435	-45.9435
6	100	100	100	16	-100	-100	-100
7	95.1057	144.2678	144.2678	17	-95.1057	-144.2678	-144.2678
8	80.9017	174.4137	174.4137	18	-80.9017	-174.4137	-174.4137
9	58.7785	187.4867	187.4867	19	-58.7785	-187.4867	-187.4867
10	30.9017	182.2073	182.2073	20	-30.9017	-182.2073	-182.2073



Plotting using Conventional Method

Fig. 7. Plots for simulated results using equations (10) and (11).

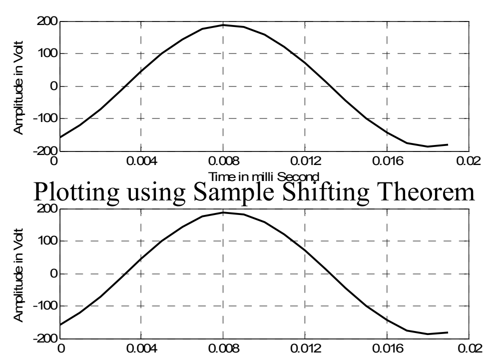


Plotting using Conventional Method

Fig. 8. Plots for simulated results using equations (15) and (16).

4.2.3. *For R-L-C parallel circuit*

Considering the same applied voltage for above impedance parameters the voltage and current samples, following equations (20) and (21), are given as per the Table-4.



Plotting using Conventional Method

Fig. 9. Plots for simulated results using equations (20) and (21).

4.2.4. *Experimental Validation*

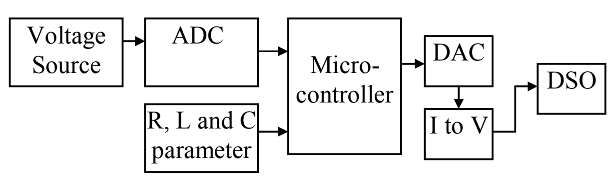


Fig. 10. Schematic diagram of experimental set-up.

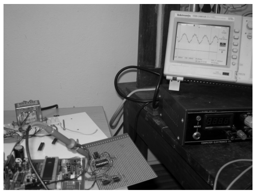


Fig. 11. Typical display of output waveform

The voltage signal is sampled at 1 kHz sampling frequency through ADC as per the experimental set-up shown in fig.10. Microcontroller collects the sample values for a full cycle period in its internal data memory. The elements of the inverse impedance matrix for series R-L-C circuit are calculated externally considering R=22 Ohm, L=2mH, C=2μF and then stored directly to the microcontroller memory. Microcontroller evaluates the required samples for the current signal from these sample values and the inverse of impedance matrix using equation (10). The evaluated samples for current are stored in another portion of the data memory. The analog representations of those evaluated samples are seen through a DSO, as shown in fig.11, with the help of DAC and I to V converters. Similarly, analog representations of the evaluated samples for R-L-C series-parallel circuit and R-L-C parallel circuit are also displayed to justify the proposed technique.

5. Conclusion

The uniqueness of this proposal is that the method of finding solution of R-L-C circuit is simplified. As the proposed technique utilizes a state-of the-art SST, this simplified technique involves only rearranging of the samples, and their multiplication and summation. This reduces the computation complexity of the processor but an ordinary microcontroller can be utilized to implement this in real world solution. The beauty of the proposed technique that the impedance matrix will always be of order 4x4, irrespective of the order of differential equation.

Another important feature of this technique is that for each higher order integral equation, the shifting will be $\pi/2$ more than its previous one. So the shifting angle will be confined to $p\pi/2$, with p=1 to q where q is the order of the equation and for every order of p=4r for r=1, 2, …, s are the same as that of original one. The only difference is in their amplitude which varies inversely with the order of the equation. The accuracy of the system largely depends on the accuracy of the shifting. As per the SST, the accurate shifting demands the number of samples over a cycle 'n' should be exactly divisible by 4. The sampling rate is to be such that 'n' should be an integral multiple of 4 i.e. 4n where n is the number of sample over a quarter cycle. The system determines only the steady state solutions of the circuit. The result of simulation for both the conventional and proposed technique shows no error for steady state solution. This justifies the proposed technique.

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