MULTISTAGE HYBRID EVOLUTIONARY COMPUTING BASED OPTIMAL PMU PLACEMENT FOR LARGE SCALE POWER GRID NETWORK

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Abstract— *The high pace rise in energy consumption across industrial-social horizon, the reliable and quality power provision have become inevitable need. Phasor measurement unit (PMU) is one of the most significant grid components that plays vital role in ensuring reliable power transmission and distribution. However, maintaining cost effective and efficient electrical grid network design have been the dominating motivations for researchers. The optimal PMU placement (OPP) in power system can not only ensure grid cost reduction, real-time monitoring and control, but can reduce the operational complexities and overheads significantly. This paper proposes a novel multistage hybrid evolutionary computing scheme for OPP solution. Our proposed model applied Adaptive Genetic Algorithm (AGA) for initial state point retrieval for OPP, which was then fed as input to the Pattern Search (PS) based PMU placement optimization. Our proposed AGA-PS scheme ensures OPP solution by retrieving minimum number of PMUs and its optimal location across grid network to make power system completely observable under varied cases such as single PMU loss and zero injection bus conditions. The simulation results with IEEE 14, IEEE 39, IEEE 118 and KPTCL 155 bus networks has exhibited that the proposed AGA-PS outperforms major existing approaches in terms of optimal OPP solutions. Performance with KPTCL 155 reveals that the AGA-PS can be useful for cost-effective large scale power grid design purpose.*

Keywords—PMU Placement, Evolutionary Computing, Adaptive Genetic Algorithm, Pattern Search, IEEE bus networks.

1. INTRODUCTION

In modern era, power system has become an inevitable part of human life. Typically, power system is defined as the collection of various electrical components to be used for generation, transmission and distribution of the electrical power. The power transmission system carries electrical power from the generation unit, while the distribution system has the responsibility to supply the electrical power to the nearest centers. Being a highly sensitive system, it comprises numerous protective components such as circuit breakers, relays, etc which play vital role in avoiding any fault probability across the system. In last few years, the advancement in technologies has motivated power industry to incorporate major transitional changes thus making enabling it more efficient and productive. The incorporation of renewable energy sources (RES) and the deregulation of the power system network has made prediction of the power flow pattern, highly intricate and even less accurate. On the other hand, the high pace rise in power demand has made present day electrical grid network to operate even under intemperately stressed situations. Measuring significant real-time operational parameters and their control can be of paramount significance to enhance power system reliability. To enable reliable and efficient power system, wide area monitoring and control system (WAMS) has emerged as a novel solution for electrical grid supervision and control. Among major advanced measuring devices phasor measurement units (PMUs) are considered as the vital component in power system that facilitate the consistent depiction of the real-time operation across the electrical grid network. It utilizes the synchronized voltage and current phasor measurements to present real time operation status of the grid network. PMU has been playing significance role in enabling real time and precise power system monitoring, analysis, and control [2]. In practice, PMU facilitates minimum Frequency measurement for 3-phase AC voltage and current waveforms. PMU measures the harmonics, the operating system state and identify or detect the fault as well as power system oscillations assessment. In wide-area interconnected systems, such real time measurements are synchronized by means of the time stamp facilitated by the Global Positioning System (GPS) [1]. The predominant ability of PMUs to facilitate synchronized current and voltage phasor measurements enables it to be the potential and most efficient alternative for electrical power system monitoring and control.

The power system turns out to be the smart grid if it functions automatically by employing smart meters , phasor measurements and integrating renewable sources like solar, wind and tidal. The GPS based communication systems are efficient enough to localize events to ensure swift and flexible process without affected by dynamic environmental conditions. The efficient implementation of these mechanisms can strengthen the smart grid to conserve energy, minimize operational cost as well as cost of establishment, enhanced power system reliability and quality [3]. PMU being one of the advanced smart metering system estimates real time phasor of the grid voltage and current phasors of all incident branches to that bus in the grid network. This is the matter of fact that the implementation of PMUs in grid network plays vital role in ensuring power reliability and quality [3]. However, enabling complete observability of the network while ensuring minimum number of PMUs in the power system can be of paramount significance as it can not only reduce the computational overheads, low complexities, swift process, but also reduced network cost. It can have numerous advantages for both the industries as well as consumers. To meet these requirements, a new research dimension has come into existence called Optimal PMU placement (OPP). The predominant objective of OPP is to estimate the minimum number of PMUs to make power system complete observable. Enabling power system complete observable by OPP can provide swift state estimation, even in a single iteration. The OPP in a power system to make power system complete observable can be derived as a constrained optimization problem. Recently, numerous researches have been made for OPP solution. A number of optimization scheme like Genetic algorithm (GA), Bacterial Foraging Optimization (BFO) algorithm, Particle Swarm Optimization (PSO) algorithm, Cuckoo search algorithm and artificial bee colony (ABC) algorithms have been applied to select the minimum number of PMUs to enable the power system complete observable and the respective optimal location across the grid network or bus architecture. However, majority of the traditional approaches suffer from local minima and convergence issue. Alleviating these issues can enable a system more precise, time and computationally efficient for real time applications.

To achieve OPP solution, an array of efforts has been made using mathematical optimization approaches [4–12]. Authors in [4–9] developed the integer linear programming (ILP) technique to resolve OPP problem. In [4–7] the emphasis was made on OPP solution retrieval to ensure complete network observability, while in [8, 9], authors emphasized on retrieving minimal PMU counts while considering channel limitation conditions. However, authors [9] could not address the issue of the PMU locations and the number of each PMU measuring channels. Furthermore, the key limitations of the ILP technique such as local minima was not take into consideration that confines it to deliver only one solution, while the might have multiple solutions. An optimization technique named branch and bound (B and B) was developed in [10, 28] where authors focused on placing PMU to make power system complete observable. Authors in [12] developed the cellular learning automata (CLA) for estimating the optimal number and the locations of PMUs to strengthen complete power system observability. However, the issue of the number of PMUs measuring channels could not be addressed. In general, mathematical optimization approaches often fail and reach local optimum that even becomes frequent with non-linear objectives and significantly large variables. In addition, these approaches are confined primarily for solving linear optimization problems. Interestingly, in function these techniques need gradient information and therefore it is infeasible to solve non-differentiable functions. The existing mathematical optimization techniques are of no significance for OPP optimization as it is inefficient in solving discrete optimization issue, which is common in OPP problem. In fact, mathematical or conventional optimization techniques employ mathematical approaches to estimate the best feasible solution where the constraints as well as the objective functions are linear with integer magnitudes of control variables. On the other hand, heuristic paradigms are typically problem dependent and are programmed to perform in the problem specific manner, where it intends to achieve the best solution on the basis of knowledge to either construct or enhance certain solutions. On the other hand, meta-heuristic optimization scheme are typically problem independent and can be applied for major optimization problems. In addition, these techniques are capable of avoiding the issue of local optimum and even achieve the global best solution. In recent years, evolutionary computing algorithms have gained global attention towards solving optimization problems.

Considering robustness of the evolutionary computing approaches, numerous efforts have been made to enhance PMU placement for better observability. Among different algorithms, GA has been applauded for its effectiveness. GA algorithm represents a type of search heuristic that imitates the natural evolution phenomenon. It is primarily used to obtain significant optimized solutions. Authors [13] suggested GA for solving OPP problem where they used varied PMU placement conditions, like the unavailability of the significant measurements and system variables, highest measurements received than the initial one, highest estimation accuracy, minimum cost of PMU placement, etc. Authors [14] used GA for OPP solution, where they applied estimator accuracy as the fitness function and to estimate the fitness function they used the inverse of the cumulative differences between measured and the really estimated power flows. A topology based approach was developed in [15] to solve OPP problem where they used B and B algorithm [10, 28] along with the GA technique. Authors [16] enhanced traditional GA and derived Minimum Spanning Tree Genetic Algorithm (MST-GA) which they applied to reduce PMU counts while ensuring complete observability in large scale power grid. In their work they applied MST algorithm to deal with impracticable solutions, and employ the power grid topology in mutation. An effort for OPP was made in [17] where they intended to enable maximum observability under probabilistic events. Researchers used PMUs installation cost as the objective function to perform OPP. Similarly, authors [18] applied GA for reliability oriented PMUs placement in smart grid. They split their problem in two parts: (i) Retrieving minimal PMU counts to enable power system observable while considering single PMU loss, and (ii) Retrieving the optimum PMU location to achieve maximum redundancy in network observability [19]. In [20] authors developed an immunity genetic algorithm (IGA) to solve OPP problem.

Authors [21] used PSO to solve PMU placement issue and evaluated the proposed system on 7, 14, 57 IEEE standard buses. Authors revealed that placing the PMUs merely at the buses with the maximum incident branches might certainly not enable OPP solution. In [22] PSO was used to estimate both the optimal number as well as location of the PMUs. However, authors could not address the issue of the loss of even a single PMU in the network. Authors [23,24] used binary PSO (BPSO) and modified BPSO schemes to estimate the number and the location of PMUs. In [25], the problem of OPP was solved while ensuring minimum number of branch current phasor measurements. An exhaustive search paradigm was used in [26] to solve OPP problem and to perform state estimation. Authors [26] considered single branch failure in assessment. A meta-heuristic paradigm was derived in [27] to perform OPP in which initially the PMUs were assigned at the vital network nodes or the grid positions, which was then followed by the implementation of the iterative local search technique so as to reduce PMU counts in the network to have complete observability. However, they could not address the issue of contingency during process. Authors [29] applied Markov process to perform OPP. However, they could not address the issue of the number of measuring channels needed for individual PMU. In [30], the heuristic paradigm was developed to perform simultaneous OPP as well as phasor data concentrators (PDCs) placement in WAMS. Authors, at first placed PMUs on all system buses, which were then followed by the implementation of a multi-stage elimination process to eliminate PMUs having no significant impact on the system observability. Authors' derived three approaches depth first search (DFS), simulated annealing (SA) and minimum spanning tree (MST) to perform OPP [31]. In [32], a multi-phase method was developed to solve OPP problem using connectivity information. In the initial two phases, PMUs were considered in all buses, while in the last phase, the PMUs entities were checked using pruning process so as to attain OPP solution. The heuristic search methods were used in [33, 34]. In [35], the grenade explosion method (GEM) was applied to solve OPP problem while ensuring complete system observability. An enhanced approach named non-dominated sorting Genetic Algorithm (NSGA) was applied in [36] where author focused on obtaining the Pareto optimal solutions. In [37] authors made the minimum PMUs required to have complete observability, as the objective functions. In [38], a hybrid two-phase paradigm was suggested for OPP solution. In this approach in the first phase, authors used the graph-theoretic paradigm to resolve the power domination (PD) issue in sparse graphs that obtains feasible PMUs quickly. In the next phase, a local search heuristic approach, named ant colony optimization (ACO) was employed to achieve the least possible PMUs to have maximum observability. A hybrid GA-PS approach was developed in [39] to solve OPP problem. In this approach GA was applied to find best initial point which was then followed by generic pattern search (PS) algorithm to estimate optimal result. Among various meta-heuristic approaches for OPP solution, the simplified chemical reaction optimization (SCRO) technique [40], the cellular GA (CGA) [41], hybrid discrete PSO (HDPSO) technique [42], binary PSO (BPSO) algorithm [43], non-dominated sorting differential evolution (NSDE) algorithm [44] and GA are the dominating techniques to solve OPP problem. Authors in [42–44] applied ACO algorithm to estimate minimum number of PMUs and their position to have maximum observability. However, the authors could not address the issue of large scale power systems.

In this paper, a novel multiphase hybrid evolutionary computing (EC) algorithm encompassing Adaptive GA (AGA) and Pattern Search Algorithm (AGA-PS) has been developed. The conventional GA algorithm can't achieve the global optimal solution as it primarily reply on the initial populations, genetic operators and defined fitness function. Varied GA designs might give different results. Unlike conventional approaches, AGA retrieves initial sub-optimal solution (initial starting points) which is then fed as input to another evolutionary approach, Pattern Search (PS) algorithm. Pattern Search (PS) achieves the minimum PMUs required and thus provides the final output as the optimal number of PMUs to be incorporated so as to achieve maximum observability. The developed algorithms have been implemented and tested with IEEE 14, IEEE 39, IEEE 118 bus systems, and KPTCL 155 bus systems.

The other sections of the presented manuscript are divided are follows. Section 2 presents the related works and literatures studied, and in Section 3 the power system observability analysis is discussed. Section 4 discusses the problem formulation and associated PMU placement rules applied. The implementation of the proposed AGA-PS algorithm for OPP problem solving is discussed in Section 5. Results obtained for the proposed system are discussed in Section 6, which is followed by the discussion of the research conclusion in Section 7. References used in this research are presented at the last of the manuscript.

2. POWER SYSTEM OBSERVABILITY ANALYSIS

In general, a power system state estimator needs complete network observability from the associated set of measurements that can form a spanning tree with "full rank" of the system [45]. In general power system observability analysis is performed in two distinct manners. These are:

1. *Numerical observability analysis (NOA),*
2. *Topological observability analysis (TOA).*

This section briefs the process of power system observability analysis.

A. Numerical Observability Analysis (NOA)

The generic measurement model applied in state estimation can be presented as:

$$z = H(x) + e \tag{1}$$

where x states the system state vector. Here, the state vector signifies the voltage phasor of all connected buses in the network. The variable z represents the measurement vector, and $H(x)$ refers the non-linear function which relates the state vector x to the measurement vector$z$. The other variable e presents the measurement error vector. As PMUs facilitate precise measurements of the phasor components and hence e is typically very small and in practice can be ignored. The elite employment of PMU measurements turns into the linear state estimation model [46]. Thus, the derived linear state estimator can be presented as (2):

$$z = Hx \tag{2}$$

where H represents the measurement function matrix. Once the grid network is entirely observable, the linear state estimator retrieves system state by solving (2). NOA verifies whether H has is full rank or not. For a N bus electric grid network the measurement sets comprising voltage as well as current phasor can enable the power system observable, if

$$Rank(H) = 2N - 1 \tag{3}$$

The optimal PMU placement for NOA based approach can be performed in two distinct ways. In first approach by introducing PMU in the power grid network sequentially so as to enhance the rank of H and then terminating PMU introduction once equation (3) is satisfied. Meanwhile, in the second approach OPP can be done by applying or placing PMU on each buses of the network, while removing PMUs one by one from different buses to fulfill condition (3). In this paradigm the elimination process can be overruled at a bus that turns H as rank-deficient. As stated, in these schemes to achieve OPP solution, numerous combinations are needed to be assessed. Here, the individual combination comprises rank assessment that eventually raises the computational complexity of NOA. Unlike this approach, the topological observability analysis (TOA) exhibits minimal computational complexity and burden. A brief of the TOA approach is given as follows.

B. Topological Observability Analysis (TOA)

In TOA approach the power system is presented in terms of a topological graph. Here, the graph possesses 'N' nodes signifying the bus bar and 'E' states the total number of edges reflecting the network's branches connecting the bus bars. Unlike NOA, in TOA scheme the OPP set is obtained in such a manner that the individual bus across the network is observable by a least single PMU. Our proposed model employs the TOA approach that completely functions on the basis of PMU estimations.

3. PROBLEM FORMULATION

This section primarily discusses the problem formulation for OPP problem. Before discussing OPP formulation, the depiction of key rules to be followed for a robust methodological formulation to have maximum observability can be of paramount significance. With this motivation, the following section briefs about the key OPP rules.

A. OPP Rules

Considering the robustness and functional effectiveness, in this paper the topological observability analysis (TOA) has been applied. A power system achieves complete network or the power system observability if all associated or the connected buses are observable. In electric grid network or the power system, a bus is stated to be observable only when its voltage can be estimated directly or indirectly using pseudo-measurements [47]. The ability of PMU to estimate the voltage phasor at the installed bus and the current phasor of all the branches connected to the PMU installed bus can play vital role in estimating other significant parameters to perform indirect measurements. Now, implementing Ohm's law and Kirchhoff's Current Law (KCL), bus adjoining the PMU installed bus can have its voltage and branch current phasor value known. Some of the key rules applicable to identify observable bus in OPP problem are given as follows:

Rule-1	A bus with a PMU installed on it would have its voltage phasor and connected branches currents incident to it measured by the phasor measurement unit (PMU).
Rule-2	Applying the Ohm's rule, the voltage phasor at one terminal (end) of a branch current may be estimated in case the voltage phasor at the other terminal of the branch current is under knowledge or known.
Rule-3	Applying Ohm's rule the branch current can be measured, if the voltages are known at both the terminals (ends) of a branch.

Another factor called the zero injection bus (ZIB) can also play significant role in reducing PMUs required to ensure complete power system observability. There exists no generator that could inject power or a load which may consume power from this bus [48]. According to the KCL rule, the cumulative addition or the sum of flows on all branch currents connected with ZIB is zero. In such cases, the power system observability can be accomplished with the presence of ZIB based on the following rules [49, 50]:

Rule-4	When grid buses incident to an observable ZIB can be completely observable except for the one, and thus the unobservable bus can be stated to be observable by employing KCL at the ZIB.
Rule-5	In case the buses are placed to certain unobservable ZIB are observable, then ZIB can be recognized as observable by introducing Ohm's rule.
Rule-6	A set of unobservable ZIB, in conjunction with the observable buses can be identified as observable by retrieving the voltage phasor of ZIB by means of KCL node equation.

These all mentioned rules enable buses incident to the ZIB to remain observable without introducing or placing a PMU on it and therefore it plays vital role in reducing PMU counts to make the power system completely observable.

The problem definition used in this research work is presented as follows:

B. Problem Definition

The OPP problem while taking into consideration of the PMUs measuring channels to make power system completely observable can be mathematically expressed as (4) [51,52]:

$$MinF(x) = \sum_{i=1}^{N_b} w_i x_i \tag{4}$$

Subject to:

$$g(x) = \sum_{i,j=1}^{N_b} A_{ij} X_i \geq b \tag{5}$$

where $F(x)$ represents the objective function (OF) which is needed to be minimized so as to achieve least count of PMUs and its optimal locations with measuring channels. In (4), the variable w_i signifies the weighting factor of the PMU placed at bus i based on system configuration, which is numerically equivalent to the total number of branches incident to the bus *i* plus one, which is again equal to the total number of PMU measuring channels. The parameter N_b represents the total number of buses in the considered grid network or architecture. Here, x_i is the binary decision variable [0,1], which can be characterized as follows (6):

$$X_i = \begin{cases} 1 & if\ a\ PMU\ is\ installed\ at\ bus\ i \\ 0 & otherwise \end{cases} \tag{6}$$

Here, the other variable $g(x)$ as presented in (5) represents the observability constraint that must verify whether each bus belongs to the system. In (5), the variable A_{ij} represents the connectivity matrix which is formed on the basis of the system line data in binary form. Mathematically, the connectivity matrix A_{ij} is presented as follows:

$$A_{ij} = \begin{cases} 1 & if\ i = j \\ 1 & if\ buses\ i\ and\ j\ are\ connected \\ 0 & otherwise \end{cases} \tag{7}$$

The parameter *b* represents the minimum limit of MR that can be considered as follows (8):

$$b = \begin{cases} 1 & for\ normal\ operating \\ 2 & for\ any\ single\ line\ or\ PMU\ outage \end{cases} \tag{8}$$

To enable power system completely observable under normal operating condition, *b* should be equal to 1 that signifies that the individual bus is observed no less than once, while in case of any contingency like single PMU failure, *b* should be equal to 2 which signifies that the individual bus is observed (minimal) twice to enable power system completely observable.

4. MULTIPHASE AGA-PS BASED OPP SOLUTION

As proposed, in this paper a two phase PMU placement model has been developed to enable power system optimally observable. In this first phase an enhanced EC algorithm called Adaptive Genetic Algorithm (AGA) has been applied to obtain the initial state points, which has been further used in second phase where Pattern Search (PS) algorithm obtains the minimum PMUs and their respective best location to enable maximum or complete network observability. The different phases of the proposed OPP mechanism are as follows:

A. *AGA Based Initial State Point Retrieval,*
B. *Pattern Search (PS) Based OPP Solution Retrieval.*

A succinct presentation of the proposed OPP scheme is presented as follows:

A. *AGA Based Initial State Point Retrieval*

In our proposed OPP model, AGA has been applied in the initial phase so as to obtain the initial state point range. Here, it should be noted that the predominant purpose of using AGA is to retrieve the optimal state points which is feasible because of its swift convergence rate that eventually enables sub-optimal solution retrieval even within a few iterations. Generally, the tentative start point retrieved from GA is fairly near the optimal solution or results. It reduces computational time and complexities incurred over unwanted iterations. It significantly assists our proposed model to facilitate additional time for further optimization, as scheduled in the second phase.

In GA approach, a population of strings, generally known as chromosomes encode each solution for optimizing the result, evolves toward more efficient solutions. Typically, these solutions are stated in the form of binary value or the strings of zeros and ones. In GA the evolution is initiated from a population where the individuals are generated randomly and keeps on generating across the generations. The fitness value of an individual in the population is estimated, and based on respective fitness value the individuals with higher fitness are selected stochastically, and are altered to generate a new generation with relatively higher fitness and closeness towards the optimal solution. Thus, the newly generated population is then processed in the next iteration and continues till the stopping criterion is met. This process may continue iterating till the number of the individuals in the population equals the specified size of the population. Here, the size of population states the total number of individuals in each generation. In our proposed OPP model, the initial population has been defined as a binary string. Thus, in the targeted grid design and OPP problem, individual grid design can be stated as an individual that eventually is represented as a binary string with 0's or 1's. These binary strings is then used for AGA based OPP optimization. In our proposed AGA based OPP optimization model, these generated 0's and 1's have been transferred to the three network variables (N_x, N_y, N_{rod}) so as to estimate the OF. In this paper, the grid construction cost has been taken as the OF that actually signifies the PMUs required enabling power system completely observable. In general, the algorithm terminates once retrieving the expected fitness level. In case, the algorithm is terminated after reaching predefined number of iterations, an expected solution may or may not be obtained [53]. To ensure optimal solution retrieval, estimating sufficient fitness level plays significance role. The following section briefs the fitness function estimation for our proposed OPP optimization.

a. *Fitness Function Estimation*

Considering the research optimization needs, in this research work the total number of PMUs, which signifies the overall cost of the power grid is considered as the OF. To estimate the fitness function for PMU placement, the following mathematical expression is used (9):

$$J(x) = W_1 * \sum_{i=1}^{Nb} fi + W_2 * N_{PMU} + W_3 * J_1 \qquad (9)$$

where $\sum_{i=1}^{Nb} \mathrm{fi}$ reflects the observability index of the considered power system network and N_{PMU} represents the PMUs needed to make network complete observable. Assuming the expected level of redundancy as 2, J_1 represents the difference existing in between the expected and the real values. In order to estimate the weights of the fitness function, the significance level of individual factor as compared to the other factor has been taken into consideration. Constructing the hierarchy framework, a pair-wise comparison matrix is obtained at the individual hierarchy and the comparison is made using a scale pair-wise comparison [54]. Table 1 represents the scale for the pair-wise comparison.

Table 1 Pair-wise comparison scale [55].

Relative intensity	Definition
1	Equal (value)
3	Slightly more (value)
5	Essential or strong (value)
7	Very Strong value
9	Extreme (value)
2,4,6,8	Intermediate values between two adjacent judgments.

Using Table 1, a 3*3 matrix is formed (10).

$$a_{ij} = \begin{bmatrix} 1 & \frac{1}{3} & 5 \\ 3 & 1 & 9 \\ \frac{1}{5} & \frac{1}{9} & 1 \end{bmatrix} \qquad (10)$$

Now, as stated in (9), W_i can be estimated using following mathematical expression:

$$W_i = \prod_{j=1}^{FFF_n} \frac{a_{ij}}{\prod_{j=1}^{n} a_{ij}} \qquad (11)$$

where a_{ij} represents an entity of the matrix, and FFF_n represents a factor called the total fitness function factors (FFF). Typically, it is also known as the Logarithmic Least Square Method (LLSM) that is a key element of the Analytical Hierarchy Process (AHP) approach [54].

b. *Selection*

In AGA, the fitness function is calculated based on the OF, which is nothing else but the cost of network. It should be noted that the cost of the network is directly propositional to the total number of PMUs applied, and hence reducing the PMUs while ensuring network completely observable is the OF. As per the fitness function value (here, the inverse of the OF is taken as fitness value), individuals in the population are ranked using the AGA's selection function. In selection, the parent individuals with higher fitness values retain its place for reproduction to form next generation. In our proposed AGA scheme, Roulette wheel mechanism has been applied to perform selection and thus the individual having higher fitness would have higher sustainability to have presence in the next generation or iteration to perform "reproduction".

C. *Reproduction*

Reproduction states the process that defines how AGA generates children (individual solution) at each generation of the evolution. It comprises two steps, crossover and mutation. In crossover process, the two parent individuals are combined so as to form a new individual for the next generation. On the contrary, mutation process simulates the influence of errors that occurs with low likelihood during replication. In GA based optimization, the optimal selection of the two genetic parameters, crossover (P_c) and mutation probability (P_m) is of paramount significance. The proper value of P_c and P_m can play vital role in preserving the diversity of GA algorithm and in addition can effectively alleviate the issue of local minima. Too high values of these genetic parameters might turn the algorithm into a primitive random search scheme. In major existing researchers, authors have used fixed value of P_c and P_m that causes system to suffer issues like local minima and convergence and in addition it introduces high computational complexity and process time. Unlike conventional GA an enhanced GA algorithm named Adaptive GA has been developed in which the GA parameters P_c and P_m are updated adaptively and the process continues till 95% of the chromosomes have the unique fitness value. The following equation (12) is used to update the GA parameters adaptively,

$$
\begin{aligned}
(P_c)_{k+1} &= (P_c)_k - \frac{C_c * C_{SF}}{5} \\
(P_m)_{k+1} &= (P_m)_k - \frac{C_m * C_{SF}}{5}
\end{aligned}
\tag{12}
$$

In equation (12), $(P_c)_k$ represents the current crossover probability in kth generation, and $(P_m)_k$ represents the mutation probability at the kth generation. The other variables are C_c and C_m are the positive constants. In our AGA model, $C_1 = 0.01$ and$C_2 = 0.001$. These constants can be any positive constant values. Here C_{SF} presents the number of chromosomes having similar fitness value. In the proposed OPP problem, our proposed AGA based optimization process continues till the stopping criterion is met or till 95% of the chromosomes achieve similar fitness value. After that the system gets saturated. The overall implementation of the proposed AGA algorithm for initial state point estimation for OPP solution is illustrated in Fig. 1. Once estimating the initial state point, in this paper the second phase of the proposed Multiphase hybrid evolutionary computing scheme has been executed where patter search heuristic has been applied to perform OPP optimization.

A. *Pattern Search Based OPP Solution*

Pattern Search (PS) has established itself as a robust evolutionary computing scheme to solve a number of optimization issues that typically remains ignored in major traditional optimization tasks. In addition, its simplicity, flexible implementation and computationally efficient functions enable it to be used for advanced optimization purposes. Unlike, GA [56, 57], PS possesses well-balanced and efficient operators that strengthen it to achieve global minima and even enhance the local search performance. Being a direct search algorithm, PS retrieves a set of points in its neighborhood as the starting point, exploring further to achieve improved results when the value of the defined OF is lower as compared to the value at the present state point. It enables PS to be used for optimizing the non-linear and differentiable functions. Considering the robustness of the PS algorithm, in our proposed work, it has been used in the second phase of OPP optimization where it functions to search an optimal solution around the approximate optimum (the initial state points obtained through AGA algorithm). The proposed PS algorithm tests multiple state points approximate to the starting points retrieved through AGA. In case any one of such multiple points generates a smaller or larger OF value (here we consider smaller value as for OPP minimum PMUs are needed while ensuring complete observability) as compared to the initial state points or the starting point, the newly retrieved start point is updated as the initial state point for the next iteration.

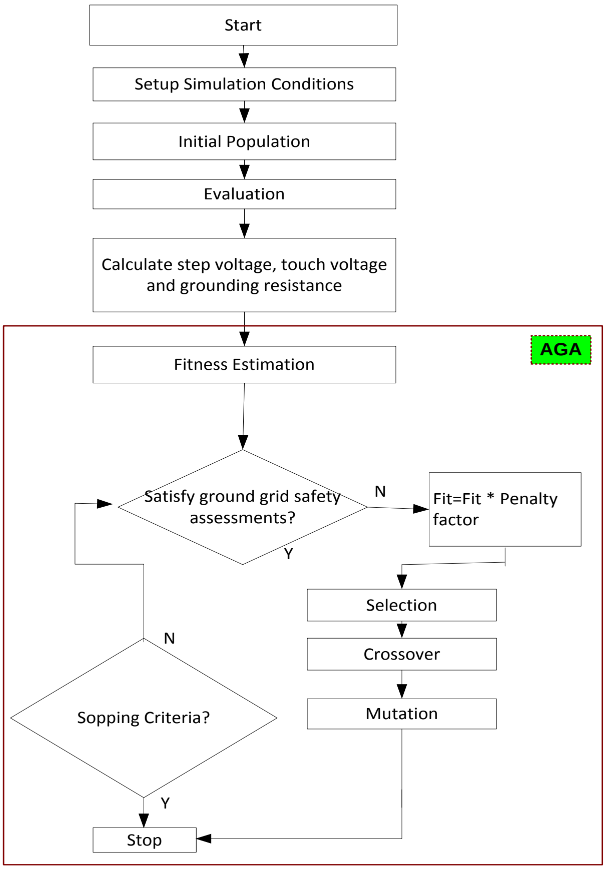

Fig. 1 Adaptive Genetic Algorithm Based Initial State Point Estimation

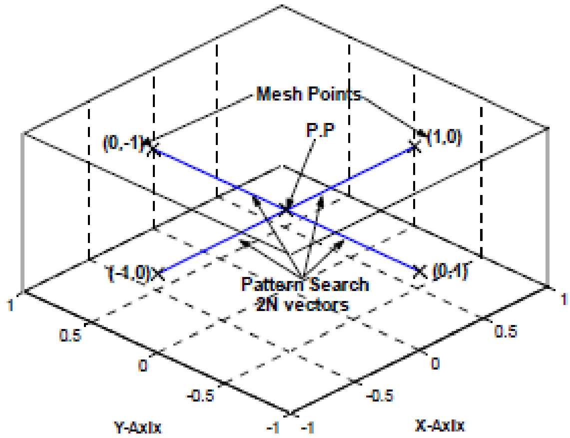

Fig.2 2N Pattern Vectors forming PS mesh points

In our proposed OPP optimization model, with the retrieved AGA based initial state points X_0, a mesh is formed around the obtained state points. In case a certain state point in the mesh is obtained to enhance the OF, the newly obtained point is updated as the current state point for the next iteration. In the first iteration, with the mesh size of 1, the directional vectors, also called as the pattern vectors are formed as [0 1], [1 0], [-1 0] and [0 -1] $+ X_0$. Thus, with the new state points the mesh is updated as depicted in Fig. 2. In this process the OF are estimated till a value lower than X_0 is obtained. In case, there exist such a point, the poll is stated to be successful and thus the PS algorithm assigns this point as equivalent to X_1. Performing successful poll, PS introduces expansion factor that multiplies the current mesh size by 2 and thus moves further for the next iteration with the following new state points: $2 * [-1 \ \ 0] + X_1, 2 * [0 \ \ 1] + X_1, 2 * [-1 \ \ 0] + X_1$ and $2 * [0 \ \ -1] + X_1$. In case a value lower than X_1 is observed, the state point X_2 is defined and thus the mesh size is raised by two and thus the process continues. In case at certain instant there is no state point having the OF lower as compared to the most recent value, then the poll is called unsuccessful then the current point remains unaltered and the size of mesh is reduced by half by applying contraction factor of value 0.5.

In case of mesh size 4, when there occurs unsuccessful poll then none of the mesh points possesses lower value of OF as compared to the state points value at X_2 and thus the poll called unsuccessful. It results into unaltered current state points for the next generation and hence $X_3 = X_2$ and to move further the contraction factor would be used to scale down the mesh size [58]. Thus, this optimization process continues till it obtains the optimal solution for the reduction of the OF, i.e., minimum PMUs to make power system complete observable. Unlike AGA in the first phase, PS terminates iteration once crosses the defined maximum iterations (200). The flow chart of the proposed PS based OPP optimization scheme is presented in Fig. 3.

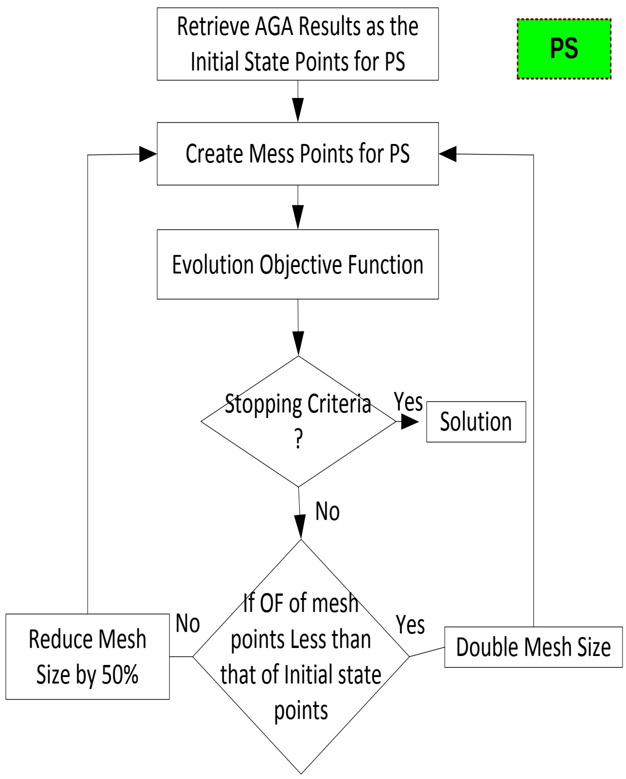

Fig. 3 Pattern Search Based OPP optimization

5. Results and Discussion

The presented research work intended to develop a robust multistage evolutionary computing scheme for PMU placement optimization. Unlike generic GA algorithm, the proposed AGA approach obtains the sub-optimal solution as the initial state points signifying the initial PMU placement configuration. To alleviate the issue of local minima and convergence, the proposed AGA scheme incorporated adaptive crossover and mutation probability P_c and P_m respectively. Here, AGA estimates minimum number of strategic bus locations while ensuring complete power system or the grid network observable. Once retrieving the initial state conditions, the retrieved state points have been fed as input to the pattern search (PS) algorithm that eventually retrieves optimal locations of the PMUs across the network and the least possible number of PMUs to enable power system complete observable. The proposed hybrid PMU placement algorithm has been examined with different standard benchmark bus systems like, IEEE 14, IEEE 39, IEEE 118 and KPTCL 155 bus architecture.

To examine the performance of the proposed PMU placement model, the assessment has been done in four conditions. These conditions and respective annotations are given as follows:

Table 1 Optimization Conditions

TC1	Hybrid GA and Pattern search Algorithm
TC2	Hybrid GA and Pattern search Algorithm considering ZIB
TC3	Hybrid GA and Pattern search Algorithm considering one PMU loss
TC4	Hybrid GA and Pattern search Algorithm by giving priority or ranking to buses or by placing at least one PMU at all generator buses. Generator buses are given highest priority followed by load buses.

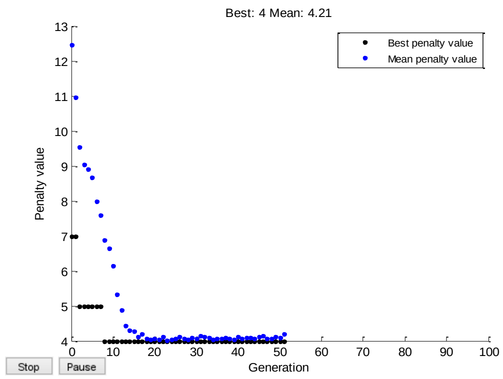

Fig. 4 Pattern Search Based OPP optimization

Fig 4 and 5 indicate the graphical plot with the objective function evolution the IEEE 14 bus power systems. Similar types of graphs are obtained for other test cases.

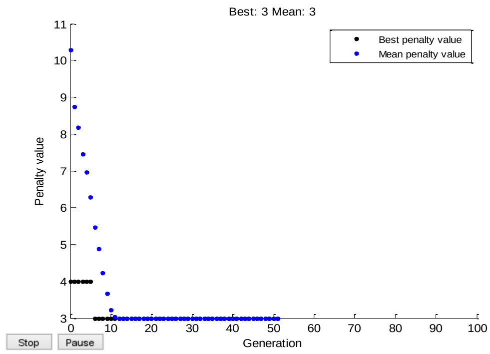

Fig. 5 Pattern Search Based OPP optimization

The results obtained for different test conditions and varied bus systems are given as follows:

Table 2 Results for different OPP algorithms

	IEEE14 Bus	IEEE39 Bus	IEEE118 Bus	KPTCL 155 bus system
TC1	4	13	39	58
TC2	3	9	29	49
TC3	9	28	74	114
TC4	6	16	46	68

As depicted in Table 2, the minimum PMUs needed to enable power system complete observable; our proposed AGA-PS algorithm requires only 4, 13 and 39 PMUs for IEEE 14, IEEE 39 and IEEE 118 bus networks respectively. In case of single PMU loss, the proposed OPP scheme needs 9, 28 and 74 PMUs to have complete observability in IEEE 14, IEEE 39 and IEEE 118 bus networks respectively. The results obtained affirm that the proposed scheme is capable of producing optimal set of PMUs for single PMU loss condition. AGA-PS with ZIB has also shown affirmative output where it needs 3, 9 and 29 PMUs to enable complete network observability of IEEE 14, 39 and 118 networks. In addition to this, the proposed AGA-PS algorithm has been tested by prioritizing generator buses or by placing at least one PMU at all generator buses, where it demands 6, 16 and 46 PMUs to make power system (IEEE 14, IEEE 39 and IEEE 118, respectively) complete observable. The overall results obtained state that the proposed AGA-PS algorithm performs better in different PMU placement conditions. Discussing as cumulative performance by the proposed AGA-PS OPP approach, it can be found that for IEEE 14 bus network a total of 4 PMUs are needed to have complete system observability; while for 39 it requires only 13 PMUs. For IEEE 118 bus network our proposed OPP approach needs 39 PMUs which seems satisfactory and hence confirms that the proposed PMU placement strategy could be effectively employed for large scale power grid network observability assurance. The PMU placements and their optimal location in different benchmark networks are presented in the following tables. Table 3 presents the locations of the PMUs to enable IEEE14 bus network completely observable. Similarly, the PMU locations for IEEE 39 and IEEE 118 are presented in Table 4 and Table 5, respectively.

Table 3 PMU placement results for IEEE 14 Bus system

IEEE14 Bus	
TC1	2,6,7,9
TC2	2,6,8
TC3	2,3,5,6,7,8,9,11,13
TC4	1,2,3,6,8,9

Table 4 PMU placement results for IEEE39 Bus system

IEEE 39 Bus	
TC1	2,6,9,10,11,14,17,19,20,22,23,25,29
TC2	2,3,5,8,9,11,13,17,23
TC3	1,2,3,6,8,9,10,11,13,14,16,17,19,20,22,23,25,26,29 30,31,32,3334,35,36,37,38
TC4	3,4,8,12,16,26,30,31,32,33,34,35,36,37,38,39

Table 5 PMU placement results for IEEE 118 Bus system

IEEE 118 Bus	
TC1	2,5,9,11,12,14,18,20,22,23,26,28,30,32,36,37,42,44 46,51,53,57,59,61, 66,68,71,75,79,82,85,86,89,92,96,100, 106,109,110,114,118
TC2	1,6,8,12,15,17,21,25,29,34,40,45,49,53 56,62, 72,75,77,80,85,86, 90,94,101,105,110,114,118
TC3	1,2,5,7,9,10,11,12,14,15,17,18,20,21,23,26,27,28,29,30, 32,34,35,37,40,42,44,45,46,49,50,52,54,56,61,62,63,64, 67,68,70,71,72,73,75,76,77,79,80,83,85,86,87,88,89,91, 92,94,96,100,101,103,105,107,108,109,110,111 112,115,116,117
TC4	3,5,10,12,15,17,20,23,25,26,28,35,37,39 ,42,44,46,47,49,51,54,56,59,60,61 65,66,68,69,71,75,78,80,83,86,89,90,92, 94,100,103,105,110,111,115,118

Considering an application aspect, the predominant contribution of this research work is its applicability for KPTCL power system or power distribution network where PMU are placed by giving weightage to all types of buses. Generator buses are given highest weight; second priority is given to buses with higher load connected buses. It is also called as the prioritized PMU placement.

KPTCL 155 Bus Power System	
TC1	1,3,4,6,10,11,13,15,16,18,23,,27,31,34,35,36,39 44,46,53,54,56,59,61,66,67,6874,76,81,,82,84,86 88,91,9495,98,100,101,105,108,109,113,114,115 119,121,124,129,131 132,137,138,143,144,145,152
TC2	1,2, 4, 6, 9, 11, 15, 17, 21, 24, 27, 30, 33, 36, 38 44, 46, 50, 52, 54, 56, 58, 61, 64, 67, 68, 70, 78 79, 80, 84, 86, 90, 93, 97, 99, 101, 103, 105,109 115, 123, 129, 134, 138, 144, 145, 152,155
TC3	1,2,4,2,6,7,8,9,10,11,12,15,16,17,18,21,22,24,26,27 28,31,32,33,34,35,36,38,39,40,43,44,45,46,47,49 50,51,52,53,54,55,56,57,58,59,,61,62,63,64,65 67,68,70,71,72,73,77,78,80,81,82,83,84,85,86,87 89,90,92,93,96,97,98,99,100,101102,105,108,109 110,112,113,115,116,117118,119,121,123,124 125,126,127128,129,131,133,134,137 138,139,143,144,145,146,147,148 149,150,152,153154,155
TC4	1,4,6,7,10,12,16,17,21,22,24,28,29,30,33,36,39 44,46,47,50,51,53,54,56,58,61 64,67,68,70,78,80,83,84,86,90,97,98,100,102 104,105,110,115,117,123,128 129,130,131,132,133,134,135,136,137,138,139,140 141,143,144,145,147,150151,152

The overall proposed model has been developed using MATLAB 2015a software tool, which has been tested on general purpose computers with 4GB RAM, 512 GB of memory space and 1.80 GHz processor.

6. CONCLUSION

This paper proposed a novel multiphase evolutionary computing based PMU placement model. Unlike conventional approaches like mathematical optimization schemes and traditional evolutionary computing based OPP systems, in this paper and enhanced Adaptive Genetic Algorithm (AGA) was used that obtained sub-optimal solutions as the initial state points for pattern search (PS) based OPP. The implementation of AGA not only alleviates the local minima but also deals with convergence and hence gives sub-optimal solution in minimum iterations without getting converged. It enables later phase (PS based OPP) to achieve optimal number of PMUs needed and its location to make power system completely observable. The performance assessment with different IEEE bus architectures has revealed that the proposed AGA-PS scheme enables complete network observability with 9, 28 and 74 PMUs for IEEE 14, IEEE 39 and IEEE 118 buses respectively, with single PMU loss. Similarly, with zero injection bus it requires only 3, 9 and 29 PMUs for IEEE 14, 39 and 118 bus networks. Considering the large scale network, the simulation of the proposed OPP algorithm has exhibited appreciable performance with KPTCL power grid network. The results exhibit that AGA-PS based OPP can be effectively used for cost effective large scale network design. The effectiveness of AGA combined with PS strengthens the proposed system to be used to cost effective grid design and optimal PMU placement to make electrical network completely observable.

REFERENCES

[1] S. M. Mazhari, H. Monsef, H. Lesani, A. Fereidunian, A multi-objective PMU placement method considering measurement redundancy and observability value under contingencies, IEEE Transactions on Power Systems 28 (2013) 2136-2146.

[2] Vahid Behravesh, and Zahra Moravej"Optimal placement of Phasor Measurement Units in Khosran Network using a Hybrid Intelligent Technique" Journal of modeling and simulation in Electrical and Electronics Engineering(MSEEE) volume1, Number 1, Feb 2015.

[3] J. Ekanayake, K. Liyanage, J. Wu, A. Yokoyama, N. Jenkins, Smart Grid: Technol-ogy and Applications, Wiley, 2012 (March).

[4] S. Azizi, G.B. Gharehpetian, A.S. Dobakhshari, Optimal integration of phasor measurement units in power systems considering conventional measurements, IEEE Trans. Smart Grid 4 (June (2)) (2013) 1113–1121.

[5] L. Huang, Y. Sun, J. Xu, W. Gao, J. Zhang, Z. Wu, Optimal PMU placement considering controlled islanding of power system, IEEE Trans. Power Syst. 29 (March(2)) (2014) 742–755.

[6] A. Enshaee, R. Hooshmand, F.H. Fesharaki, A new method for optimal placement of phasor measurement units to maintain full network observability under various contingencies, Electr. Power Syst. Res. 89 (August) (2012) 1–10.

[7] S.M. Mahaei, M.T. Hagh, Minimizing the number of PMUs and their optimal placement in power systems, Electr. Power Syst. Res. 83 (February (1)) (2012)66–72.

[8] F. Aminifar, A. Khodaei, M. Fotuhi-Firuzabad, M. Shahidehpour, Contingency-constrained PMU placement in power networks, IEEE Trans. Power Syst. 25 (February (1)) (2010) 516–523.

[9] R. Kumar, V.S. Rao, Optimal placement of PMUs with limited number of chan-nels, in: North American Power Symposium (NAPS) Conf., 4–6 August, 2011,pp. 1–7.

[10] B. Mohammadi-Ivatloo, Optimal placement of PMUs for power system observability using topology based formulated algorithms, J. Appl. Sci. 9 (13) (2009)2463–2468.

[11] R.J. Albuquerque, V.L. Paucar, Evaluation of the PMUs measurement channels availability for observability analysis, IEEE Trans. Power Syst. 28 (August (3))(2013) 2536–2544.

[12] S.M. Mazhari, H. Monsef, H. Lesani, A. Fereidunian, A multi-objective PMU placement method considering measurement redundancy and observability value under contingencies, IEEE Trans. Power Syst. 28 (August (3)) (2013)2136–2146.

[13] A. Z. Gamm, I. N. Kolosok, A. M. Glazunova, and E. S. Korkina, "PMU placement criteria for EPS state estimation," in *2008 Int. Conf. on Electric Utility Deregulation and Restructuring and Power Technologies*, pp. 645-649.

[14] M. Gavrilas, I. Rusu, G. Gavrilas, and O. Ivanov, "Synchronized phasor measurements for state estimation", *Revue Roumaine des Sciences Techniques*, no. 4, pp. 335-344, 2009.

[15] B. Mohammadi-Ivatloo, "Optimal placement of PMUs for power system observability using topology based formulated algorithm," *Journal of Applied Sciences*, vol. 9, no. 13, pp. 2463-2468, 2009.

[16] Z. Hui-Ling, D. Yuan-Xiu, Z. Xiao-Pan, Q. Huan and H. Cheng-Xun, "Hybrid of MST and Genetic Algorithm on Minimizing PMU Placement," *Intelligent System Design and Engineering Applications (ISDEA), 2013 Third International Conference on*, Hong Kong, 2013, pp. 820-823.

[17]A. Kimiyaghalam, M. Mahdavi, A. Ashouri and M. Bagherivand, "Optimal placement of PMUs for reliable observability of network under probabilistic events using BABC algorithm," *Electricity Distribution (CIRED 2013), 22nd International Conference and Exhibition on*, Stockholm, 2013, pp. 1-4.

[18] V. Khiabani, E. Erdem, K. Farahmand and K. Nygard, "Genetic algorithm for instrument placement in smart grid," *Nature and Biologically Inspired Computing (NaBIC), 2013 World Congress on*, Fargo, ND, 2013, pp. 214-219.

[19] P. P. Bedekar, S. R. Bhide and V. S. Kale, "Optimum PMU placement considering one Line/ One PMU outage and maximum redundancy using genetic algorithm," *Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON), 2011 8th International Conference on*, Khon Kaen, 2011, pp. 688-691.

[20] F. Aminifar, C. Lucas, A. Khodaei and M. Fotuhi-Firuzabad, "Optimal Placement of Phasor Measurement Units Using Immunity Genetic Algorithm," in *IEEE Transactions on Power Delivery*, vol. 24, no. 3, pp. 1014-1020, July 2009.

[21] A. Sadu, R. Kumar and R. G. Kavasseri, "Optimal placement of Phasor Measurement Units using Particle swarm Optimization," *Nature & Biologically Inspired Computing, 2009. NaBIC 2009. World Congress on*, Coimbatore, 2009, pp. 1708-1713.

[22] A. Sadu, R. Kumar, R.G. Kavasseri, "Optimal placement of phasor measurement units using particle swarm", Proceedings of the World Congress on Nature and Biologically Inspired

[23] A. Ahmadi, Y. Alinejad-Beromi, M. Moradi, "Optimal PMU placement for power system observability using binary particle swarm optimization and considering measurement redundancy", Expert Systems with Applications, Vol.38, pp. 7263–7269, 2011.

[24] M. Hajian, A. M. Ranjbar, T. Amraee, B. Mozafari, "Optimal placement of PMUs to maintain network observability using a modified BPSO algorithm", Electrical Power and Energy Systems, Vol. 33, pp. 28–34 2011.

[25] K. S. Sajan and B. Tyagi, "Optimal placement of PMU with optimal branch current phasors for complete and incomplete observability," *2011 IEEE Power and Energy Society General Meeting*, San Diego, CA, 2011, pp. 1-5.

[26] S. Chakrabarti, E. Kyriakides, "Optimal placement of phasor measurement units for power system observability", IEEE Transactions on Power Systems, Vol. 23, pp. 1433–1440, 2008.

[27] M. Hurtgen, J.C. Maun, "Optimal PMU placement using iterated local search", Electric Power and EnergySystems, Vol. 32, pp. 857–860, 2010.

[28] B. Mohammadi-Ivatloo, Optimal placement of PMUs for power system observability using topology based formulated algorithms, J. Appl. Sci. 9 (13) (2009)2463–2468.

[29] R.J. Albuquerque, V.L. Paucar, Evaluation of the PMUs measurement channels availability for observability analysis, IEEE Trans. Power Syst. 28 (August (3))(2013) 2536–2544.

[30] F.H. Fesharaki, R. Hooshmand, A. Khodabakhshian, A new method for simultaneous optimal placement of PMUs and PDCs for maximizing data transmission reliability along with providing the power system observability, Electr. PowerSyst. Res. 100 (July) (2013) 43–54.

[31] A.Y. Abdelaziz, A.M. Ibrahim, R.H. Salem, Power system observability with mini-mum phasor measurement units placement, Int. J. Eng. Sci. Technol. 5 (3) (2013)1–18.

[32] B.K. Saha Roy, A.K. Sinha, A.K. Pradhan, An optimal PMU placement technique for power system observability, Int. J. Electr. Power Energy Syst. 42 (November(1)) (2012) 71–77.

[33] R.H. Shewale, B.K. Kethineni, U.P. Balaraju, S.K. Bhil, P.D. More, Optimal placement of phasor measurement unit for power system observability by heuristicsearch method, Int. J. Adv. Technol. Eng. Res. 2 (May (2)) (2012) 128–133.

[34] C. Rakpenthai, S. Premrudeepreechacharn, S. Uatrongjit, N.R. Watson, An optimal PMU placement method against measurement loss and branch outage, IEEETrans. Power Deliv. 22 (January (1)) (2007) 101–107.

[35] F. Khavari, V. Naseri, Ali H. Naghshbandy, Optimal PMUs placement for power system observability using grenade explosion algorithm, Int. Rev. Electr. Eng.6 (June (3)) (2011) 1332–1338.
[36] Milosevic B. D. and Begovic M., "Nondominated Sorting Genetic Algorithm for Optimal Phasor Measurement Placement", IEEE Power Engineering Review, Vol. 22, No. 12, pp. 61-61,2002
[37] Gao Y., Hu Z., He X. and Liu D., "Optimal Placement of PMUs in Power Systems based on Improved PSO Algorithm", IEEE Conference on Industrial Electronics and Applications, pp. 2464-2469, 2008.
[38] Chung-Shou Liao, Tsung-Jung Hsieh, Xian-Chang Guo, Jian-Hong Liu and Chia-Chi Chu, "Applying power domination with hybrid search to optimal PMU placement problems," *2013 IEEE Power & Energy Society General* Meeting, Vancouver, BC, 2013, pp. 1-5.
[39] Q. Zhang and X. Wu, "Software development of optimal substation ground grid design based on genetic algorithm and pattern search," North American Power Symposium, 2014, Pullman, WA, 2014, pp. 1-6.
[40] M.H.F. Wen, J. Xu, V.O.K. Li, Optimal multistage PMU placement for wide-areamonitoring, IEEE Trans. Power Syst. 28 (November (4)) (2013) 4134–4143.
[41] Z. Miljani´c, I. Djurovi´c, I. Vujoˇsevi´c, Optimal placement of PMUs with limitednumber of channels, Electr. Power Syst. Res. 90 (September) (2012) 93–98.
[42] Y. Alinejad-Beromi, A. Ahmadi, H. Rezani Soleymanpour, Optimal PMUplacement considering contingencies by using a hybrid discrete particleswarm optimization technique, Int. Rev. Electr. Eng. 6 (Aug (4)) (2011)1927–1938.
[43] A. Ahmadi, Y. Alinejad-Beromi, M. Moradi, Optimal PMU placement for powersystem observability using particle swarm optimization and considering mea-surement redundancy, Expert Syst. Appl. 38 (2011) 7263–7269.
[44] C. Peng, H. Sun, J. Guo, Multi-objective optimal PMU placement using a non-dominated sorting differential evolution algorithm, Int. J. Electr. Power EnergySyst. 32 (October (8)) (2010) 886–892.
[45] Krumpholz GR, Clements KA, Davis PW. Power system observability: a practical algorithm using network topology. IEEE Trans Power Ap Syst 1980;99(4):1534–42.
[46] Yoon YJ. Study of utilization and benefits of phasor measurement units for large scale power system state estimation, MS dissertation, Texas A&M University; December 2005.
[47] Su C, Chen Z. Optimal placement of phasor measurement units with new considerations. In: 2010 Asia-Pacific power and energy engineering conference. IEEE; 2010. p. 1–4.
[48] Sanchez-Ayala G, Aguerc JR, Elizondo D, Lelic M. Current trends on applications of PMUs in distribution systems. 2013 IEEE PES Innovative Smart Grid Technologies Conference (ISGT). IEEE; 2013.
[49] Aminifar F, Khodaei A, Fotuhi-Firuzabad M, Shahidehpour M. Contingency-constrained PMU placement in power networks. IEEE Trans Power Syst 2010;25(1):516–23.
[50] Esmaili M. Inclusive multi-objective PMU placement in power systems considering conventional measurements and contingencies. Int Trans Electr Energy Syst 2016;26(3):609–26.
[51] A. Enshaee, R. Hooshmand, F.H. Fesharaki, A new method for optimal placementof phasor measurement units to maintain full network observability undervarious contingencies, Electr. Power Sys. Res. 89 (August) (2012) 1–10.
[52] S.M. Mahaei, M.T. Hagh, Minimizing the number of PMUs and their optimalplacement in power systems, Electr. Power Syst. Res. 83 (February (1)) (2012)66–72.
[53] Akbari R. and Ziarati K., "A Multilevel Evolutionary Algorithm for Optimizing Numerical Functions", Journal of Industrial Engineering Computations, Vol. 2, No. 2, pp. 419-430, 2011.
[54] P. S. Shelokar, V. K. Jayaraman and B. D. Kulkarni, An Ant Colony Approach for Clustering, Journal of Analytica Chimica Acta, Vol.509, pp.187-195, 2004.
[55] H. Ariff, M. S. Salit, N. Ismail and Y. Nukman, Use of Analytical Hierarchy Process (AHP) for Selecting the Best Design Concept, Asian Journal of Teknologi, pp.1-18, 2008.
[56] Goldberg DE. Genetic algorithms in search, optimization, and machine learning. Harlow: Addison-Wesley; 1989.
[57] Michalewicz Z. Genetic algorithms + data structures = evolution programs, 3rd rev. and extended ed. Berlin, New York: Springer-Verlag; 1996.
[58] C. Thitithamrongchai, B. Eua-arporn, Selfadaptive Differential Evolution Based Optimal Power Flow for Units with Non-smooth Fuel Cost Functions, *J. Electrical Systems* 3-2 (2007), pp.88-99.
[59] Likith Kumar. M. V., MaruthiPrasanna. H. A, Dr. T. Ananthapadmanabha. A novel approach for Distribution system state Estimation with renewable Energy sources based distributed Generations, Journal of Electrical Engineering Romania Volume 14-3 (2014), pp.295-302.