ROBUST H_∞ STATE FEEDBACK CONTROLLER FOR DISCRETE-TIME TAKAGI-SUGENO SYSTEMS

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Abstract— This paper develops a robust state feedback control for discrete-time Takagi-Sugeno fuzzy model system. The state H_∞ feedback controller is synthesized based on a fuzzy model control technique called Parallel Distributed Compensation (PDC). The new algorithm is proposed straightforward controller to robustly stabilize the highly nonlinear systems and to reduce the conservatism. So, the algorithm is used to improve the performance and to guarantee the steady state condition of the nonlinear system and ensures the robustness of the fuzzy model system. Moreover, a design condition and criteria for the controller are given by the feasibility of linear matrix inequalities (LMIs). So the simulations verify that the designed technique will assure H_∞ performance of the nonlinear system and attenuate the effect of external disturbances.

Keywords: Takagi–Sugeno fuzzy (TSF) system, Parallel Distributed Compensator (PDC), Linear Matrix Inequality (LMI).

I. INTRODUCTION

As the main objectives of the control theory is designing a controller to be able to robustly stabilize the real dynamic systems which are nonlinear. Therefore, a lot of studies are trying hard to find a simple and effective technique to control real systems, where the stability conditions and control techniques for these systems are generally challenging. In the last decades, the fuzzy logic control (FLC) is regarded to be a powerful technique to control complex nonlinear system. In this direction, FLC using Takagi-Sugeno fuzzy (TSF) system gains great attentions in the field of nonlinear control systems as a result of its simplicity, systematically and effectiveness [1-8]. In TSF systems, the nonlinear model is blending of linear time-invariant subsystems connected by weighted membership functions. This allows the nonlinear system to be presented in the form of local linear models set. Therefore, linear systems approaches can be employed to study and design closed-loop controlled systems. Based on TSF model, the technique of Parallel Distributed Conpensator (PDC) is used to control and stabilize complex nonlinear systems by using state feedback controller for each rule in TSF model [8–11].

Nowadays, a PDC scheme has been intensively used in solving many academic and industrial problems because the concept of PDC is simple and easy to develop. Such as, the TSF model of an inverted pendulum is presented and the PDC scheme is developed to stabilize it in [12]. In [13] switched PDC is synthesized for R/C Hovercraft. Baranyi et al. [14] propose a PDC controller for stabilization of a real 3-DOF RC helicopter to get a good speed response. A practical approach of a Takagi-Sugeno fuzzy (TSF) modeling and control for F16 aircraft is presented in [15]. The design and implementation of PDC controller for temperature control are introduced in [16]. In [17], the fuzzy controller is presented utilizing the PDC technique and it is synthesized in the design of Maximum Power Point Tracking (MPPT) for the solar PV power system. The PDC fuzzy based controller has been proposed for wind energy conversion systems in [18].

Recently, a study of the stability of TSF system has been a major and important problem in this regard, therefore considerable efforts are being exerted in order to reach and find the finest and easiest solutions using Lyapunov function approach. Also, the optimal procedure using linear matrix inequality (LMI) stability problem formulation is very attractive, because the optimal procedure of the LMI approach can solve the difficulties in finding control design conditions, for TSF systems. Despite the prosperity of LMI-based approach, there is a significant issue that the solutions related to LMIs are quite conservative. Hence, considerable researches on relaxed criteria and stability conditions for TSF system have been carried out to reduce the conservatism. So some improvements have been reported to relax the conditions for TSF systems exploiting a common quadratic Lyapunov function [19,20]. Whereas, other studies were utilizing Piece-wise Lyapunov function [22-24] and fuzzy Lyapunov function [24–30] to construct LMI relaxations conditions for TFS systems. Moreover, such different kind of relaxing techniques like non-PDC scheme [32-34] and switched PDC techniques [3,5,6,30,34] are frequently used to reduce the conservatism. The relaxed conditions and schemes are worthy addressed not only for ensuring the stability of TSF system but also for improving other requirements such as speed performance, the domain of attraction, input-output constraint and so on.

The scope of this work is to propose a robust H∞ controller for TSF model. The proposed technique has a slack matrix which provides less conservative conditions, therefore, yields good results. The main purpose of proposed PDC is to robustly stabilize the closed loop system under different conditions. Based on LMI technique, the state feedback controller's gain will be obtained. Finally, Two dynamic systems are simulated to prove the ability of the proposed technique to ensure the stability properties of the nonlinear system and to guarantee robustness against external disturbance. The structure of this paper is as follows. The discrete TSF models are introduced in Section II. The proposed PDC controller is reported in Section III. Section IV, simulation results are shown to emphasize the efficiency and explore the effectiveness of the proposed method and compare it with PDC scheme. Finally in section V, the conclusion is given.

Notation: The superscript "T" represents the transpose of a matrix. The notation "*" is used as an ellipsis for terms that are induced by symmetry.

II. TSF MODEL

The TSF model is given by fuzzy IF-THEN rules and represents nonlinear system by linear time-invariant subsystems connected by weighted membership functions [1]. So the i-th rule of the discrete fuzzy models (DFS) is represented by:

DFS model rule i:

If $\delta_1(z)$ is M_{i1} ... and $\delta_P(z)$ is M_{iP},

$$Then \begin{cases} x(z+1) = A_i x(z) + B_{1i} w(z) + B_{2i} u(z) \\ y(k) = C_i x(z) + D_{1i} w(z) + D_{2i} u(z) \end{cases} \qquad (1)$$

Here i=1,2,...,r where r is the number of model rules; M_{ij} is the fuzzy set ; $x(z) \in R^n$ is the state vector ; $u(z) \in R^m$ is the control input; $y(z) \in R^q$ is the output vector ; $w(z) \in R^s$ is the energy-bounded disturbance. $A_i, B_{1i}, B_{2i}, C_i, D_{1i}$ and D_{2i} are of appropriate dimensions ; $\delta_1(z),..., \delta_P(z)$ are known premise variables.

The final outputs of the fuzzy systems are inferred as follows:

$$x(z+1) = \sum_{i=1}^{r} \eta_i(\delta(z))(A_i x(z) + B_{1i} w(z) + B_{2i} u(z)) \qquad (2)$$

$$y(z) = \sum_{i=1}^{r} \eta_i(\delta(z))(C_i x(z) + D_{1i} w(z) + D_{2i} u(z)) \qquad (3)$$

Where $\delta(z) = [\delta_1(z) \;\; \delta_2(z) \cdots \delta_P(z)]^T$

$$\eta_i(\delta(z)) = \frac{\prod_{j=1}^{P} M_{ij}(\delta_j(z))}{\sum_{i=1}^{r} \prod_{j=1}^{P} M_{ij}(\delta_j(z))} \qquad (4)$$

For all z. The term $M_{ij}(\delta_j(z))$ is the grade of membership function of $(\delta_j(z))$ in M_{ij} .For brief expression, we will denote $\eta_i(z) = \eta_i(\delta(z))$. We have

$$\begin{cases} \sum_{i=1}^{r} \eta_i(z) = 1 \\ \eta_i(z) \geq 0 \end{cases} \qquad \forall i \qquad (5)$$

III. FUZZY CONTROLLER DESIGN

For the above TSF model, fuzzy controller can be constructed using the PDC technique. The PDC control concept is synthesized by designing a linear compensator to control each of fuzzy rules. So the fuzzy controller uses the same fuzzy sets of TSF model.

Control rule i:

$$IF\ \delta_1(z)\ is\ M_{i1}\ and \cdots and\ \delta_r(z)\ is\ M_{ir}\ , then$$
$$u(z) = -F_i\, x(z), \quad i = 1,2,\cdots,r$$

Where $F_i\ (i = 1,2,\cdots,r)$ is the gain of feedback controller. Hence, the PDC controller can be obtained by

$$u(z) = -\sum_{i=1}^{r} \eta_i(\delta(z)) F_i\, x(z), \qquad (6)$$

By combining (2),(3) and (6) then the closed-loop discrete fuzzy system is:

$$x(z+1)=\left(A(\delta(z))+B_1(\delta(z))\right)x(z)+B_2(\delta(z))w(z)$$
$$y(z)=\left(C(\delta(z))+D_1(\delta(z))\right)x(k)+D_2(\delta(z))w(z)$$
(7)

Where

$$A(\delta(z))=\sum_{i=1}^{r}\delta_i(z)A_i,\ B_1(\delta(z))=\sum_{i=1}^{r}\delta_i(z)B_{1i}$$
$$B_2(\delta(z))=\sum_{i=1}^{r}\delta_i(z)B_{2i},\ C(\delta(z))=\sum_{i=1}^{r}\delta_i(z)C_i$$
$$D_1(\delta(z))=\sum_{i=1}^{r}\delta_i(z)D_{1i},\ D_2(\delta(z))=\sum_{i=1}^{r}\delta_i(z)D_{2i}$$

Now, a robust H_∞ state feedback controller is proposed using PDC technique. The new controller will be used to control and stabilize unstable system. The proposed PDC will exploit an LMI-based scheme to conduct a solution to the controller's gain of state feedback controller for TSF systems in discrete form. So the main objective of the proposed controller is to stabilize the overall system if there is no disturbance. Also, under zero initial conditions, the controlled system fulfill the exponential H_∞ control performance γ.

Theorem: For a given $\mathcal{H}_\infty$ performance level $\gamma > 0$, if there exists symmetric matrix $G = G^T > 0$, $\Psi_{ij}=\Psi^T_{ij}$, $E > 0$, Y_j, $1\le i,j\le q$, satisfying:

$$\Psi_{ii} \ge 0, \tag{10}$$
$$\varepsilon\Psi_{ij} > 0,\quad 1\le i,j\le q,\ \varepsilon\in\{-1,1\} \tag{11}$$
$$\Gamma_{ii} < 0,\ 1\le i\le q, \tag{12}$$
$$\frac{1}{q-1}\Gamma_{ii}+\frac{1}{2}\left(\Gamma_{ji}+\Gamma_{ij}\right)<0, 1\le i\ne j\le q \tag{13}$$

Where

$$\Gamma_{ij}=\begin{bmatrix} G-E-E^T & * & * & * \\ 0 & -\gamma I & * & * \\ A_iE+B_{2i}Y_j & B_{1i} & -G & * \\ C_iE+D_{2i}Y_j & D_{1i} & 0 & -\gamma I \end{bmatrix}+\Psi_{ij},\ 1\le i,j\le q \tag{14}$$

Then the controller (6) with

$$K_i = Y_i G^{-1} \tag{15}$$

Provides the TSF system is asymptotically stable satisfying H_∞ performance level γ.

The slack matrix Ψ_{ij} is used to reduce the conservatism results from utilizing the LMI technique. So, the theorem provides a relaxed stability criteria to calculate the controller's gain of the PDC scheme. The algorithm is explicit to analysis the TSF control system and there no much LMI's variables to evaluate.The main advantage of this scheme that it is very easy and simple to implement the proposed controller practically because there is no additional information that you must know about the nonlinear system and there is no complexity in the procedures like other nonlinear control techniques.

Remark: If we choose $\Psi_{ii} = \Psi_{ij} = 0$, $1\le i, j\le q$ in the theorem, then the proposed PDC will be the same PDC scheme as in corollary in [35].

IV. SIMULATION RESULTS

In this section, discrete-time TSF control system is employed to two dynamic systems to make them stable and more robust. The TFS system model is presented. Then the proposed PDC controller is implemented and the corresponding simulations are given.

With changing the amplitude of the disturbance, the proposed PDC controller is compared with PDC scheme. It should be mentioned that the proposed controller has the same response or better than PDC scheme.

A. *Example 1*

Consider a following discrete-time TSF model of steering a model car[11]. The system model with sampling time T=0.5 exposed to disturbance:

$$x(z+1)=\sum_{i=1}^{2}\delta_i(z)\left[A_i x(z)+B_{wi}w(z)+B_{ui}u(z)\right]$$
$$y(z)=\sum_{i=1}^{2}\delta_i(z)\left[C_i x(z)\right]$$

Where the grade of membership functions are:

$$M_1(x(z))=1-\frac{|x_1(z)|}{\pi},\ M_2(x(z))=1-M_1(x(z))$$
$$A_1=\begin{bmatrix}1 & 0\\ 0.5 & 1\end{bmatrix},\ B_{w1}=\begin{bmatrix}0\\ 0.1\end{bmatrix}$$
$$A_2=\begin{bmatrix}1 & 0\\ 0.0016 & 1\end{bmatrix},\ B_{w2}=\begin{bmatrix}0\\ 0.1\end{bmatrix}$$
$$B_{u1}=B_{u2}=\begin{bmatrix}0.0412\\ 0.4123\end{bmatrix},$$
$$C_1=C_2=\begin{bmatrix}0.1708 & 0.0804\end{bmatrix}$$

Table 1
Some Cases of Initial Position of the Model Car

	$x_1(0)$(deg)	$x_2(0)$(m)
Case1	0	6.4
Case2	-180	5.5
Case3	-90	-6.4
Case4	180	-5.5

Where $x_1(z)$ is the angle of the car with respect to horizontal, while $x_2(z)$ is the vertical position of the rear end of the car. The nonlinear system will be controlled and the response will be shown under different initial condition. The model car is exposed to external disturbance $\mathbf{w} = \rho * \mathbf{sin}(10z)/20(1+2z)$ to notice the effect of proposed PDC on the output and compare the results with PDC scheme[35].Applying the theorem of the proposed technique (14), we get:

$$F_1 = [-6.2666 \quad -1.3332], \quad F_2 = [-5.6020 \quad -1.3331]$$

Fig. 1. and Fig. 2. display the response of the controlled system from the initial positions in [11]. In cases of changing the initial state of the system, we try to verify that the proposed controller will stabilize the system under different positions. It is realized that the both controller are effective to bring the car back to zero position very quickly. But despite the fact that the states in PDC are faster than the proposed controller, however, it is more smoothly without distortions in the proposed PDC. The state trajectories of the controlled system with change in the value of disturbances are shown in Fig. 3. and Fig. 4. It is noticed that the proposed PDC controller and PDC scheme [35] stabilize and control the system. The obtained H_∞ performance via proposed PDC equal 0.0088, whilst the value using PDC equal 0.736. So from the above computational we noticed that the proposed PDC gives better response.

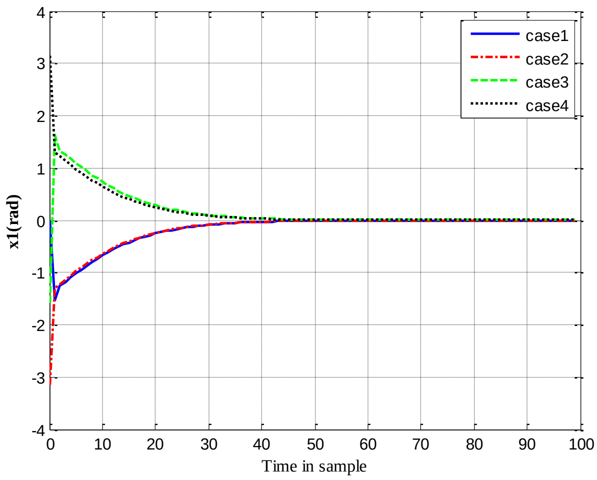

Fig. 1. Response of horizontal angle x_1

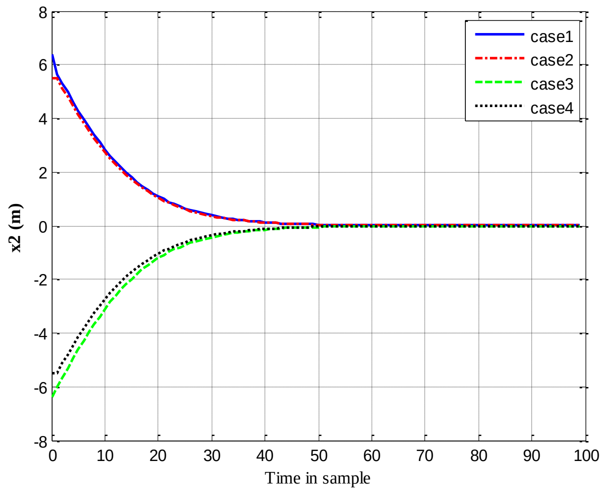

Fig. 2. Response of vertical position

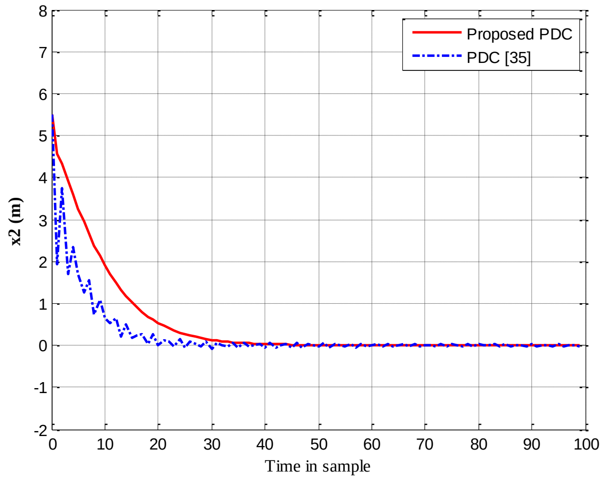

Fig. 3. State response of x_2 with ρ=50

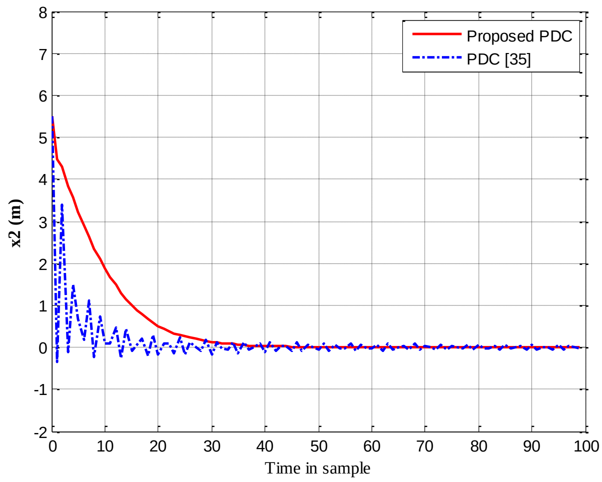

Fig. 4. State response of x_2 with ρ=100

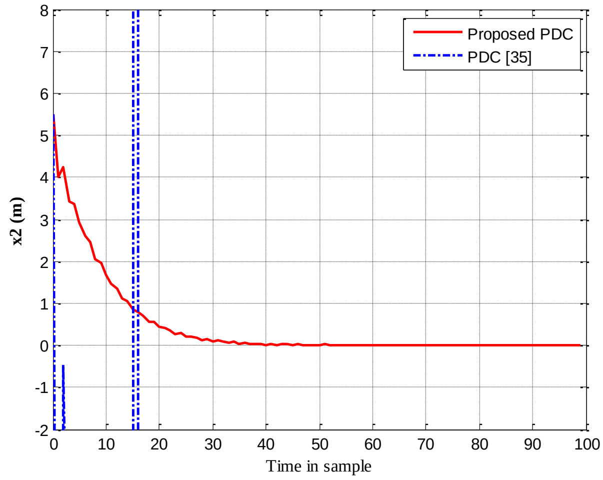

Fig. 5. State response of x_2 with ρ=400

From Fig.5, the gain of disturbance is much increased and both controllers try to stabilize the response. Nevertheless, it is depicted that in case of PDC the system is unstable. Where, in the proposed scheme the system is still stable with perfect response without deformity. This is because the proposed algorithm has slack matrix variables to minimize the conservatism arises from using of Lyapunov function.

B. *Example 2*

The second example also presents the performance and robust stabilization of the proposed H_∞ controller. So consider a discrete-time of a DC motor controlling an inverted pendulum via a gear train [12]:

$$A_1 = \begin{bmatrix} 2.3576 & 0.6244 & 0.0469 \\ 6.1187 & 1.8883 & 0.1551 \\ -4.5991 & -1.5506 & -0.1222 \end{bmatrix},$$

$$B_{u1} = B_{w1} = \begin{bmatrix} 0.0916 \\ 0.4693 \\ 0.6529 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0.4217 & 0.0350 \\ 0 & 0.6503 & 0.0720 \\ 0 & -0.7196 & -0.0693 \end{bmatrix},$$

$$B_{u2} = B_{w2} = \begin{bmatrix} 0.0916 \\ 0.4693 \\ 0.6529 \end{bmatrix}$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The grade of membership functions is given:

$$M_1(x(z)) = \frac{\sin(x_1(z))}{x_1(z)},$$

$$M_2(x(z)) = 1 - M_1(x(z))$$

The disturbance to be w=2 $(1+z)^{-1}\sin(10z)$ and apply to the system to analysis the controlled system with and without disturbance. The simulation results are depicted in Figs 6-8. These figures will explore the effectiveness of proposed PDC in attenuation the disturbance's influence on the output of the closed loop system.

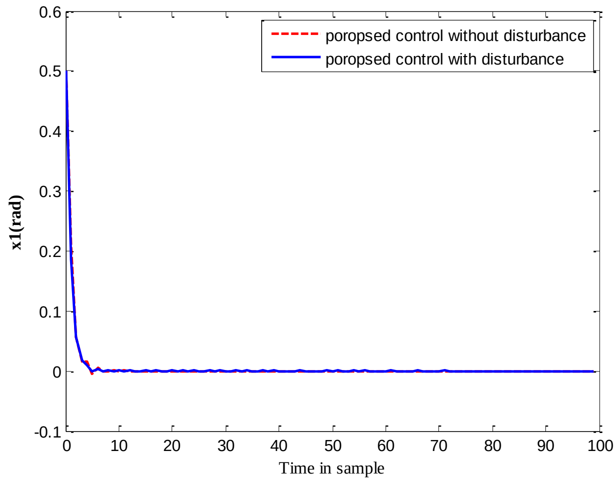

Fig. 6. State trajectory of x_1

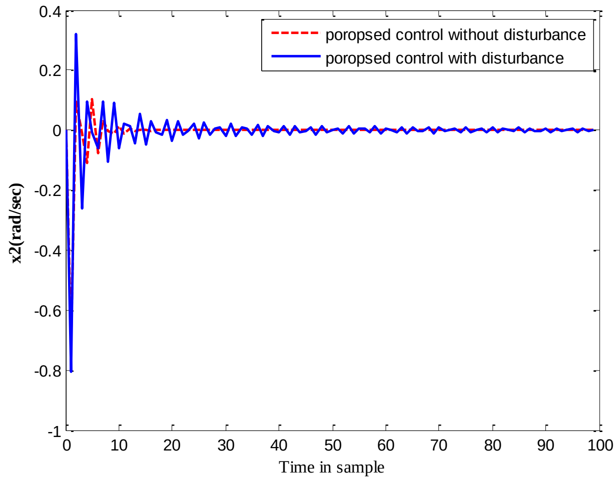

Fig. 7. State trajectory of x_2

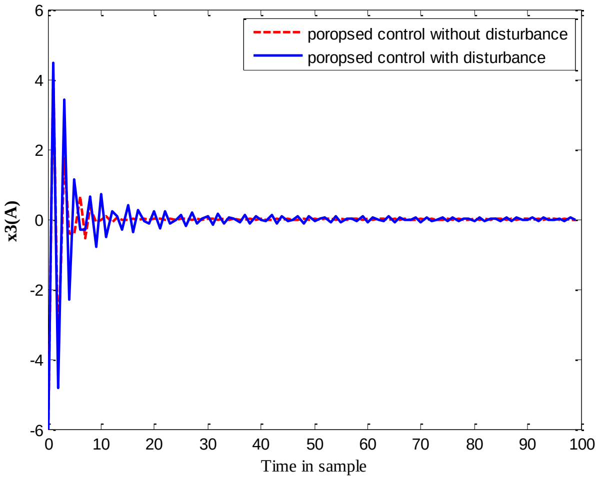

Fig. 8. State trajectory of x_3

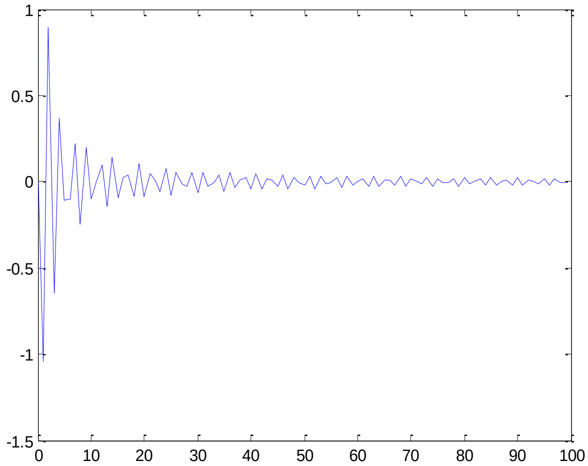

Fig. 9. The Disturbance

The responses indicate that the proposed PDC control and robustly stabilizes the system. The state response of the proposed controller with disturbance is close to the nominal response without disturbance. This addresses that the proposed technique ensures the robustness of the system with H_∞ performance equal to 0.0538 which is less than one, which indicates that the new controller gives good performance with ensuring the robustness of the closed loop system.

V. CONCLUSIONS

This paper analyses discrete-time TSF system control. A new algorithm is addressed to design an H_∞ state feedback controller. The proposed algorithm is a simple controller to robustly stabilize the highly nonlinear systems under different conditions. The proposed technique not only guarantees the stability, but also reduces the conservative and improves performance of the closed loop nonlinear system and ensures robustness to external disturbances. The stabilization criteria and robust stability are formulated directly as LMIs and it can be solved simply by utilizing convex programming tools. Numerical results report that the proposed algorithm provides better results than the other controller scheme.

REFERENCES

[1] Tanaka K, Ikeda T, Wang HO.: *Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: quadratic stabilizability, H∞ control theory, and linear matrix inequalities*. In: IEEE Trans Fuzzy Syst, 4(1996) , No.1 , 1996, pp.1–13.

[2] Kazuo T, Wang HO.: *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. John Wiley & Sons, Inc., 2001.

[3] Wang T, Tong S.: *H∞ control design for discrete-time switched fuzzy systems*. In: Neurocomputing, 151(2015), 2105, pp.782–789.

[4] Tognetti ES, Oliveirab RCLF, Peres PLD.: *H∞ and H2 nonquadratic stabilisation of discrete-time Takagi–Sugeno systems based on multi-instant fuzzy Lyapunov functions*. In: Int J Syst Sci , 46(2015), 2015 , pp. 76–87.

[5] Xiang W, Xiao J, Han L.: *Switching PDC control for discrete-time T–S fuzzy system: A membership function ranking approach*. In: J Franklin Inst 351(2014), No.7, 2014,pp.3536–3558.

[6] Xiang W, Xiao J.: *H∞ control synthesis of switched discrete-time fuzzy systems via hybrid approach. Optim Control Appl Methods* 312(2013), 2013, pp.298–312.

[7] Hsiao FH.: *Robust H∞ fuzzy control of dithered chaotic systems*. In: Neurocomputing, 99(2013) , 2013 ,pp.509–520.

[8] Hoang Duong T, Apkarian P, Narikiyo T, Kanota M.: *New fuzzy control model and dynamic output feedback parallel distributed compensation.* In: IEEE Trans Fuzzy Syst, 12 (2004) , No.1 , 2004 , pp.13–21.

[9] Feng G.: *A Survey on Analysis and Design of Model-Based Fuzzy Control Systems.* In: IEEE Trans Fuzzy Syst, 14 (2006), No.5 , 2006, pp. 676–697.

[10] Rajesh R, Kaimal MR.: *T–S fuzzy model with nonlinear consequence and PDC controller for a class of nonlinear control systems.* In: Appl Soft Comput, 7(2007), No.3 , 2007, pp. 772–782.

[11] Nachidi M, Benzaouia A, Tadeo F, Rami MA.: *LMI-Based Approach for Output-Feedback Stabilization for Discrete-Time Takagi – Sugeno Systems.*In: IEEE Trans Fuzzy Syst, 16(2008), No.5 , 2008 , pp.1188–1196.

[12] Yoon T-S, Wang F, Park S-K, Kwak G-P, Ahn H-K, Gao H, et al.: *Linearization of T-S fuzzy systems and robust H∞ control*. In: J Cent South Univ Technol (Engl Ed) , 18 (2011), No.1, 2011,pp. 140–145.

[13] Tanaka K, Iwasaki M, Wang HO.: *Switching control of an R/C hovercraft: Stabilization and smooth switching*. In: IEEE Trans Syst Man, Cybern Part B Cybern, 31(2001), No.6, 2001, pp. 853–863.

[14] Baranyi P, Korondi P, Tanaka K.: *Parallel distributed compensation based stabilization of a 3-DOF RC helicopter: A tensor product transformation based approach*. In: J Adv Comput Intell Intell Informatics 13(2009), No.1 , 2009, pp. 25–34.

[15] Tanaka M, Yamaguchi K, Ogura D, Chen Y, Tanaka K.: *Nonlinear Control of F16 Aircraft via Multiple Nonlinear Model Generation for Any Trimmed Equilibriums*. In: Int J Fuzzy Syst 16 (2014), No.2 , 2014, pp. 140–152.

[16] Yordanova S, Sivchev Y.: *Design and Tuning of Parallel Distributed Compensation-based Fuzzy Logic Controller for Temperature*. In: J Autom Control, 2(2014), No.3, 2014, pp.79–85.

[17] Abid H, Toumi A, Chaabane M.: *MPPT Algorithm for Photovoltaic Panel Based on Augmented Takagi-Sugeno Fuzzy Model*. In: ISRN Renew Energy, 2014, pp.1–10.

[18] Kamal E, Aitouche A, Ghorbani R, Bayart M.: *Robust fuzzy fault-tolerant control of wind energy conversion systems subject to sensor faults.* In: IEEE Trans Sustain Energy, 3(2012), No.2, 2012 , pp. 231–41..

[19] Wang WJ, Sun CH.: *Relaxed stability and stabilization conditions for a T-S fuzzy discrete system*. In: Fuzzy Sets Syst, 156 (2005), No.2, 2005 , pp.208–225.

[20] Montagner VF, Oliveira RCLF, Peres PLD.: *Convergent LMI Relaxations for Quadratic Fuzzy Systems*. In: IEEE Trans Fuzzy Syst, 17 (2009),No.4 , 2009, pp. 863–873.

[21] Chen YJ, Ohtake H, Tanaka K, Wang WJ, Wang H.:*Relaxed Stabilization Criterion for T-S Fuzzy Systems by Minimum-type Piecewise Lyapunov Function Based Switching Fuzzy Controller*. In: IEEE Trans Fuzzy Syst, 20(2012), No.2,2012,pp.1166–1173.

[22] Wang L, Feng G.: *Piecewise H infinity: controller design of discrete time fuzzy systems*. In: IEEE Trans Syst Man Cybern B Cybern, 34(2004), No.1, 2004, pp. 682–686.

[23] Wang W-J, Chen Y, Sun C-H.: *Relaxed stabilization criteria for discrete-time fuzzy systems based on a switching fuzzy model and piecewise Lyapunov functions*. In: J Control Theory Appl, 37(2007), No.3, 2007,pp. 551–559.

[24] Rhee B-J, Won S.: *A new fuzzy Lyapunov function approach for a Takagi–Sugeno fuzzy control system design*. In: Fuzzy Sets Syst 157 (2006) , No.9, 2006, pp. 1211–1228.

[25] Chang Y, Tsai Z.: *A Fuzzy Lyapunov Function Approach for Robust Fuzzy Control Design of Nonlinear Systems with Model Uncertainties*. In: Tamkang J Sci Eng, 10(2007), No.3, 2007, pp. 201–210.

[26] Li J, Zhou S, Xu S.: *Fuzzy control system design via fuzzy Lyapunov functions*. In: IEEE Trans Syst Man Cybern B Cybern, 38(2008), No.6, 2008 , pp. 1657–61.

[27] Lee DH, Park JB, Joo YH.: *A new fuzzy lyapunov function for relaxed stability condition of continuous-time takagi-sugeno fuzzy systems.* In : IEEE Trans Fuzzy Syst, 19 (2011), No.4, 2011, pp. 785–91.

[28] Faria F a., Silva GN, Oliveira V a.: *Reducing the conservatism of LMI-based stabilisation conditions for TS fuzzy systems using fuzzy Lyapunov functions*. In: Int J Syst Sci 44(2013), No.10 , 2013, pp. 1956–1965.

[29] Campos VCS, Souza FO, Torres L a B, Palhares RM.: *New stability conditions based on piecewise fuzzy lyapunov functions and tensor product transformations*. In: IEEE Trans Fuzzy Syst 21(2013) , No.4, 2013, pp. 748–760.

[30] Faria F a., Valentino MC, Oliveira V a. *A fuzzy Lyapunov function approach for stabilization and H∞ control of switched TS fuzzy systems.* In: Appl Math Model 38 (2014), 2014, pp. 4817–4834.

[31] Lam HK, Leung FHF. *LMI-based stability and performance conditions for continuous-time nonlinear systems in Takagi-Sugeno's form*. In: IEEE Trans Syst Man Cybern B Cybern, 37(2007), No.5 , 2007, pp.1396–406.

[32] Lee DH, Park JB, Joo YH.: *A fuzzy Lyapunov function approach to estimating the domain of attraction for continuous-time Takagi-Sugeno fuzzy systems*. In: Inf Sci (Ny), 185(2012), No.1, 2012, pp. 230–48.

[33] Baocang D.: *Stabilization of Takagi–Sugeno Model via Nonparallel Distributed Compensation Law*. In: IEEE Trans Fuzzy Syst, 18(2010), No.1, 2010 ,pp.188–94.

[34] Yang H, Zhang L, Liu X.: *Robust tracking control for switched fuzzy systems with fast switching controller*.In: Math Probl Eng , 2012, pp1-21.

[35] Jiuxiang Dong, Guang-Hong Yang. *H∞ Controller Synthesis via Switched PDC Scheme for Discrete-Time T--S Fuzzy Systems*. In: IEEE Trans Fuzzy Syst, 17(2009), No.3, 2009, pp. 544–555.