Maximum Power Control for Photovoltaic Power based on Fuzzy Takagi-Sugeno Model

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Abstract: This paper deals with an intelligent control method for the Maximum Power Point Tracking (MPPT) of a photovoltaic system under variable temperature and radiation conditions. The MPPT is based on the Takagi-Sugeno (T-S) fuzzy model approach. A DC/DC buck converter has been connected between the photovoltaic panel and the load to regulate the output power of the PV panel array. A fuzzy MPPT controller based on TS fuzzy model is proposed in which controller gains are obtained by solving Linear Matrix Inequalities (LMIs). Numerical simulations are presented and compared to other types of control to check the performance of this type of control.

Keywords: Photovoltaic (PV), Maximum Power Point Tracking (MPPT), Takagi-Sugeno (TS) fuzzy controller, Linear Matrix Inequality (LMI).

1. Introduction

Photovoltaic (PV) power generation is expanding very rapidly as an important solution due to energy crisis and environmental issues such as pollution and global warming effect. While PV is a commercially available and reliable technology with a significant potential for long-term growth in nearly all world regions, PV systems are becoming a very attractive alternative for many terrestrial applications [1].

Unfortunately the actual energy conversion efficiency of PV module is rather low. So to overcome this problem and to get the maximum possible efficiency, the design of all the elements of the PV system has to be optimized [2]. Since solar power uses the photovoltaic effect to transform solar energy into electrical energy, the PV panel is a nonlinear power source. There are different approaches to control a nonlinear system. The feedback stabilization of nonlinear systems where a linear feedback control is

designed for the linearization of the system about a nominal operating point is considered as a typical approach. Other approaches such as feedback linearization [3], [36] and [4] are rather involved and tend to result in rather complicated controllers.

In recent years, there have been significant advances in the study of the stability analysis and controller synthesis for the so-called Takagi-Sugeno (TS), also known as the TS fuzzy systems, which have been used to represent certain or uncertain complex nonlinear systems. In fact, TS fuzzy model has been greatly used to describe renewable energy systems such as photovoltaic systems and wind energy systems, e.g., [6] and [34].

In the TS fuzzy model, the local dynamics in different state-space regions are represented by linear state space models. The overall model of the system is obtained by the fuzzy blending of these local models.

Stability and design issues of fuzzy control systems have been discussed in an extensive literature, e.g., [5]-[7]. The control design is carried out based on the fuzzy model by the so-called parallel distributed compensation (PDC) scheme [5]. For each local linear model, a linear feedback control is designed. The resulting overall controller is again a fuzzy blending of the individual linear controllers. Originally, Tanaka and Sugeno [5], [7] have provided certain conditions those are sufficient for the stability of the T-S fuzzy systems in the sense of Lyapunov [5], [8]. The conditions for the existence of a common Lyapunov function are obtained by solving linear matrix inequalities (LMIs).

At a given temperature and radiation level, PV cells supply maximum power at one particular operation point called the Maximum Power Point (MPP). Unlike

conventional energy sources, it is desirable to operate PV systems at its MPP.

Tracking the maximum power point (MPP) of a photovoltaic module/array is an essential task in a PV control system, since it maximizes the power output of the PV system, and therefore maximizes the PV module's efficiency [9]. However, the MPP locus varies over a wide range, depending on PV array temperature and radiation intensity. Maximum power point trackers (MPPTs) play an important role in PV power systems because they maximize the power output from a PV system for a given set of conditions, and therefore maximize the array efficiency. Thus, an MPPT can minimize the overall system cost. Many techniques have been proposed [30],[31], such as Perturb-and-Observe (P&O) which is the most commonly used; it is an iterative method of obtaining MPP, modified P&O, Three Point Weight Comparison [10],[11], Constant Voltage (CV) [12], Incremental Conductance (IC); it is an alternative to the PO method and based on differentiating the PV power with respect to voltage and setting the result to zero [12], Short Current Pulse [13], Open Circuit Voltage [14];this method is based on the voltage of PV generator at MPP which is approximately linearly proportional to its open circuit voltage, the Temperature Method [15] which uses the temperature to determine the MPP voltage, and methods derived from it [15]. These techniques are easily implemented and have been widely adopted for low-cost applications.

As is well known, each technique has advantages and drawbacks. Simplicity and ease of implementation present the main factors that can make some methods popular such as P&O method. However, P&O has some limitations like it fails under rapidly changing environment conditions.

On the other hand, speed and accuracy are considered as the principle parameters provided by some other methods like fuzzy logic and neural network.

Still, most of MPPT approaches lack strict convergence analysis, subsequently, only approximate tracking of maximum power is accomplished. Even if neural network technique provides better performance than traditional methods, its disadvantage is the necessity of solar radiation and cell temperature measurement. Furthermore, two control loops are required in the case that the dynamics of the converter are considered. In fact, we need to determine the maximum power of the PV array and then, to control it according to the reference voltage.

Several practical applications of fuzzy logic control (FLC) and classical stability analysis methods for fuzzy

systems have been presented [37],[38],[39] and [40], but FLC methods still lack stability and performance analysis. To achieve the MPPT under strict theoretical analysis, the TS fuzzy model-based control is applied.

In this paper, the solar power generation system is described by nonlinear equations, and then it is transformed into an augmented system which is described by T-S fuzzy model. We develop an MPPT based on Parallel Distributed Compensation (PDC) method which allows transferring the maximum power from the panel to the load. We have used Lyapunov approach to prove the stability of the system and Linear Matrix Inequalities (LMI) to calculate the control parameters.

This paper is organized as follows. In section 2, we introduce the solar power generation system. In section 3 we present MPPT using traditional methods; (P&O) and Mamdani fuzzy logic. The forth section deals with the T-S fuzzy model of the system. In section 5, T-S fuzzy controller is presented. First, we describe the system by an augmented T-S fuzzy model. Then, we present the stability analysis of the closed loop system. To show the control performance, numerical simulation results of PV energy system are discussed in section 6. Finally, conclusions are drawn in the final section.

2. PV System Characteristics

In this paper, the solar power generation system considered consists of a photovoltaic array and a DC/DC buck converter. A schematic overview of the PV energy system is shown in Fig.1.
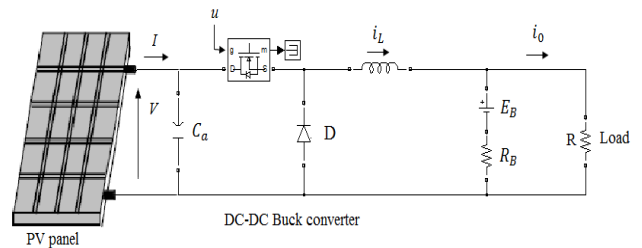

Fig.1. Configuration of the PV energy system

2.1 Photovoltaic Array:

We consider a PV panel array arranged in np-parallel solar cells and ns - series cells. The output current and the PV array voltage are denoted respectively as I and V. The current-voltage characteristic equation of a PV array can be described by a light-generated current source and a diode. If internal shunt and series resistances are neglected, the

output current of the PV array is described by the following equation [6]:

$$I = n_p I_{ph} - n_p I_{rs} (e^{\frac{k_{pv}V}{n_s}} - 1) \qquad (1)$$

where n_p and n_s are the number of the parallel and series cells $k_{pv}=q/(pKT)$, with $q=1.6\ 10^{-19}\ C$ is the electronic charge, $K=1.38\ 10^{-23}\ JK^{-1}$ is the Boltzmann's constant, T is the cell temperature, p is an ideal factor, I_{ph} and I_{rs} are respectively the photocurrent and reverse saturation current which are given by

$$I_{ph} = (I_{sc} + K_I (T - Tr)) \frac{\lambda}{100} \qquad (2)$$

$$I_{rs} = I_{rr} (\frac{T}{T_r})^3 \exp(q E_{gp} (1/T_r - 1/T)) / pK \qquad (3)$$

Where I_{sc} is the short-circuit cell current at reference temperature and radiation and I_{rr} is the reverse saturation current at the reference temperature T_r, E_{gp} is the band-gap energy of the semiconductor used in the cell, K_I is the short-circuit current temperature coefficient and λ is the solar radiation in W/m^2. The power generation of the PV array [6]:

$$P = I.V$$
$$P = n_p I_{ph} V - n_p I_{rs} V (\exp(K_{pv} V / n_s) - 1) \qquad (4)$$

The system operates at the maximum power point if the condition $dP/dV = 0$ is satisfied.
At the maximum power point and by taking the partial derivate of the power array with respect to the PV voltage we have:

$$\frac{dP}{dV} = 0 \Rightarrow I + V \frac{dI}{dV} = 0 \qquad (5)$$

$$\frac{dP}{dV} = I - \frac{n_p k_{pv}}{n_s} I_{rs} V \exp(\frac{k_{pv}V}{n_s}) \qquad (6)$$

Due to the high nonlinearity, the maximum power point is difficult to be solved from (6). For this reason it is difficult to achieve MPPT by traditional methods.

2.2 DC/DC buck converter

When a direct connection is carried out between the source and the load, the output of the PV module is seldom maximum and the operating point is not optimal [16] .To overcome this problem, it is necessary to add an adaptation device, an MPPT controller with a DC–DC converter, between the source and the load [17].The Buck DC-DC converter is applied so in order to adjust the array photovoltaic to the voltage range.

Consider the PV power control system using a buck converter, as shown in Fig.1, the dynamic model can be described by the following state equations:

$$\begin{cases} \dfrac{di_L}{dt} = -\dfrac{1}{L}(R_L + R_B(1 - \dfrac{i_0}{i_L}))i_L + \dfrac{uV}{L} - \dfrac{E_B}{L} \\ \dfrac{dV}{dt} = \dfrac{1}{C_a}(I - ui_L) \end{cases} \qquad (7)$$

Where V is the PV array voltage and i_L is the current on the inductance L.

R_L and R_B are the internal resistances on the inductance L and the battery, i_0 is a measurable current on the load, u is the duty ratio of the Pulse-Width-Modulated (PWM) signal to control the switching MOSFET, $u \in \{0,1\}$ defines the switch position.

3. MPPT Using Traditional Methods

3.1 Perturb & observe MPPT Technique

The Perturb and Observe method is considered as the most algorithms used in practice by the majority of authors to track the maximum power point [18],[19],[20],[21],[22] and [23] among others . Its principle is based on the perturbation of the system by the increase/decrease of the duty-cycle of the converter and the observation of the effect on the output power [24], [25] and [26].

If a given perturbation leads to an increase (decrease) in the output power of the PV, then the subsequent perturbation is generated in the same (opposite) direction, otherwise the perturbation is inverted. The duty cycle is varied and the process is repeated until the maximum power point has been reached, that leads the system to oscillate about the MPP.

In Fig.2 it is given a flowchart which describes the P & O technique.
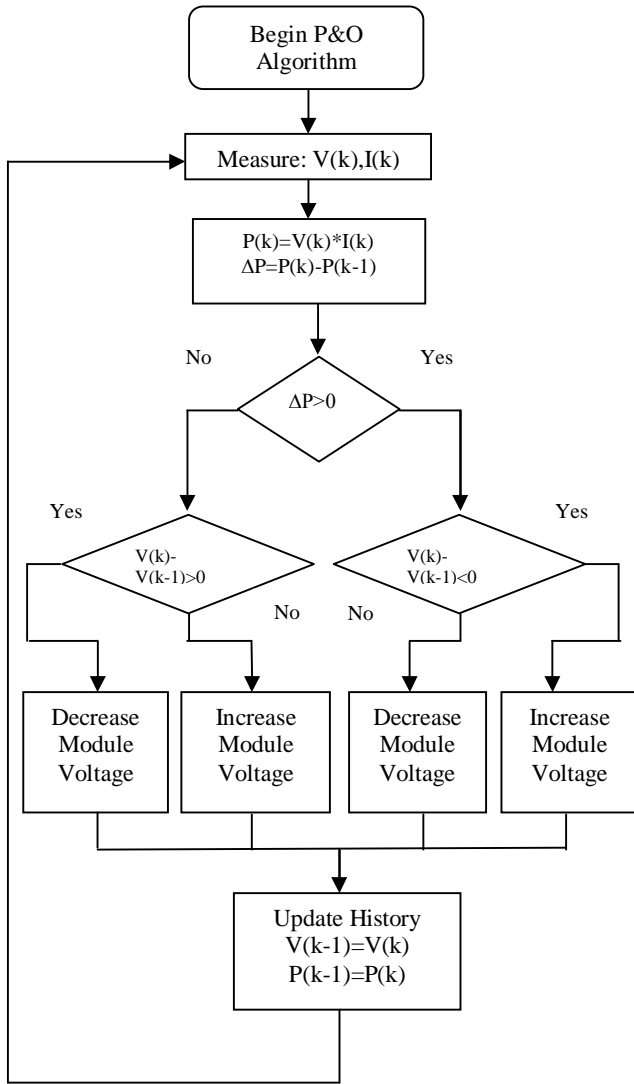
Begin P&O
Algorithm

Measure: V(k),I(k)

P(k)=V(k)*I(k)
ΔP=P(k)-P(k-1)

ΔP>0

No   Yes

Yes   Yes

V(k)-
V(k-1)>0

V(k)-
V(k-1)<0

No   No

Decrease
Module
Voltage

Increase
Module
Voltage

Decrease
Module
Voltage

Increase
Module
Voltage

Update History
V(k-1)=V(k)
P(k-1)=P(k)

Fig.2. Flowchart of the Perturb and Observe method

3.2 *Mamdani's fuzzy inference method*

Fuzzy logic controllers have been recently introduced in the tracking of the MPP in PV systems [27], [28] , [29]and [35].
The fuzzy controller has four main components: (i) the "rule-base" holds the knowledge, in the form of a set of rules, of how best to control the system; (ii) the inference mechanism evaluates which control rules are relevant at the current time and then decides what the input to the plant should be; (iii) the fuzzification interface simply modifies the inputs so that they can be interpreted and compared to the rules in the rule-base and (iiii) the defuzzification interface converts the conclusions reached by the inference mechanism into the inputs to the plant [27].

In our Mamdani controller, we use the PV current and voltage as inputs. Output is the duty cycle u of the buck converter.

The variable inputs are expressed as linguistic variables denoted ZE (Zero), S (Small), M (Medium), B (Big) and VB (Very Big). The first step is the fuzzification of the input variables by triangular MFs.
The second step, inference rules, where the fuzzified variables are compared with predefined sets in order to get the appropriate response. The last stage is defuzzication of the rules in order to obtain the crisp values of the duty cycle perturbations.
In Table I, it is summarized the different fuzzy rules used in Mamdani fuzzy controller to track the maximum power point.

Table 1
FLC Rules Base

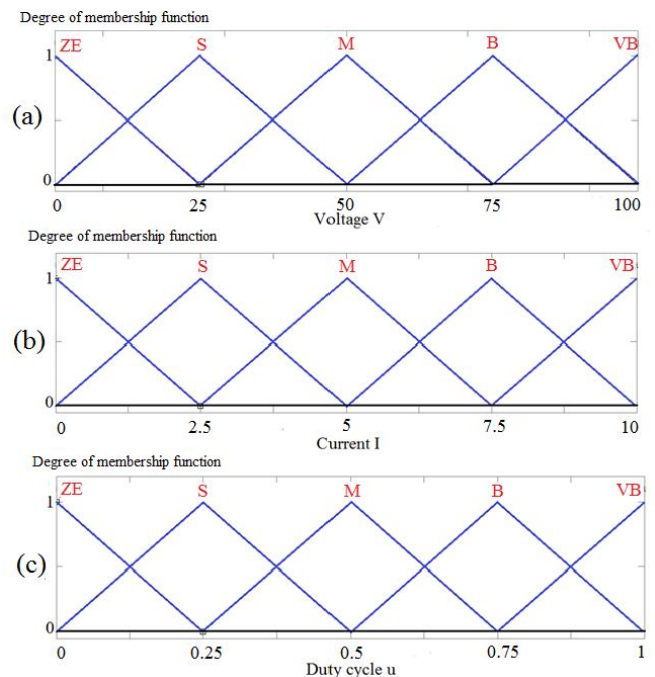| V\I	ZE	S	M	B	VB
ZE	ZE	S	S	M	M
S	S	S	M	M	B
M	M	M	B	B	B
B	M	B	B	B	VB
VB	B	B	B	VB	VB

Fig.3. Membership functions of

(a)PV voltage V – (b) PV current I– (c) Duty cycle u

The input variables and the control action for tracking of the maximum power point are illustrated in Fig.3.

4. T-S Fuzzy model representation of PV power control system

In this study we take the partial derivate *dP/dV* as the control output y(t) because the maximum power point occurs at *dP/dV=0*, i.e.[6]

$$y(t) = \frac{dP}{dV} \qquad (8)$$

Since T-S fuzzy model describes nonlinear systems by combining local linear dynamic subsystems in IF-THEN fuzzy rules [28], we will represent the solar power system in a T-S fuzzy model. The overall solar power system dynamics can be expressed by the following:

$$\begin{bmatrix} \dfrac{di_L}{dt} \\ \dfrac{dV}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{L}(R_L + R_B(1 - \dfrac{i_0}{i_L})) & 0 \\ 0 & \dfrac{1}{C_a}\dfrac{I}{V} \end{bmatrix} \begin{bmatrix} i_L \\ V \end{bmatrix} +$$

$$\begin{bmatrix} \dfrac{V}{L} \\ \dfrac{-i_L}{C_a} \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix}(-\dfrac{E_B}{L})$$

$$\dot{x} \equiv A(x)x + B(x)u + B_o b_d \qquad (9)$$

$$y = \begin{bmatrix} 0 & (\dfrac{I}{V} - \dfrac{n_p k_{pv}}{n_s} I_{rs} \exp(\dfrac{k_{pv}V}{n_s})) \end{bmatrix} \begin{bmatrix} i_L \\ V \end{bmatrix}$$

$$y \equiv C(x)x \qquad (10)$$

Where $B_0 = [1\ 0]^T$, $b_d = -E_B/L$ and the state variable vector is defined as $x = [i_L\ V]^T$.
According to the previous expression and the fuzzy modeling method [32], we have to fuzzify the matrices *A(x)*, *B(x)* and *C(x)*, we choose the fuzzy premise variables as follow:

$$q_1 = 1 - \frac{i_0}{i_L}$$

$$q_2 = i_L$$

$$q_3 = \frac{I}{V}$$

$$q_4 = V$$

and $q_5 = \dfrac{n_p k_{pv}}{n_s} I_{rs} \exp(\dfrac{k_{pv}V}{n_s})$.

The syntax "if-then" is always used so the system can be described by the following T-S rules:

If $q_1(t)$ is F_{1i} and …and $q_5(t)$ is F_{5i}, then
$$\dot{x} = A_i x(t) + B_i u(t) + B_o b_d$$
$$y = C_i x(t)$$
$$i = 1,2,\ldots,r \qquad (11)$$

F_{ji} denote the fuzzy sets, r is the number of fuzzy rules (r=32) and A_i, B_i and C_i are appropriate subsystem matrices.

The response of a TS model is a weighted sum:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(q(t))\{A_i x(t) + B_i u(t) + B_0 b_d\}$$

$$y(t) = \sum_{i=1}^{r} \mu_i(q(t))C_i x(t) \qquad (12)$$

The vector q(t) contains the premise variables;

$$q(t) = \begin{bmatrix} q_1(t) & q_2(t) & \ldots & q_5(t) \end{bmatrix}^T$$

The degree of activation for rule i is normalized as

$$\mu_i(q(t)) = \frac{w_i(q(t))}{\sum_{i=1}^{r} w_i(q(t))} \text{ with } w_i(q(t)) = \prod_{j=1}^{5} F_{ji}(q(t))$$

This normalization implies that $\sum_{i=1}^{r} \mu_i(q(t)) = 1$ for all t.

To obtain an exact fuzzy representation of the dynamic the membership functions of premises should be chosen

such as $A(x) = \sum_{i=1}^{r} \mu_i(q(t))A_i$, $B(x) = \sum_{i=1}^{r} \mu_i(q(t))B_i$

and

$$C(x) = \sum_{i=1}^{r} \mu_i(q(t))C_i \ .$$

For simplification, the membership functions are written in the general form as [6]:

$$f_{aj} = \frac{q_j(t) - qm_j}{qM_j - qm_j}$$

$$f_{bj} = 1 - f_{aj} \qquad (13)$$

Where qM_j and qm_j are the upper and the lower bounds of the variable $q_j(t)$ for j=1...5.

The subsystem matrices are given as:

$$A_i = \begin{bmatrix} -\dfrac{1}{L}(R_L + R_B q_{1i}) & 0 \\[2mm] 0 & \dfrac{1}{C_a}q_{3i} \end{bmatrix},$$

$$B_i = \begin{bmatrix} \dfrac{q_{4i}}{L} \\[2mm] \dfrac{-q_{2i}}{C_a} \end{bmatrix}$$

$$C_i = \begin{bmatrix} 0 & q_{3i} - q_{5i} \end{bmatrix} \tag{14}$$

In tables 2 and 3 we present the setting of the parameters q_{ji} with respect to each rule for j=1...5 and i=1...32.

Table 2
Fuzzy Rules from 1 to 16

Fuzzy Sets of rules					Parameters of then- part				
F_{1i}	F_{2i}	F_{3i}	F_{4i}	F_{5i}	q_{1i}	q_{2i}	q_{3i}	q_{4i}	$q_{3i}-q_{5i}$
f_{a1}	f_{a2}	f_{a3}	f_{a4}	f_{a5}	qM_1	qM_2	qM_3	qM_4	qM_3-qM_5
f_{a1}	f_{a2}	f_{a3}	f_{b4}	f_{a5}	qM_1	qM_2	qM_3	qm_4	qM_3-qM_5
f_{a1}	f_{a2}	f_{b3}	f_{a4}	f_{a5}	qM_1	qM_2	qm_3	qM_4	qm_3-qM_5
f_{a1}	f_{a2}	f_{b3}	f_{b4}	f_{a5}	qM_1	qM_2	qm_3	qm_4	qm_3-qM_5
f_{a1}	f_{b2}	f_{a3}	f_{a4}	f_{a5}	qM_1	qm_2	qM_3	qM_4	qM_3-qM_5
f_{a1}	f_{b2}	f_{a3}	f_{b4}	f_{a5}	qM_1	qm_2	qM_3	qm_4	qM_3-qM_5
f_{a1}	f_{b2}	f_{b3}	f_{a4}	f_{a5}	qM_1	qm_2	qm_3	qM_4	qm_3-qM_5
f_{a1}	f_{b2}	f_{b3}	f_{b4}	f_{a5}	qM_1	qm_2	qm_3	qm_4	qm_3-qM_5
f_{b1}	f_{a2}	f_{a3}	f_{a4}	f_{a5}	qm_1	qM_2	qM_3	qM_4	qM_3-qM_5
f_{b1}	f_{a2}	f_{a3}	f_{b4}	f_{a5}	qm_1	qM_2	qM_3	qm_4	qM_3-qM_5
f_{b1}	f_{a2}	f_{b3}	f_{a4}	f_{a5}	qm_1	qM_2	qm_3	qM_4	qm_3-qM_5
f_{b1}	f_{a2}	f_{b3}	f_{b4}	f_{a5}	qm_1	qM_2	qm_3	qm_4	qm_3-qM_5
f_{b1}	f_{b2}	f_{a3}	f_{a4}	f_{a5}	qm_1	qm_2	qM_3	qM_4	qM_3-qM_5
f_{b1}	f_{b2}	f_{a3}	f_{b4}	f_{a5}	qm_1	qm_2	qM_3	qm_4	qM_3-qM_5
f_{b1}	f_{b2}	f_{b3}	f_{a4}	f_{a5}	qm_1	qm_2	qm_3	qM_4	qm_3-qM_5
f_{b1}	f_{b2}	f_{b3}	f_{b4}	f_{a5}	qm_1	qm_2	qm_3	qm_4	qm_3-qM_5

Table 3
Fuzzy Rules from 17 to 32

Fuzzy Sets of rules					Parameters of then- part				
F_{1i}	F_{2i}	F_{3i}	F_{4i}	F_{5i}	q_{1i}	q_{2i}	q_{3i}	q_{4i}	$q_{3i}-q_{5i}$
f_{a1}	f_{a2}	f_{a3}	f_{a4}	f_{b5}	qM_1	qM_2	qM_3	qM_4	qM_3-qm_5
f_{a1}	f_{a2}	f_{a3}	f_{b4}	f_{b5}	qM_1	qM_2	qM_3	qm_4	qM_3-qm_5
f_{a1}	f_{a2}	f_{b3}	f_{a4}	f_{b5}	qM_1	qM_2	qm_3	qM_4	qm_3-qm_5
f_{a1}	f_{a2}	f_{b3}	f_{b4}	f_{b5}	qM_1	qM_2	qm_3	qm_4	qm_3-qm_5
f_{a1}	f_{b2}	f_{a3}	f_{a4}	f_{b5}	qM_1	qm_2	qM_3	qM_4	qM_3-qm_5
f_{a1}	f_{b2}	f_{a3}	f_{b4}	f_{b5}	qM_1	qm_2	qM_3	qm_4	qM_3-qm_5
f_{a1}	f_{b2}	f_{b3}	f_{a4}	f_{b5}	qM_1	qm_2	qm_3	qM_4	qm_3-qm_5
f_{a1}	f_{b2}	f_{b3}	f_{b4}	f_{b5}	qM_1	qm_2	qm_3	qm_4	qm_3-qm_5
f_{b1}	f_{a2}	f_{a3}	f_{a4}	f_{b5}	qm_1	qM_2	qM_3	qM_4	qM_3-qm_5
f_{b1}	f_{a2}	f_{a3}	f_{b4}	f_{b5}	qm_1	qM_2	qM_3	qm_4	qM_3-qm_5
f_{b1}	f_{a2}	f_{b3}	f_{a4}	f_{b5}	qm_1	qM_2	qm_3	qM_4	qm_3-qm_5
f_{b1}	f_{a2}	f_{b3}	f_{b4}	f_{b5}	qm_1	qM_2	qm_3	qm_4	qm_3-qm_5
f_{b1}	f_{b2}	f_{a3}	f_{a4}	f_{b5}	qm_1	qm_2	qM_3	qM_4	qM_3-qm_5
f_{b1}	f_{b2}	f_{a3}	f_{b4}	f_{b5}	qm_1	qm_2	qM_3	qm_4	qM_3-qm_5
f_{b1}	f_{b2}	f_{b3}	f_{a4}	f_{b5}	qm_1	qm_2	qm_3	qM_4	qm_3-qm_5
f_{b1}	f_{b2}	f_{b3}	f_{b4}	f_{b5}	qm_1	qm_2	qm_3	qm_4	qm_3-qm_5

5. TS Fuzzy Controller

After obtaining the TS fuzzy model of the system, we propose to design the TS fuzzy controller.

5.1 PDC Fuzzy controller

In order to achieve the MPPT control we have to drive the control output to zero. Based on the T-S fuzzy models, the PDC fuzzy controller is designed as follows.

Controller rule i:

If $q_1(t)$ is F_{1i} and ...and $q_5(t)$ is F_{5i}, then

$$\begin{cases} \dot{x}_y(t) = y(t) - y_0 \\ u(t) = K_{1i}x(t) + K_{2i}x_y(t) \end{cases} \tag{15}$$

$i = 1..32$

Where $x_y(t)$ denotes the integral of the controlled output, y_0 is a null reference, and K_{1i} and K_{2i} represent the control gains.

The global fuzzy inferred controller is obtained as follows:

$$\begin{cases} \dot{x}_y(t) = \sum_{i=1}^{r} \mu_i(q(t))C_i x(t) \\ u(t) = \sum_{i=1}^{r} \mu_i(q(t))\left\{ K_{1i}x(t) + K_{2i}x_y(t) \right\} \end{cases} \quad (16)$$

By substituting the control law (16) in the fuzzy model (12), the closed loop system is given by:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(q(t))\mu_j(q(t))\left\{ (A_i + B_i K_{1j})x(t) \right.$$
$$\left. + B_i K_{2j}x_y(t) + B_o b_d \right\} \quad (17)$$

In this subsection, we transform the average dynamic model of the photovoltaic system into an augmented model. So we can express the overall controlled system in terms of a new coordinate as follows:

$$\overline{x}(t) = \begin{bmatrix} x^T(t) & x_y^T(t) \end{bmatrix}^T \text{ as}$$

$$\dot{\overline{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(q(t))\mu_j(q(t))\left\{ \overline{A}_{ij}\overline{x}(t) + \overline{B}_o b_d \right\} \quad (18)$$

Where $\overline{A}_{ij} = \begin{bmatrix} A_{ci} + B_{ei}K_j \end{bmatrix}$

$$A_{ci} = \begin{bmatrix} A_i & 0 \\ C_i & 0 \end{bmatrix} \quad B_{ei} = \begin{bmatrix} B_i & 0 \end{bmatrix}^T$$

$$K_j = \begin{bmatrix} K_{1j} & K_{2j} \end{bmatrix} \text{ and } \overline{B}_o = \begin{bmatrix} B_0 & 0 \end{bmatrix}^T \quad (19)$$

5.2 Stability analysis

To analyze the convergence of the maximum tracking, we consider the quadratic Lyapunov function $V(\overline{x}) = \overline{x}^T P \overline{x}$ with P is a positive-definite symmetric matrice $P = P^T \succ 0$. The time derivative of Lyaunov function along the control dynamics results in:

$$\dot{V}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(q(t))\mu_j(q(t))\left\{ \overline{x}^T(P\overline{A}_{ij} + \overline{A}_{ij}^T P)\overline{x} \right.$$
$$\left. + \overline{x}^T P \overline{B}_o b_d + b_d^T \overline{B}_o^T P \overline{x} \right\} \quad (20)$$

$$\dot{V}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(q(t))\mu_j(q(t))\overline{x}^T \left\{ P\overline{A}_{ij} + \overline{A}_{ij}^T P + \right.$$
$$\left. P\overline{B}_o \overline{B}_o^T P \right\} \overline{x} + b_d^T b_d \quad (21)$$

For $\overline{Q} = \overline{Q}^T \succ 0$ if

$$P\overline{A}_{ij} + \overline{A}_{ij}^T P + P\overline{B}_o \overline{B}_o^T P + \overline{Q} \prec 0$$
(22)
Then

$$\dot{V}(t) \leq -\overline{x}^T \overline{Q}\overline{x} + b_d^T b_d$$
(23)

this implies that $\dot{V}(t) \leq 0$ for

$$-\lambda_{\min}(\overline{Q})\|\overline{x}(t)\|^2 \geq \|b_d\|^2 \quad (24)$$

where $\lambda_{\min}(\overline{Q})$ denotes the minimum eigenvalue operator.

Using the T-S fuzzy control law, the maximum power generation of the solar system is achieved if the controller gains are set to $K_i = M_i X^{-1}$ with the matrices X and M_i satisfying the following LMIs:

$$X \succ 0$$

$$\begin{bmatrix} A_{ci}X + XA_{ci}^T + B_{ei}M_j + M_j^T B_{ei}^T + \overline{B}_o \overline{B}_o^T & XQ_1 \\ Q_1 X & -X \end{bmatrix} \prec 0$$
(25)

For all i, j and given positive-definite symmetric matrices $Q_1 \succ 0$.

If the LMI have a feasible solution then $V > 0$ and $\dot{V} \leq 0$ with $\overline{Q} = Q_1 X^{-1} Q_1$. Therefore, due to the fact that b_d is a constant, $\lim_{t\to\infty} \dot{x}_y(t) = \lim_{t\to\infty} y(t) = 0$ is guaranteed.

In other words, the condition $dP/dV=0$ is guaranteed and the maximum power point is achieved.

6. Numerical Simulation

This section presents the main results. The PV power system was simulated with the software package Matlab/Simulink.

In the following table we state the characteristics of the PV array panel under 398 K and 1000 W/m².

Table 4
PV Array Parameters

Parameters	Values
Maximum power P_{max}	606 W
Voltage at P_{max}	90 V
Current at P_{max}	6,7 A
Short circuit Current I_{sc}	7,8 A
Series-Parallel cells	(36 , 2)

The control gains are found by solving the LMI (25), here, we state some partial gains:

K_1= [0.0323 0.0337 21.1528]
K_{16}= [0.0377 0.0222 24.7630]
K_{32}= [0.0015 0.0083 0.2227]

The aim of simulation is to present the electrical behavior of the PV system under rapid changes in radiation levels.

So to demonstrate the performance of the MPPT, we apply a sudden change in radiation with a fixed temperature. The solar radiation level starts from 800 W/m², then decreases to 600 W/m², after that increases to 700 W/m² and finally, increases to 900 W/m², while the temperature is set to the value T=398 K. The evolution of the varying radiation is given by the following figure.
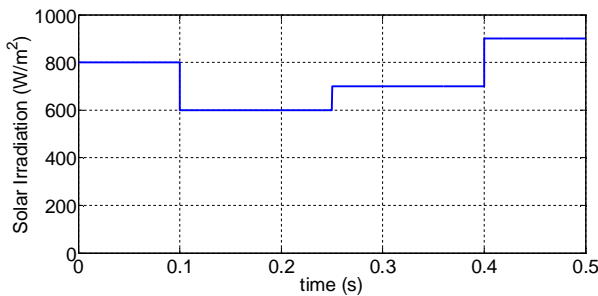
Fig.4. Solar radiation levels

In order to better understand the behavior of the PV array when controlled by our proposed MPPT control strategy, it is worthwhile to carry out simulation comparisons with Perturb and Observe method and fuzzy logic control method.

The power regulation response generated by applying the PO technique, Mamdani fuzzy controller and the proposed T-S controller are illustrated respectively in Fig. 5, 6 and 7.
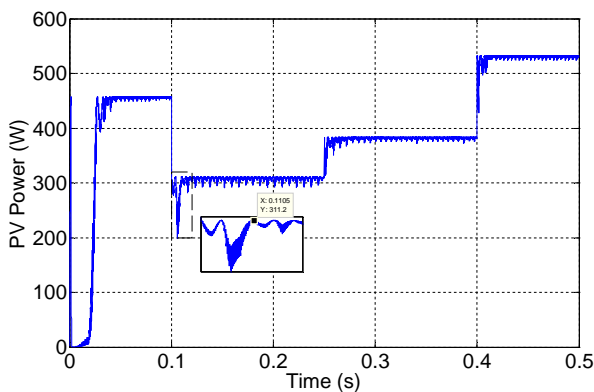

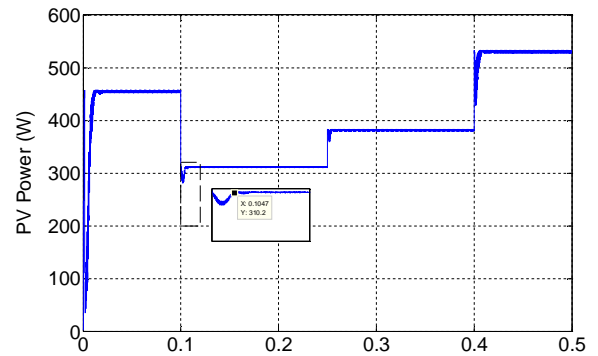
Fig.5. PV Power response based on PO method

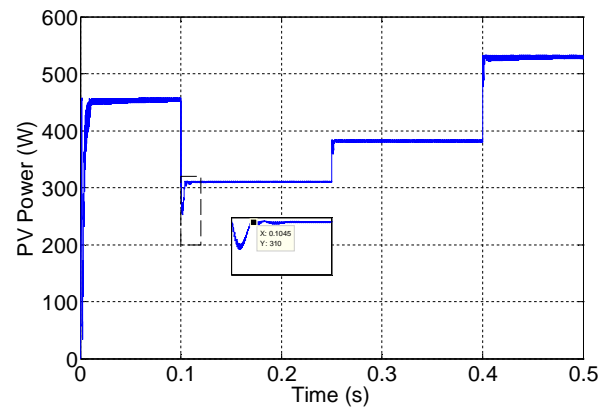Fig. 6: PV Power response based on Mamdani controller

Fig.7. PV Power response based on TS controller

We can observe that the proposed controller has a faster time response to track the MPP than PO and Mamdani controllers and presents fewer oscillations.

It is also clear that the generated solar power achieves rapidly the maximum value under the radiation conditions considered and maintains the real MPP after each step of radiation variation, hence a good stabilization is obtained thanks to T-S controller acts.

The evolution of PV voltage and current are respectively given in Fig. 8 and Fig. 9.
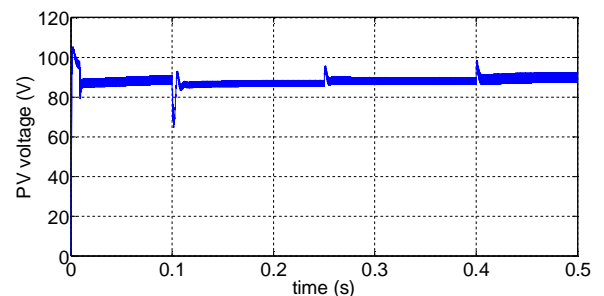

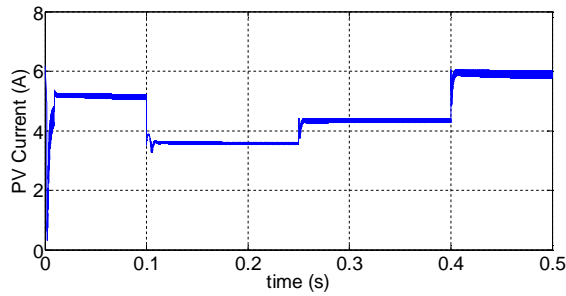Fig.8. PV voltage response based on MPPT control

Fig.9. PV current response based on MPPT control

In Figures 8 and 9, we observe momentary peaks which are resulted from sudden and significant change in solar radiation.

In this step, we consider a varying sinusoidal radiation $500+500\sin(\pi t)$ in W/m², the P-V diagrams responses of the controlled system with the use of the three methods are shown in the following figures.
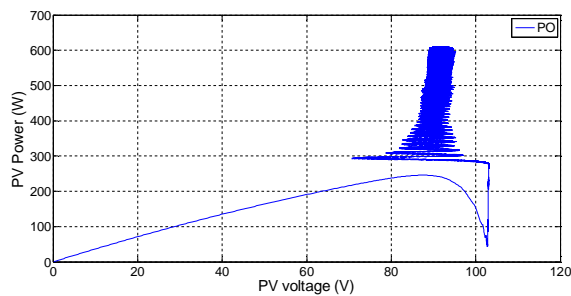

Fig.10. P-V diagram using the Perturb and Observe method under radiation changes
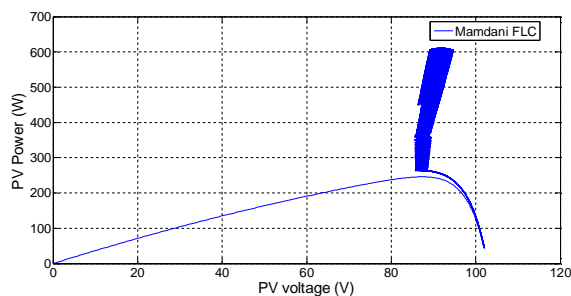

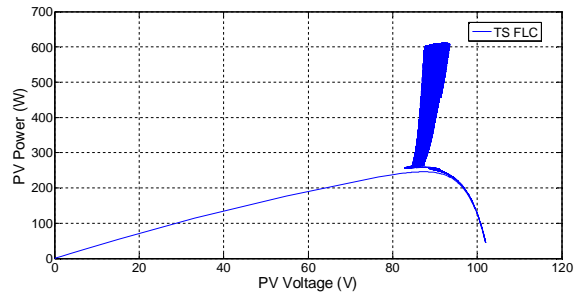Fig.11. P-V diagram using Mamdani fuzzy controller under radiation changes

Fig.12. P-V diagram using the TS fuzzy controller under radiation changes

With reference to Figs 10, 11 and 12, we can notice that for different values of radiation, the PV array would exhibit different characteristic curves. Each curve has its maximum power point. It is at this point, where the corresponding maximum voltage is supplied to the converter.

It can be seen that the first method (PO) results in important oscillations. However, the fuzzy methods present less power chattering phenomenon. Thus, it can be deduced that the proposed TS fuzzy controller gives a better performance than both PO method and Mamdani controller and the effectiveness of the proposed maximum-power-point tracking control method is ensured.

7. Conclusion

In this work, we have presented a nonlinear fuzzy controller for the maximum power point tracking of a solar power generation system. At first, a Takagi-Sugeno (TS) fuzzy modeling for the considered nonlinear system has been proposed. Based on the elaborated fuzzy model, a T-S fuzzy controller has been designed with guaranteed stability and good dynamic behavior. The control gains are obtained by solving a set of LMIs. The MPPT is achieved even when we consider varying atmosphere. In order to verify the performance of the proposed controller, we have compared it with PO method and Mamdani fuzzy logic controller and we can deduce that TS controller can track the maximum power point with less chattering phenomenon or approximate MPPT than the traditional methods.

References

1. IEA Technology roadmap solar photovoltaic energy, 2010.

2. M.Cheikh, A., Larbes, C., Kebir, G.T., and Zerguerras, A., *Maximum Power Point Tracking Using a Fuzzy Logic Control Scheme*, Revue des energies renouvelables, vol.10, no.3, pp.387-395, 2017.

3. J.-J. E. Slotine and W. Li, *Applied Nonlinear Control, Englewood Cliffs*, New Jersey: hentice-Hall, 1991.

4. J. K. Hedrick and A. Girard, *Control of Nonlinear Dynamic Systems:* Theory and Applications, 2005.

5. Tanaka, K. and M. Sugeno, Stability analysis and design of fuzzy control systems, *Fuzzy Sets Syst.*, vol. 45, no. 2, pp. 135–156, 1992.

6. C.S. Chiu, *TS fuzzy maximum power point tracking control of solar power generation systems*, IEEE Trans. Energy Convers, vol. 25, no 4, pp. 1123-1132, 2010.

7. C.-L. Chen et al., *Analysis and design of fuzzy control systems*, Fuzzy Sets Syst., vol. 57, pp. 125–140, 1993.

8. K. Tanaka, T. Ikeda, and H.O. Wang, *Robust stabilization of a class of uncertain nonlinear systems via fuzzy control:* Quadratic stabilizability, H∞ control theory, and linear matrix inequalities, IEEE Trans. Fuzzy Sys., vol. 4, pp. 1-13, 1996.

9. A. Mellit and Kalogirou, Soteris A, MPPT-based artificial intelligence techniques for photovoltaic systems and its implementation into field programmable gate array chips, Energy, 2014

10. V. Salas, E. Olias, A. Barrado, and A. Lazaro, *Review of the maximum power point tracking algorithms for stand-alone photovoltaic systems*, Solar Energy Mater, Solar Cells, vol. 90, pp. 1555–1578, 2006.

11. K. H. Hussein, I. Muta, T. Hoshino, and M. Osakada*, Maximum photovoltaic power tracking: An algorithm for rapidly changing atmospheric condition*, Proc. Inst. Electr. Eng., Gen. Transmiss. Distrib., vol. 142, no. 1, pp. 59–64, 1995.

12. G.J.Yu, Y.S.Jung, J.Y.Choi, I.Choy, J.H.Song and G.S.Kim*, A Novel Two-Mode MPPT Control Algorithm Based on Comparative Study of Existing Algorithms*, Proc. Photovoltaic Specialists Conference, pp. 1531-1534, 2002.

13. T. Noguchi, S. Togashi and R. Nakamoto, *Short-Current Pulse-Based Maximum-Power-Point Tracking Method for Multiple Photovoltaic-and-Converter Module System*, IEEE Trans. Ind. Electron., vol. 49, no.1, pp. 217-223, 2002.

14. D.Y. Lee, H.J. Noh, D.S. Hyun and I.Choy*, An Improved MPPT Converter Using Current Compensation Method for Small Scaled PV-Applications*, Proc. APEC, pp. 540-545, 2003,.

15. M. Park and I.K. Yu, *A Study on Optimal Voltage for MPPT Obtained by Surface Temperature of Solar Cell,* Proc. IECON, pp. 2040-2045, 2004.

16. C. Larbes , A. Cheikh, S.M Obeidi, T. and Zerguerras, *Genetic algorithms optimized fuzzy logic control for the maximum power point tracking in photovoltaic system*, Renewable Energy,vol.34, no. 10 ,pp. 2093-2100, 2009

17. A.F. Boehinger, *Self adaptive DC converter for solar spacecraft power supply*, IEEE Trans. Aerospace and Electronic Sys., AES-4, (1), pp. 102–111, 1968.

18. Z. Salameh, D. Taylor, *Step-up maximum power point tracker for photovoltaic arrays*, Solar Energy, vol. 44, no. 1 ,pp. 57–61,1990.

19. W. Denizinger, *Electrical power subsystem of globlastar,* Proceedings of the European Space Conference, pp. 171–174, 1995.

20. W.J.A. Teulings, J.C. Marpinard, A. Capel, A maximum power point tracker for a regulated power bus, *Proceedings of the European Space Conference*, 1993.

21. Y. Kim, H. Jo, D. Kim , *A new peak power tracker for cost-effective photovoltaic power systems*, IEEE Proc. Energy Conversion Eng. Conf. IECEC 96, vol.3, pp. 1673–1678, 1996

22. Ch. Hua, J. Lin, Ch. Shen , *Implementation of a DSP-controlled PV system with peak power tracking*, IEEE Trans. Ind. Electron., vol. 45, no.1, pp. 99–107,1998.

23. H. Al-Atrash, I. Batarseh, K. Rustom, *Statistical modeling of DSP-based hill-climbing MPPT algorithms in noisy environments*, Applied Power Electronics Conference and Exposition, APEC 2005, Twentieth Annual IEEE, vol. 3, 2005, pp. 1773–1777 , 2005.

24. A. Chouder, F. Guijoan and S. Silvestre , *Simulation of fuzzy-based MPP tracker and performance comparison with perturb & observe method*, Revue des Energies Renouvelables, vol. 11, no.4, pp.577 – 586, 2008.

25. J.L. Santos, F. Antunes, A. Chehab, C. Cruz , *A maximum power point tracker for PV systems using a high performance boost converter ,* Solar Energy, vol. 80, no.7, pp. 772–778, 2006.

26. X. Liu, L.A.C. Lopes, *An improved perturbation and observation maximum power point tracking algorithm for PV arrays*, IEEE 35th Annual Power Electronics Specialists Conference, PESC 04, vol. 3, pp. 2005–2010, 2004.

27. K.M. Passino, S. Yurkovich, *Fuzzy control*, Reading MA : Addison Wesley, vol. 42 , 1998.

28. T. Takagi and M. Sugeno, *Fuzzy identification of systems and its applications to modeling and control*, IEEE Trans. Syst., Man, Cybern., no. 1, pp. 116–132, 1985.

29. Wilamowski BM, Li X., *Fuzzy system based maximum power point tracking for PV system*, 28th

Annual Conf. of the IEEE Ind. Electron. Society, vol.4, pp. 3280–84, 2002.

30. V.Salas,E. Olias, A.Barrado and A.Lazaro, *Review of the maximum Power Point Tracking Algorithms for Stand-Alone Photovoltaic systems*, Solar Energy Materials and Solar Cells, no.11, vol. 90, pp.1555-1578, 2006.

31. H. Kopf, Analysis, *Simulation and Evaluation of Maximum Power Point Tracking (MPPT) Methods for a Solar Powered vehicle*, Master Thesis, Portland State university, 1999.

32. C., Y. Haw and C. Chia-Yu, *A maximum power point tracking of PV system by scaling fuzzy control*, International MultiConference of Engineers and Computer Scientists, Hong Kong, 2010.

33. K. Y. Lian, T. S. Chiang, C. S. Chiu, and P. Liu, *Synthesis of fuzzy model-based designed to synchronization and secure communication for chaotic systems*, IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 31, no. 1, pp. 66–83, 2001.

34. E. Kamal A. Aitouche, *Robust Scheduler fuzzy Controller of DFIG Wind Energy Systems*, IEEE Trans. Sustainable Energy, vol. 4, no. 3, pp.706-715, 2013.

35. H. Boumaaraf, A. Talha and O. Bouhali , *Modeling and control of grid-connected photovoltaic systems*, Journal of Electrical Engineering vol. 14 , no. 3, 2014.

36. H. Abouobaida, M. Khayat and M. Cherkaoui, *Combination of RCC MPPT and backstepping controller to design a standard continuous source (12-24V) supplied by a PV panels,* Journal of Electrical Engineering, vol. 14 , no. 3, 2014.

37. J. F. Baldwin, and J. Lawry, *A new approach to learning linguistic control rules*, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, vol. 8, no. 1, pp. 21-44, 2000.

38. I. Škrjanc, S. Blažič, and D. Matko, *Direct fuzzy model-reference adaptive control*, International Journal of Intelligent Systems, vol. 17, no. 10, pp. 943-963, Oct. 2002.

39. R. E. Precup, and S. Preitl, *Stability and sensitivity analysis of fuzzy control systems*, Mechatronics applications, Acta Polytechnica Hungarica, vol. 3, no. 1, pp. 61–76, 2006.

40. A. Chatterjee, R. Chatterjee, F. Matsuno and T. Endo, *Augmented stable fuzzy control for flexible robotic arm using LMI approach and neuro-fuzzy state space modeling*, IEEE Transactions on Industrial Electronics, vol. 55, no. 3, pp. 1256-1270, 2008.