A DIRECT ANALYTICAL APPROACH FOR DETERMINATION OF OPERATING FREQUENCY OF A SELF-EXCITED INDUCTION GENERATOR FEEDING AN INDUCTION MOTOR

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Abstract: *This paper presents a direct analytical method for the determination of the operating frequency of a system composed of a self-excited induction generator (SEIG) feeding an induction motor (IM). In the method presented, results are obtained for different operating conditions of the SEIG/IM system. The presented analytical approach was verified by comparing the results obtained from it, at different operating conditions, with previously published results obtained numerically, with the induction motor considered as equivalent to a static R-L load. The two sets of results were found to be almost identical.*
Keywords: *Induction generator, Induction motor, Steady-state analysis, Numerical solution, Equivalent circuit.*

1. Introduction

Interest in renewable energy sources such as wind, photovolatic and hydro for generating electricity has increased [1-4]. Self-excited induction generators have been investigated extensively to be used as a generator in renewable energy systems [2-11]. This is because of many advantages of SEIG, such as robustness and cost.

In the analysis of the SEIG the equivalent circuit of the generator is derived using either the nodal admittance method [10, 11] or the loop impedance method [12, 13].

Analysis of the SEIG feeding an induction motor has been previously investigated [14-17].

In all of the previous investigations, numerical techniques have been used to obtain the operating frequency, and consequently the performance of the system.

In this paper the operating frequency of a system composed of a SEIG and an induction motor is obtained in terms of parameters and operating conditions of the system by deriving an eleventh order equation in the frequency, which can be solved numerically to obtain it. Also, an analytical approach in which a second order equation relating the operating frequency of the system to the rest of the parameters is derived, and its validity has been verified.

2. Method of analysis

At first, a brief analysis of a SEIG feeding a static R-L load is presented, and then the new proposed approach is developed for analyzing the SEIG when feeding an induction motor.

2.1 R-L load

The steady-state performance of a SEIG feeding a static R-L load has been investigated before [12, 13, 18].

In [18], the steady-state equivalent circuit of the SEIG feeding an R-L load is given as shown in Fig. 1.

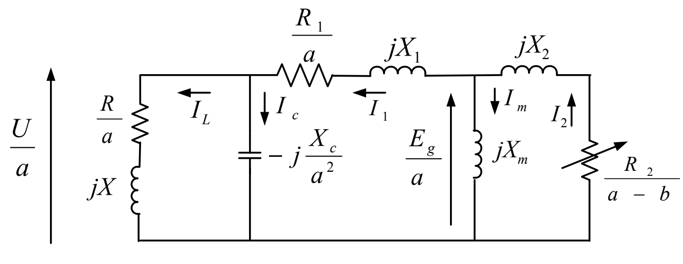

Fig. 1. Equivalent circuit of induction generator.

In this figure R and X are the resistance and reactance of the load (at base frequency). In this circuit all the parameters of the machine are referred to the rated per-unit frequency, and are assumed to be independent of saturation except for the magnetizing reactance and they are also referred to the stator. Core losses and effect of harmonics are neglected.

In [18] the equation relating the per-unit frequency of operation to the rest of parameters, for a static R-L load, is derived as [10, 11]:

$$P_7\, a^7+P_6\, a^6+P_5\, a^5+P_4\, a^4+P_3\, a^3+P_2\, a^2+ P_1\, a+P_0=0 \tag{1}$$

The coefficients P_0 to P_7 are given in Appendix (I) For certain SEIG speed and R-L load, eqn. (1) can be solved numerically to obtain the operating per-unit frequency (a). Consequently, the performance of the system can be obtained using the equivalent circuit in Fig. 1.

and the equivalent reactance is obtained as:

$$X_{em}=R_{2m}^{\,2}(X_{mm}+X_{1m})+(a-b_m)^2(X_{mm}+X_{2m})[X_{mm}X_{2m}+ X_{1m}(X_{mm}+X_{2m})] \,/\, [R_{2m}^{\,2}+(a-b_m)^2(x_{mm}+X_{2m})^2] \tag{3}$$

Replacing R and X in the coefficients P_0 to P_7 of eqn. (1), by the expressions of R_{em} and X_{em}, eqns. (2) and (3), an equation of the eleventh order in the per-unit frequency of the SEIG/IM system is obtained as:

$$P_{11m}\, a^{11}+P_{10m}\, a^{10}+P_{9m}\, a^9+P_{8m}\, a^8+P_{7m}\, a^7+P_{6m}\, a^6+ P_{5m}\, a^5+P_{4m}\, a^4+P_{3m}\, a^3+P_{2m}\, a^2+P_{1m}\, a+P_{0m}=0 \tag{4}$$

The expressions of P_{11m} to P_{0m} are very large. Therefore if the magnetizing reactance of the induction motor, X_{mm} in Fig. 2., is considered to

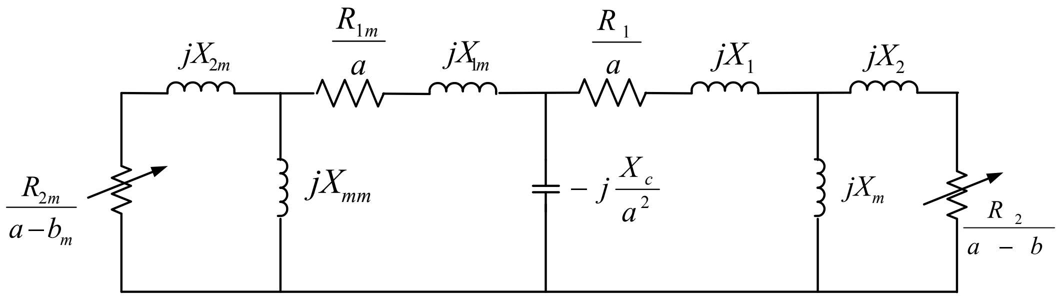

Fig. 2. Equivalent circuit of SEIG / IM system

2.2 Induction motor load

Fig. 2. shows the equivalent circuit of SEIG feeding an induction motor [14, 15, 19]

From this circuit, the equivalent resistance of the induction motor is obtained as:

$$R_{em}= [aX_{mm}^{\,2}R_{2m}(a-b_m)+R_{1m}R_{2m}^{\,2}+ R_{1m}\,(X_{mm}+X_{2m})^2(a-b_m)^2] \,/\, [R_{2m}^{\,2}+ (a-b_m)^2(X_{mm}+X_{2m})^2] \tag{2}$$

be open- circuited, the equivalent circuit of Fig. 2. becomes as shown in Fig. 3., and the induction motor will have an equivalent resistance as:

$$R_e = (R_{1m}/a) + (R_{2m} \,/\, (a-b_m)) \tag{5}$$

and an equivalent reactance as:

$$X_e = X_{1m}+X_{2m} \tag{6}$$

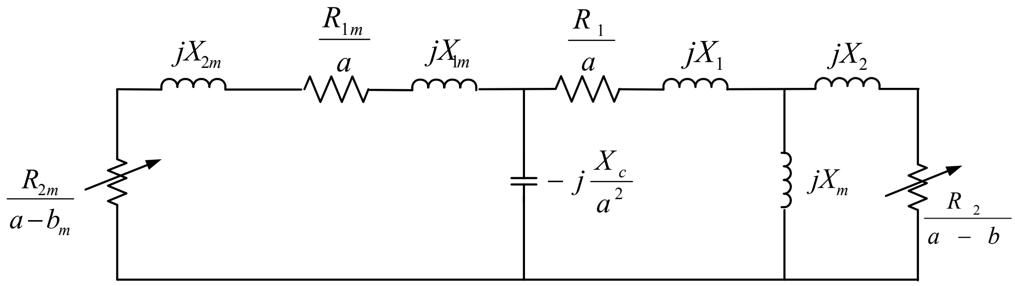

Fig. 3. Equivalent circuit of SEIG / IM system X_{mm} not included.

Therefore, eqn. (4) becomes:

$$P_{11n}\,a^{11}+P_{10n}\,a^{10}+P_{9n}\,a^{9}+P_{8n}\,a^{8}+P_{7n}\,a^{7}+P_{6n}\,a^{6}+ P_{5n}\,a^{5}+P_{4n}\,a^{4}+P_{3n}\,a^{3}+P_{2n}\,a^{2}+P_{1n}\,a+P_{0n}=0 \qquad (7)$$

The above assumption will be shown later that it will not affect the accuracy of results, and will be shown that it is valid in the section of results.

Eqn. (7) is used to derive a second order equation in the operating frequency of the system as follows.

The difference between the per-unit speed, b, of the SEIG and the per- unit frequency, *a*, can be expressed by the relationship [18]:

$$\xi = a - b$$

ξ has always very small value [9, 11, 18, 20].
The per-unit value of the frequency, *a*, in eqn. (7) can then be replaced by:

$$a = \xi + b$$

Neglecting the values of ξ^n with n>2 [18], eqn. (7) can be rewritten as:

$$A_2\,\xi^2 + A_1\,\xi + A_0 = 0 \qquad (8)$$

where A_2, A_1, A_0 are functions of the parameters and speeds of the generator and the motor, and are given in Appendix (II)
Solving eqn. (8), the per-unit operating frequency, *a*, can be obtained in a closed-form expression as:

$$a = b - \frac{A_2 - \sqrt{(A_2^2 - 4A_0A_2)}}{2\,A_2}$$

$$= b - \frac{A_2 - \sqrt{RR}}{2\,A_2} \qquad (9)$$

The per-unit frequency, *a*, is directly obtained from eqn. (9) instead of using numerical techniques with eqn. (7).

3. Results

To validate the approach presented in Subsection (2.2), in which a second order analytical equation was obtained from which the operating frequency of the SEIG/IM system, whose parameters are given in Appendix (III), can be obtained, the numerical solution of eqn. (4) was compared with that obtained from the numerical solution of eqn. (7) for several operating conditions as shown in Fig. 4.
From this figure it is evident that the numerical results obtained from the solution of eqns.(4) and (7) are almost identical although X_{mm} is not taken into consideration in eqn. (7).
It is important to notice that for each operating per-unit speed of the generator, b, the operating

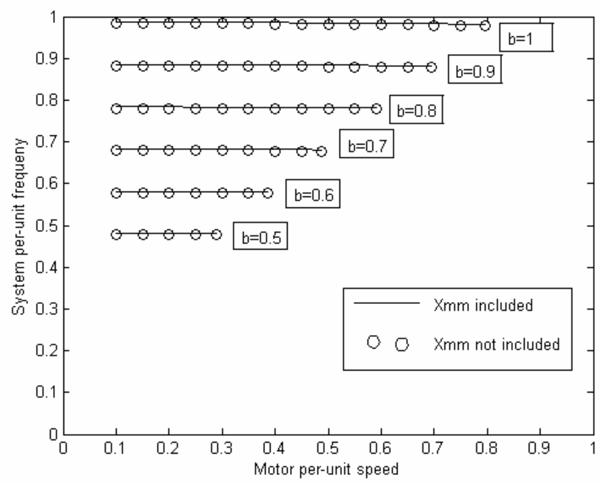

Fig. 4. Comparison between numerical solutions with X_{mm} included and numerical solution without X_{mm}.

speed of the motor should not exceed a boundary value otherwise the per-unit frequency, *a*, obtained from the solution of either eqn. (4) or eqn. (7) will be a complex quantity.

To determine the boundary motor speed above which the frequency will be complex, the expression RR in eqn. (9) (i.e. $A_1^2 - 4 A_2 A_0$) is equated to zero and solved for certain system parameters and generator speed to obtain the required boundary motor speed. The values of the motor speed used in obtaining the results should be equal to or less than that boundary speed as shown in Fig. 5.

The simplified equation from which a closed-form analytical expression for the operating frequency of the system, eqn. (9), is used and its results for different operating conditions are compared with the results obtained numerically from eqn. (4), Fig. 6.

Also, these results are compared with the results obtained from eqn. (7) as shown in Fig. 7. In both of Figs. 6 and 7 it is evident that the results obtained from the analytical approach presented are almost identical with those obtained numerically. This proves that the analytical approach presented is valid.

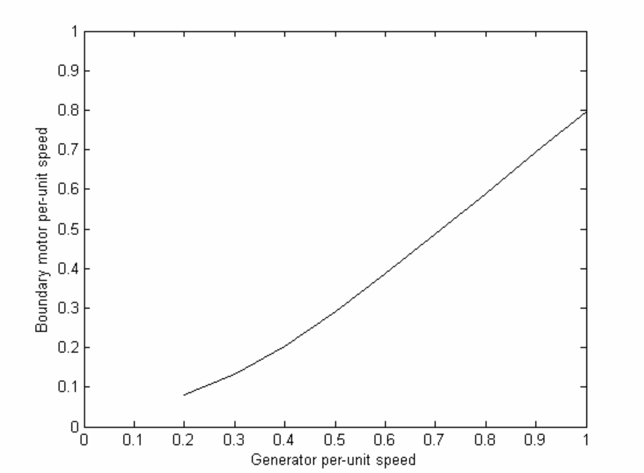

Fig. 5. Relationship between the generator per-unit speed and the boundary value of the motor speed.

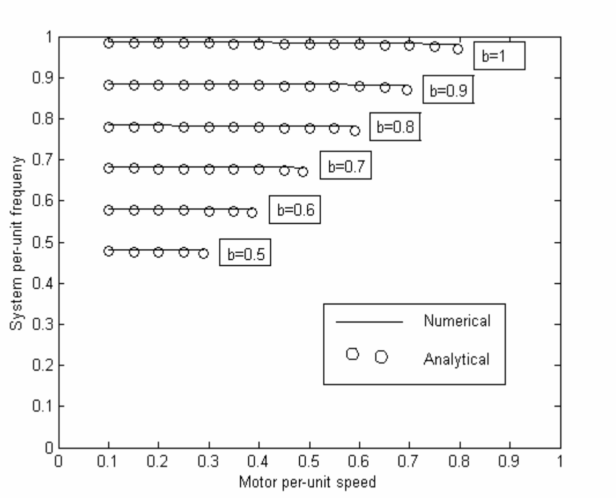

Fig. 6. Comparison between numerical solution with X_{mm} included and analytical solution with X_{mm} not included.

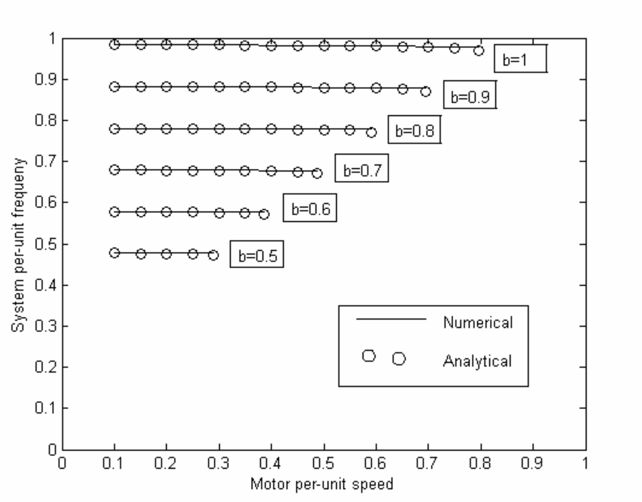

Fig. 7. Comparison between numerical and analytical solution with X_{mm} not included.

4. Conclusions

The paper presented a direct analytical method to obtain the per-unit frequency of a system composed of a self-excited induction generator feeding an induction motor. This method is simple and direct instead of using numerical methods to determine the frequency of the system. The presented approach is verified by comparing its results, at different operating conditions, with results obtained using a previously published reference. The result of comparison showed that the sets of results are almost identical which proves the validity of the presented approach.

LIST OF SYMBOLS

a system per-unit frequency, generated frequency/ rated frequency.

b induction generator per-unit speed.

b_m induction motor per-unit speed.

X_1, X_2 induction generator stator and rotor leakage reactances per phase respectively referred to stator.

X_{1m}, X_{2m} induction motor stator and rotor leakage reactances per phase respectively referred to stator.

X_m induction generator unsaturated magnetizing reactance per phase.

X_{mm} induction motor magnetizing reactance per phase.

R_1, R_2 induction generator stator and rotor resistances per phase respectively referred to stator.

R_{1m}, R_{2m} induction motor stator and rotor resistances per phase respectively referred to stator.

X_c induction generator capacitive reactance per phase.

U output rms terminal voltage per phase.

E_g air-gap rms voltage.

I_1 stator current.

I_2 rotor current.

I_c capacitor current.

I_L load current.

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Appendix (I)

$P_0 = -b\, R_2\, X_c^2\, (R+R_1)^2$

$P_1 = R_2\, X_c^2\, (R+R_1)^2 + X_c^2\, (R_2^2 + b^2\, X_2^2)\, (R+R_1),$

$P_2 = -b\, R_2\, (X_c^2\, (X+X_1)^2 - 2\, X_1\, X_c\, (R^2+R_1^2) + R_1^2\, R^2) - (2b\, X_2^2\, X_c^2\, (R+R_1))$

$P_3 = R_2\, (X_c^2\, (X+X_1)^2 - 2\, X_c\, (X_1\, R^2 + X\, R_1^2) + R_1^2\, R^2) + X_2^2 X_c^2\, (R+R_1) + R_1\, (R_2^2 + b^2\, X_2^2)\, (R^2 - 2\, X\, X_c)$

$P_4 = -b\, R_2\, (X_1^2\, (R^2 - 2\, X\, X_c) + X^2\, (R_1^2 - 2\, X_1\, X_c)) - (2\, bX_2^2\, R_1\, (R^2 - 2\, X\, X_c))$

$P_5 = R_2\, (X_1^2\, (R^2 - 2\, X\, X_c) + X^2\, (R_1^2 - 2\, X_1\, Xc)) + X_2^2\, R_1 (R^2 - 2XX_c) + R_1 X^2\, (R_2^2 + b^2\, X_2^2)$

$P_6 = (-b\, X^2)\, (R_2\, X_1^2 + 2\, R_1\, X_2^2)$

$P_7 = X^2\, (R_2\, X_1^2 + R_1\, X_2^2)$

Appendix (II)

$$A_2 = (kp_{20}+kp_{11}+kp_{02}+21\, kp_{70}\, b^5+21\, kp_{61}\, b^5+21kp_{52}\, b^5+21\, kp_{43}\, b^5+21\, kp_{34}\, b^5+15\, kp_{51}\, b^4 +15\, kp_{42}\, b^4+15\, kp_{33}\, b^4+10\, kp_{50}\, b^3+10\, kp_{41}\, b^3+10\, kp_{32}\, b^3+10\, kp_{23}\, b^3+10\, kp_{14}\, b^3+ 6\, kp_{40}\, b^2 +6\, kp_{31}\, b^2+6\, kp_{22}\, b^2+6\, kp_{13}\, b^2+6\, kp_{04}\, b^2+3\, kp_{30}\, b+3\, kp_{21}\, b+3kp_{12}\, b+ 15\, kp_{24}\, b^4+3\, kp_{03}\, b +15\, kp_{60}\, b^4+55\, kp_{74}\, b^9+45\, kp_{73}\, b^8+45\, kp_{64}\, b^8+36\, kp_{72}\, b^7+ 36\, kp_{63}\, b^7+36\, kp_{54}\, b^7+28\, kp_{71}\, b^6 + 28\, kp_{62}\, b^6+28\, kp_{53}\, b^6+28\, kp_{44}\, b^6)$$

$$A_1 = (kp_{10}+kp_{01}+8\, kp_{44}\, b^7+7\, kp_{61}\, b^6+7\, kp_{52}\, b^6+7kp_{43}\, b^6+7\, kp_{34}\, b^6+6\, kp_{60}\, b^5+6\, kp_{51}b^5+ 6\, kp_{42}\, b^5 +6\, kp_{33}\, b^5+5\, kp_{50}\, b^4+5\, kp_{41}b^4+5\, kp_{32}\, b^4+5\, kp_{23}\, b^4+7\, kp_{70}\, b^6+5kp_{14}b^4+ 4\, kp_{40}\, b^3+4\, kp_{31}\, b^3 +4\, kp_{22}\, b^3+4\, kp_{13}b^3+ 4\, kp_{04}\, b^3+3\, kp_{30}\, b^2+3\, kp_{21}\, b^2+6\, kp_{24}\, b^5+11\, kp_{74}\, b^{10}+3\, kp_{12}\, b^2+3\, kp_{03}\, b^2 +2\, kp_{20}\, b+2\, kp_{11}\, b+2\, kp_{02}\, b+10\, kp_{73}b^9+10\, kp_{64}\, b^9+ 9\, kp_{72}\, b^8+9\, kp_{63}\, b^8+9\, kp_{54}\, b^8+8\, kp_{71}\, b^7 +8\, kp_{62}\, b^7+8\, kp_{53}\, b^7)$$

$$A_0 = kp_{00}+kp_{74}\, b^{11}+kp_{30}\, b^3+kp_{21}\, b^3+kp_{12}\, b^3+kp_{03}\, b^3+kp_{20}\, b^2+kp_{11}\, b^2+ kp_{02}\, b^2+kp_{10}b+kp_{01}\, b +kp_{60}\, b^6+kp_{51}\, b^6+kp_{42}\, b^6+kp_{33}b^6+kp_{24}\, b^6+kp_{50}\, b^5+kp_{41}\, b^5+kp_{32}\, b^5+kp_{23}\, b^5+kp_{14}\, b^5+kp_{40}\, b^4 + kp_{31}\, b^4+kp_{22}b^4+kp_{13}\, b^4+kp_{04}\, b^4+kp_{73}\, b^{10}+kp_{64}b^{10}+kp_{72}\, b^9+kp_{63}\, b^9+kp_{54}\, b^9+kp_{71}\, b^8 +kp_{62}\, b^8+kp_{53}\, b^8+kp_{44}\, b^8+ kp_{70}\, b^7+kp_{61}\, b^7+kp_{52}\, b^7+kp_{43}\, b^7+kp_{34}\, b^7$$

where:

$kp_{74} = k_1^2\, (R_2\, X_1^2 + R_1\, X_2^2)$

$kp_{73} = -\, 4\, b_m\, k_1^2\, (R_2 X_1^2 + R_1\, X_2^2)$

$kp_{72} = (2\, k_1^2)(3\, b_m^2\,)(R_2\, X_1^2 + R_1\, X_2^2)$

$$kp_{71}= - 4\, b_m^3\, k_1^2\, (R_2\, X_1^2+R_1\, X_2^2)$$

$$kp_{70}= b_m^4\, k_1^2\, (R_2\, X_1^2+R_1\, X_2^2)$$

$$kp_{64}= - b\, k_1^2\, (R_2 X_1^2+2\, R_1\, X_2^2)$$

$$kp_{63}= 4\, b_m\, b\, k_1^2\, (R_2\, X_1^2+2\, R_1\, X_2^2)$$

$$kp_{62}=-6\, b\, k_1^2\, b_m^2 (R_2\, X_1^2+2\, R_1\, X_2^2)$$

$$kp_{61}=4\, b_m^3\, b\, k_1^2\, (R_2\, X_1^2+2\, R_1\, X_2^2)$$

$$kp_{60}= - b\, b_m^4\, k_1^2\, (R_2\, X1^2+2\, R_1\, X_2^2)$$

$$kp_{54}= (R_1\, X_2^2\, k_1^2)\, b^2+R_1\, (R_1\, R_2\, X_{2m}^2-2\, X_2^2\, X_c\, X_{1m})-R_2 (2\, X_c\, k_1-k_2^2)\, X_1^2+R_1\, X_2^2\, (R_{1m}^2-2\, X_c X_{2m}+R_{2m}^2)$$
$$+ 2\, R_1\, X_{2m}\, X_{1m}\, R_2^2+R_1\, (R_2^2\, X_{2m}^2+2 X_2^2\, R_{2m}\, R_{1m})+ R_1\, R_2\, X_{1m}\, (R_2\, X_{1m}+R_1 X_{1m}+2\, R_1\, X_{2m})-2\, R_2\, X_c\, k_1^2\, X_1$$

$$kp_{53}= -2\, b_m\, (-4\, k_1\, (X_1\, R_2\, k_1+R_1\, X_2^2+R_2\, X_1^2))\, X_c-4\, b_m R_1\, X_2^2\, k_1^2\, b^2-2\, b_m\, (2\, R_1\, (R_{1m}^2\, X_2^2+ R_2^2 k_3+ R_1\, R_2\, X_{2m}^2) +$$
$$4\, R_1\, R_2\, X_{2m}\, X_{1m}\, (R_2+R_1)+2\, R_2 X_1^2\, R_{1m}^2+3\, R_{2m}\, R_{1m}\, k_4+2\, R_2\, X_{1m}^2 R_1^2+R_{2m}^2\, k_4)$$

$$kp_{52}= (6\, R_1\, X_2^2\, b_m^2\, k_1^2)\, b^2+6\, R_1\, b_m^2\, (R_2^2\, k_1^2+X_2^2 R_{1m}^2)+ 6\, R_1\, b_m^2\, (-2\, X_2^2\, X_c\, X_{2m}+R_1\, R_2\, X_{2m}^2$$
$$+R_{1m}\, R_{2m}\, X_2^2) + (-12\, R_2\, X_c\, b_m^2\, k_1^2)\, X_1+(-R_2\, b_m^2\, (-6\, R_{1m}\, k_2+12\, X_c\, k_1-R_{2m}^2))\, X_1^2+R_1\, b_m^2 (R_{2m}^2\, X_2^2$$
$$-12\, X_2^2\, X_c\, X_{1m}+6\, R_1\, R_2\, X_{1m}^2+12\, R_1 R_2\, X_{2m}\, X_{1m})$$

$$kp_{51}= -2\, b_m^3\, (2\, R_1\, X_2^2\, k_1^2)\, b^2-2\, b_m^3\, ((-4\, k_1\, (X_1\, R_2 k_1+R_1\, X_2^2+R_2\, X_1^2))\, Xc+R_{1m}\, (2\, R_2\, X_1^2 R_{1m}+R_1\, X_2^2\, R_{2m}$$
$$+R_2\, X1^2\, R_{2m})+2\, R_2\, k_3\, R_1^2+2\, R_1 (k_3\, R_2^2+ X_2^2\, R_{1m}^2)+4\, R_1\, R_2\, X_{2m}\, X_{1m} (R_2+R_1))$$

$$kp_{50}= b_m^4\, (-2\, k_1\, (X_1\, R_2\, k_1+R_2\, X_1^2+R_1\, X_2^2))\, X_c+b_m^4 (R_1\, X_2^2\, k_1^2\, b^2+R_1\, R_2\, (R_2+R_1)\, X_{2m}^2+2\, R_1 R_2\, X_{1m}\, (R_2+R_1)\, X_{2m}$$
$$+R_2\, X_{1m}^2\, R_1^2+R_1\, X_2^2 R_{1m}^2+R_1\, X_{1m}^2\, R_2^2+ R_2\, X_1^2\, R_{1m}^2)$$

$$kp_{44}= (-b\, (-R_2\, (2\, X_c\, k_1-k_2^2))\, X_1^2+(2\, R_2\, X_c\, k_1^2)\, b X_1+((4\, R_1\, X_2^2 k_1)\, X_c-R_1\, (2 X_2^2\, k_2^2+2 R_2 X_{2m} X_{1m}\, R_1\, +R_1\, R_2\, k_3))\, b)$$

$$kp_{43}= (2\, b\, b_m\, (-4\, R_2\, k_1)\, X_1^2+2\, b\, b_m\, (-4\, R_2\, k_1^2)\, X_1+ 2\, b b_m\, (-8\, R_1\, X_2^2\, k_1))\, X_c+2\, b\, b_m\, R_2 (k_2+R_{1m})\, k_2)\, X_1^2+$$
$$2\, b\, b_m (4\, R_1\, X_2^2\, R_{1m}\, k_2+2\, R_1 X_2^2\, R_{2m}\, k_2+2\, R_2\, R_1^2\, k_1^2)$$

$$kp_{42}= (-b\, (-12\, R_2\, b_m^2\, k_1)\, X_1^2-b\, (-12\, R_2\, b_m^2\, k_1^2)\, X_1-b(-24\, R_1\, X_2^2\, b_m^2\, k_1))\, X_c-b\, (R_2\, b_m^2\, (6\, R_{1m} k_2+ R_{2m}^2))\, X_1^2$$
$$-b\, (2\, R_1\, b_m^2\, (6\, X_2^2\, R_{1m}\, k_2\, +3\, R_1 R_2\, k_3+ R_{2m}^2\, X_2^2+6\, R_1\, R_2\, X_{2m}\, X_{1m}))$$

$$kp_{41}= ((-2\, R_2\, b_m^3\, (-R_{2m}\, R_{1m}+4\, X_c\, k_1-2\, R_{1m}^2))\, X_1^2\, b-b(8\, R_2\, X_c\, b_m^3\, k_1^2)\, X_1+((-16\, R_1\, X_{22}\, X_c\, k_1+ 4 R_2\, X_{1m}^2\, R_1^2$$
$$+ 8\, R_1\, (X_2^2\, R_{1m}^2+R_1\, R_2\, X_{2m}\, X_{1m})\, +4 R_1\, (R_1\, R_2\, X_{2m}^2+ X_2^2\, R_{2m}\, R_{1m}))\, b_m^3)\, b)$$

$$kp_{40}= b\, (-(-R_2\, b_m^4\, (2\, X_c\, k_1-R_{1m}^2))\, X_1^2+(2 R_2\, X_c\, b_m^4\, k_1^2)\, X_1+(-R_1\, b_m^4\, (-4\, X_2^2\, X_c\, k_1+2\, R_1\, R_2\, X_{2m}\, X_{1m}+$$
$$2\, R_{1m}^2\, X_2^2+R_1\, R_2\, k_3)))$$

$$kp_{34}= (R_2\, X_1^2+(2\, R_2\, k_1)\, X_1+R_2\, k_3+2\, R_2\, X_{1m}\, X_{2m}+X_2^2 R_1+X_2^2\, k_2)\, X_c^2+(-2\, R_2\, R_1^2\, k_1-2\, R_1\, b^2 X_2^2\, k_1+(-2\, R_2\, k_2^2)\, X_1$$
$$-2\, R_1\, R_2^2\, k_1)\, X_c+R_1\, X_2^2\, (R_{1m}^2+R_{2m}\, (R_{2m}+2\, R_{1m}))\, b^2+R_1\, R_2\, k_2^2\, (R_2+R_1)$$

$$kp_{33}= (-4\, b_m\, R_2\, X_c^2\, X_1^2-b_m\, (4\, R_2\, X_c\, (2 X_c\, k_1-3\, R_{2m} R_{1m}-R_{2m}^2-2\, R_{1m}^2))\, X_1-b_m\, ((3\, X_2^2\, R_{2m}+8\, R_2\, X_{2m}\, X_{1m}$$
$$+4\, X_2^2\, k_5+4\, R_2\, k_3)\, X_c^2+(-8\, R_1\, k_1 (R_2^2+R_1\, R_2+ b^2\, X_2^2))\, X_c+(2\, R_1\, X_2^2\, (R_{2m}+2\, R_{1m})\, k_2)\, b^2$$
$$+2\, R_1\, R_2\, (R_{2m}+2\, R_{1m})\, k_2\, (R_2+R_1)))$$

$$kp_{32}= -b_m^2\, (-6\, R_2\, X_{2m}^2+(-12\, R_2\, (X_1+X_{1m}))\, X_{2m}-3\, X_2^2\, R_{2m}-12\, X_1\, R_2\, X_{1m}-6\, R_2\, (X_{1m}^2+X_1^2)-6\, X_2^2\, k_5)\, X_c^2$$
$$-b_m^2\, ((12\, R_1\, X_2^2\, k_1)\, b^2+12\, R_1\, R_2\, k_1\, k_6+2\, X_1\, R_2\, R_{2m}^2+12\, X_1\, R_2\, R_{1m}\, k_2)\, X_c$$
$$- b_m^2\, (- R_1\, (6\, R_{1m}\, k_2+R_{2m}^2)\, (R_2\, k_6+b^2\, X_2^2))$$

$$kp_{31}= -b_m\, (b_m^2\, (X_2^2\, R_{2m}+4\, R_2\, (X_{2m}^2+X_1^2)+4\, X_2^2\, k_5+4 R_2\, X_1\, X_{1m}+4\, R_2\, (X_{2m}+k_1)\, (X_1+X_{1m})))\, X_c^2$$
$$- b_m\, (-4\, b_m^2\, (2\, R_1\, (R_2\, k_6+b^2\, X_2^2))\, X_{2m}- 4\, b_m^2\, (X_1\, R_2\, R_{1m}\, (2\, R_{1m}+R_{2m})+2\, R_1\, X_{1m}\, (R_2 k_6+b^2\, X_2^2)))\, X_c$$
$$-b_m\, (2\, R_1\, b_m^2\, R_{1m}\, (2\, R_{1m}+R_{2m})(R_2^2+R_1\, R_2+b^2\, X_2^2))$$

$$kp_{30}= ((X_2^2\, k_5+R_2\, X_{2m}\, (k_1+X_{1m})+(2\, R_2\, k_1)\, X_1+R_2\, (X_{1m}^2+X_1^2))\, X_c^2+(-2\, R_1\, b^2\, X_2^2\, k_1$$
$$-2\, R_1\, R_2\, k_1\, k_6-2\, R_2\, R_{1m}^2\, X_1)\, X_c+ b^2\, X_2^2\, R_1\, R_{1m}^2+R_1\, R_2\, R_{1m}^2\, k_6)\, b_m^4$$

$$kp_{24}= (-b\, R_2\, X_1^2+b\, (-2\, R_2\, k_1)\, X_1+b\, (-R_2\, k_3-2\, X_2^2 R_{2m}-2\, X_2^2\, k_5-2\, R_2\, X_{2m}\, X_{1m}))\, X_c^2$$
$$+b\, (2\, R_2\, (k_2^2+R_1^2))\, X_1\, X_c+ b\, (-R_2\, R_1^2\, k_2^2)$$

$$kp_{23}= 2\, b\, b_m\, (2\, R_2\, X_1^2+4\, R_1\, X_2^2+4\, R_2\, X_1\, k_1+3\, X_2^2\, k_2+2\, R_2\, k_3+4\, R_2\, X_{2m}\, X_{1m}+X_2^2\, R_{1m})\, X_c^2$$
$$+2\, b\, b_m\, (-2\, R_2\, X_1\, (2\, k_7+R_{2m}\, (3\, R_{1m}+R_{2m})))\, X_c+ 2\, b\, b_m\, (R_2\, R_1^2\, (R_{1m}+k_2)\, k_2)$$

$$kp_{22}= (-6\, b\, R_2\, b_m^2\, X_1^2+b\, (-12\, R_2\, b_m^2\, k_1)\, X_1+b\, (-6\, b_m^2\, (R_2\, k_3+2\, X_2^2\, k_5+2\, R_2\, X_{2m}\, X_{1m}+X_2^2\, R_{2m})))\, X_c^2$$
$$+b\, b_m^2\, (2\, R_2\, (R_{2m}\, k_2+ 5\, R_{2m}\, R_{1m}+6\, k_7))\, X_1\, X_c+b\, b_m^2\, (-R_2\, R_1^2\, (6\, R_{1m}\, k_2+R_{2m}^2))$$

$$kp_{21}= (4\, b\, R_2\, b_m^3\, X_1^2+b\, (8\, R_2\, b_m^3\, k_1)\, X_1+b\, b_m^3\, (4\, R_2\, k_3+2\, X_2^2\, R_{2m}+8\, R_2\, X_{1m}\, X_{2m}+8\, X_2^2\, k_5))\, X_c^2$$
$$+b\, b_m^3\, (-4\, R_2\, (R_{2m}\, R_{1m}+2\, k_7))\, X_1\, X_c+ b\, b_m^3 (2\, R_2\, R_1^2\, R_{1m}\, (R_{1m}+k_2))$$

$$kp_{20}= (-b\, R_2\, X_c^2\, b_m^4\, X_1^2+b\, b_m^4\, (-2\, R_2\, X_c^2\, k_1+2\, R_2\, X_c\, k_7)\, X_1$$
$$+b\, b_m^4\, (-R_2\, R_1^2\, R_{1m}^2-2\, R_2\, X_c^2\, X_{1m}\, X_{2m}-R_2\, X_c^2\, k_3-2\, X_2^2\, X_c^2\, k_5))$$

$$kp_{14}= X_c^2\, (k_5+R_{2m})\, (R_2\, (R_2+k_5+R_{2m})+b^2\, X_2^2)$$

$$kp_{13}= - X_c^2\, b_m\, (4\, b^2\, X_2^2\, k_5+3\, b^2 X_2^2\, R_{2m}+4\, R_2^2\, k_5+8\, R_2\, R_1\, R_{1m}+2\, R_2\, R_{2m}^2+4\, R_2\, k_7+3\, R_2^2\, R_{2m}\, +6\, R_2\, R_{2m}\, k_5)$$

$kp_{12}=X_c^2\ b_m^2\ (6\ b^2\ X_2^2\ k_5+6\ R_2\ k_5\ (k_5+R_{2m})+3\ b^2\ X_2^2 R_{2m}+3\ R_2^2\ (R_{1m}+k_2)+6\ R_2^2\ R_1+R_2\ R_{2m}^2)$
$kp_{11}= -X_c^2\ b_m^3\ (4\ R_2^2\ k_5+R_{2m}\ R_2^2+4\ R_2\ k_7+R_{2m}\ b^2\ X_2^2+8\ R_2\ R_1\ R_{1m}+2\ R_2\ R_{2m}\ k_5+4\ b^2\ X_2^2\ k_5)$
$kp_{10}= X_c^2\ k_5\ b_m^4\ (R_2\ k_5+R_2^2+b^2\ X_2^2)$
$kp_{04}= -\ b\ R_2\ X_c^2\ (k_5+R_{2m})^2$
$kp_{03}= 2\ b\ R_2\ X_c^2\ b_m\ (k_5+R_{2m})\ (R_{2m}+2\ k_5)$
$kp_{02}= -bR_2\ X_c^2\ b_m^2\ (6\ k_5^2+6\ R_{2m}\ k_5+R_{2m}^2)$
$kp_{01}= 2\ b\ R_2\ X_c^2\ b_m^3\ k_5\ (2\ k_5+R_{2m})$
$kp_{00}= -b\ R_2\ X_c^2\ b_m^4\ k_5^2$

and
$k_1=X_{1m}+X_{2m}$, $k_2=R_{2m}+R_{1m}$, $k_3=X_{1m}^2+X_{2m}^2$, $k_4=R_1\ X_2^2+R_2\ X_1^2$, $k_5=R_1+R_{1m}$, $k_6=R_1+R_2$, $k_7=R_1^2+R_{1m}^2$

Appendix (III)

Parameters of the System:

Three-phase induction generator, star-connected,
460 V, 1180 rpm, 40 kW, 50 Hz with the following parameters:
$R_1 = 0.191\ \Omega$, $L_1 = 0.0012\ H$, $R_2 = 0.0707\ \Omega$,
$L_2 = 0.00179\ H$,
L_m (unsaturated magnetizing inductance) = 0.0448 H, C =300 μF

Three-phase induction motor, star-connected, 460 V, 1180 rpm, 20 kW, 50 Hz with the following parameters:
$R_{1m} = 0.455\ \Omega$, $L_{1m} = 0.00159\ H$, $R_{2m} = 0.149\ \Omega$,
$L_{2m} = 0.00239\ H$, $L_{mm} = 0.0653\ H$.