OPTIMAL POWER FLOW SOLUTION USING EFFICIENT PARTICLE SWARM OPTIMIZATION ALGORITHM

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Abstract-*This paper presents a particle swarm optimization efficient algorithm (PSO) for the solution of the optimal power flow (OPF). The objective is to minimize the total fuel cost of generating units with optimal setting of control variables without violating inequality constraints and satisfying equality constraint. Control variables are both continuous and discrete. The continuous control variables are unit active power outputs and generator bus voltage magnitudes, while the discrete variables are transformer tap settings and reactive power of shunt compensators. The PSO algorithm solution has been tested on the standard IEEE 30-Bus test system with different cases of objective function such as simple quadratic fuel cost, simple fuel cost with voltage profile improvement with both continuous and discrete control variables. The results have been compared to other methods.*

Key words-*Optimal power flow (OPF), particle swarm Optimization (PSO), voltage profile improvement.*

I-Introduction

The main objective of the economic dispatch (ED) of electric power generation or optimal power flow (OPF) is to minimize a selected objective function such as the fuel cost via the optimal adjustment of the power system control variables while at the same time satisfying various equality and inequality constraints. The equality constraints are the power flow balance equations, while the inequality constraints are the limits on the control variables and the operating limits of the power system dependent variables. The problem control variables include the generator real powers, the generator bus voltages, the transformer tap settings, and the reactive power of switchable VAR sources, while the problem dependent variables include the load bus voltages, the generator reactive powers, and the power line flows. Generally, the OPF problem is a large-scale highly constrained nonlinear non convex and multimodal optimization problem.

To solve the OPF problems, the optimization methods are classified into classical and heuristic optimization methods.

Classical optimization methods such as nonlinear programming, quadratic programming, linear programming, Newton-based techniques, and interior point methods are highly sensitive to starting points and frequently converge to local optimum solution or diverge altogether in non-convex OPF problem.

Linear programming methods are fast and reliable but their main disadvantage is associated with the piecewise linear cost approximation. Nonlinear programming methods are known to suffer from the complex algorithms. The problem encountered in Newton based algorithms resides in the fact that the inequality constraints are added as quadratic penalty terms to the problem objective, multiplied by appropriate penalty multipliers [3]. Interior point (IP) methods convert the inequality constraints to equalities by the introduction of nonnegative slack variables. A logarithmic barrier function of the slack variables is then added to the objective function and multiplied by a barrier parameter, which is gradually reduced to zero during the solution process [15].

Most of these methods are based on the combination of the objective function and the constraints by Lagrange formulation and Kuhn Tucker condition and applying sensitivity analysis and gradient-based optimization algorithm [3].

Heuristic methods such as genetic algorithm [13], evolutionary programming algorithm [9], particle swarm optimization (PSO) [6], and differential evolution (DE) [16] have been proposed for solving the

OPF problem. GA and DE are parallel and global search techniques emulating natural genetic operators such as selection, crossover and mutation. A GA and DE methods is more likely to converge toward the global solution because it, simultaneously, evaluates many points in the parameter space. The PSO algorithm is also a global search method which explores search space to get to the global optimum, the PSO is a stochastic, population-based computer algorithm modeled on swarm intelligence, PSO finds the global minimum of a multidimensional, multimodal function with best optimum. It does not need to assume that the search space is differentiable or continuous. In reference [6], a particle swarm optimization method has been proposed to minimize the total cost function with continuous control variables.

In the present paper a PSO algorithm method is used to improve the quality of solution, leading to the global optimum in each generation with both continuous and discrete control variables.

The continuous control variables are unit active power outputs and generator bus voltage magnitudes, while the discrete variables are transformer tap settings and reactive power of shunt compensators.

The control variables are multimodal and have not an approximate value. To improve the quality of solution you need to normalize vector of control variables to the approximate values.

This method has been tested on the IEEE 30-bus standard system with different type of objective function such as total simple quadratic fuel cost and total cost with voltage profile improvement effect witch are multimodal optimization problem. The results are compared with other methods with both continuous and discrete control variables.

II-Problem Formulation

The OPF problem is considered as a general minimization problem with constraints, and can be written in the following form:

Minimize $f(x,u)$ (1)

Subject to $g(x,u) = 0$ (2)

and $h(x,u) \leq 0$ (3)

Where $f(x,u)$ is the objective function, $g(x,u)$ and $h(x,u)$ are respectively the set of equality and inequality constraints, x is the vector of state variables and u is the vector of control variables.

The state variables are the load buses voltages and angles, the generator reactive powers and the slack active generator power.

$$x^T = (P_{g1}, \theta_2,..\theta_N, V_{L1},..V_{LNL}, Q_{g1},..Q_{gng})^T \quad (4)$$

The control variables are the generator active power outputs except active power of slack bus, the bus voltages, the shunt capacitors/reactors and the transformers tap-settings.

$$U = (P_g, V_g, T, Q_c)^T \quad (5)$$

Or:

$$U = (P_{g2},;.....P_{gng}, V_{g1},......V_{gng},$$
$$T_1,....T_{Nt}, Q_{c1},..Q_{cNC})^T \quad (6)$$

Where N_g, NT, NC are the number of generators, number of tap transformers and the number of shunt compensators respectively.

A-Objective Function

The objective function for the OPF reflects the costs associated with the system power generation. The quadratic cost model is used where the objective function for the entire power system can then be written as the sum of the generators quadratic cost models:

$$F_T = \sum_{i=1}^{Ng} C_i(P_{gi}) \quad (7)$$

1- Simple quadratic fuel cost model

In this case a cost of unit i take a simple quadratic form:

$$C_i(P_{gi}) = a_i + b_i P_{gi} + c_i P_{gi}^2 \quad (8)$$

Where N_g the number of units, P_{gi} is the generator active power at unit i and a_i, b_i and c_i are the cost coefficients of the ith unit.

B-Equality Constraint

The equality constraint $g(x,u)$ of the OPF problem is represented by the power balance equation, where the total power generation must cover the total power demand and the power losses:

$$P_G = P_D + P_L \tag{9}$$

This equation is solved by nonlinear load flow method to calculate the active power of slack bus and active power loss.

C-Inequality Constraints

The inequality constraints $h(x,u)$ reflect the limits on physical devices in the power system as well as the limits created to ensure system security:
Upper and lower bounds on the active and reactive power of generations:

$$P_{gi\,min} \le P_{gi} \le P_{gi\,max} \tag{10}$$

$$Q_{g\,min} \le Q_{gi} \le Q_{g\,max} \quad i = 1,..., N_g \tag{11}$$

Upper and lower bounds on the bus voltage magnitudes of all buses:

$$V_{i\,min} \le V_i \le V_{i\,max} \quad i = 1,.., N \tag{12}$$

Upper and lower bounds on the transformers tap ratios:

$$T_{i\,min} \le T_i \le T_{i\,max} \quad i = 1,..., N_T \tag{13}$$

Upper and lower bounds on the compensators reactive powers:

$$Q_{c\,min} \le Q_c \le Q_{c\,max} \quad i = 1,...., N_c \tag{14}$$

Where N is the number of buses, N_T is the number of Transformers, N_c is the number of shunt reactive compensators,
Security constraints describe the maximum capability of transmission line, then the power flow of each line does not exceed its limit then we have:

$$\left| Lf_i \right| \le Lf_{i\,max} \quad i = 1,..., Nbr \tag{15}$$

Where *Nbr* is the number of transmission lines in the power network and Lf_i is the maximum power.
In PSO search algorithm all control variables stand in there limits except active power in slack bus
By adding the inequality constraints to the objective function, the augmented form to be minimized becomes:

$$F = F_T + \lambda_P (P_{gslack} - P_{gslack}^{lim})^2 + \lambda_V \sum_{i:=1}^{NL} (V_i - V_i^{lim})^2 \tag{16}$$

where λ_P, λ_V are the penalty factors and both penalty factors are large positive constants; *NL* is a number of load buses.
P_{gslack}^{lim} and V_i^{lim} are defined as

$$P_{gslack}^{lim} = \begin{cases} P_{gslack}^{min} & \text{if } P_{gslack} < P_{gslack}^{min} \\ P_{gslack}^{max} & \text{if } P_{gslack} > P_{gslack}^{max} \end{cases}$$

$$V_i^{lim} = \begin{cases} V_i^{min} & \text{if } V_i < V_i^{min} \\ V_i^{max} & \text{if } V_i > V_i^{max} \end{cases}$$

The equality constraint and reactive power inequality constraints are of generators is handling in solution of load flow problem.
In problem of OPF with voltage profile improvement the objective is to minimize the load bus voltage magnitude deviation   from 1.0 per unit. Then the objective function can be expressed as:

$$F = F_T + \lambda_P (P_{gslack} - P_{gslack}^{lim})^2 + \lambda \sum_{i:=1}^{NL} (V_i - 1)^2 \tag{17}$$

where λ is a weighting factor and weighting factor is large positive constant; *NL* is the number of load buses.

III- Overview of PSO

Particle Swarm Optimization was introduced by R, Eberhart and J, Kennedy in 1995 [2], inspired by social behavior of bird flocking or fish schooling. It is a part of modern heuristic optimization algorithm, it work on population or group in witch individuals called particles move to reach the optimal solution in the multidimensional search space. Differ as the genetic algorithm PSO use directly a real value of control variables .The number of particles in the group is *Np*. the initial population of a PSO algorithm is randomly generated within the control variables bounds. Each particle adjusts its position through its present velocity, previous positions and the positions of its neighbors.
In *d* dimensional search space the position and velocity of particle *i* are represented as the vectors

$X_i = (x_{i1},....,x_{iD})$ and $Vt_i = (vt_{i1},....vt_{iD})$ respectively where $i \in Np$, and d is the number of members in a particle, it represent in general the number of control variables in the objective function .

Let $Xpbest_i = (x_{i1}^{best},...x_{id}^{best})$ the best previous position of particle i, and $Xgbest = (x_1^{gbest},....x_d^{gbest})$ the best particle among all the particles in the swarm. The updated velocity of particle i is modified under the following equation:

$$vt_i^{k+1} = \omega vt_i^k + c_1 rand_1 \times (Xpbest_i^k - X_i^k) + c_2 rand_2 (Xgbest^K - X_i^K) \quad (18)$$

where

Vt_i^K : velocity of particle i at iteration k;

ω : inertia weight factor ;

c_1, c_2 : acceleration constant;

k : current iteration;

$rand_1, rand_2$: random numbers between 0 and 1;

$Xpbest_i^k$:best position of particle i until iteration k;

$Xgbest^k$: best position of the swarm until iteration k;

Each particle changes its current position to the new position by adding the modified velocity (18) using the following equation:

$$X_i^{k+1} = X_i^k + Vt_i^{K+1}, i = 1,2,..Np \quad (19)$$

In general inertia weight factor ω decreases linearly from ω_{max} to ω_{min} according to the following equation:

$$\omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{k_{max}} \times k \quad (20)$$

where k_{max} is the maximum number of iterations.

IV- implementation of the OPF PSO algorithm:

IV-1 Initialization:

Initial value of each particle is generated randomly between $[u_{min}, u_{max}]$ $X_i^0 = (x_{i,1}^0,....,x_{i,j}^0)$ then $x_{i,j}^0 = random(u_j^{min}, u_j^{max})$.

Also initials values of velocity of each particle is generated randomly between $[Vt_{min}, Vt_{max}]$

$vt_{i,j}^0 = random(vt_{i,j}^{min}, vt_{i,j}^{max})$

$vt_{i,j}^{max} = -Vt_{i,j}^{min} = (u_j^{max} - u_j^{max})/Nv$ Where Nv is an integer value representing the number of intervals.

where $i = 1,..N_P, j = 1,..d$ and u_j^{max}, u_j^{min} are maximum and minimum values of control variables respectively.

IV-2 Algorithm OF OPF-EPSO

The steps of the proposed algorithm are listed as follow:

Step 1: give PSO parameters and $k=1$; Np; ω_{min}, ω_{max} , k_{max}, c1,c2, d=dimension of vector of control Variables U.

Step 2: Initialize at random Np particles within their limits, and initialize the velocities of each particle within their limits.

Step 3: Calculate fitness function of each initial particle X_i^0 using objective function F giving in (16) or (17).

Step 4: set $Xpbest_i = X_i^0$ as a previous Xi and $Xgbest$ to the best a particle have the best fitness of all particles $Xpbest_i$

Step 5: set iteration K=1;

Step 6: update velocity of each particle using equation (18), If $vt_{i,j} < vt_{i,j}^{min}$ then $vt_{i,j} = vt_{i,j}^{min}$, or if $vt_{i,j} > vt_{i,j}^{max}$ then $vt_{i,j} = vt_{i,j}^{min}$

Step 7: adjusts the position of each particle using equation (19) if the element of vector of particle Xi exceeds its limits, enforce it in within boundary.

Step 8: calculate new fitness function of each particles X_i using objective function F.

Step 9: if the evaluation value of each particle is better than previous $Xpbest_i$, the current is set to be $Xpbest_i$ if the best particle of all $Xpbest_i$ is better than $Xgbest$, the current is set to $Xgbest$.

Step 10: if $k < Kmax$ set $K=K+1$ and go to step 6, otherwise go to step 11.

Step 11: take $U^{best} = Xgbest$ and running load flow to calculate real slack power, and other elements of state variables.

To evaluate fitness function of each particle X_i set the vector of control variables $U = X_i$ and running load flow to evaluate real slack power, and other elements of state vector.

If the values of the control variables have not an approximate value; for affecting their values to the particle it may be normalized by the operators: multiplying or dividing to constant values. To extract the values from particle to control variables you must inverse the operators.

IV-3- Handling of discrete Variables:

The discrete control variables are adjusting by 0.01 step size. Then each transformer tap setting is rounded to its nearest decimal integer value of 0.01, by utilizing the rounding operator. The same principle applies to the discrete reactive power injection of shunt compensators.

V- Numerical Results

The PSO algorithm has been tested on the IEEE 30-bus, 41 branch system [6]. It has a total of 24 control variables as follows: 5 unit active power outputs, 6 generator-bus voltage magnitudes, 4 transformer-tap settings, and 9 bus shunt reactive compensators.

The security constraints considered are the voltage magnitudes of all buses, the reactive power limits of the shunt VAR compensators and the transformers tap settings limits. The variables limits are listed in Table 1. The transformer taps and the reactive power source installation are discrete with the changes step of 0.01.

TABLE 1
VARIABLES LIMITS IN PU

V^{min}	V^{max}	T^{min}	T^{max}	Q_c^{max}	Q_c^{max}
0.95	1.1	0.9	1.1	0	0.05

The power limits and cost coefficients of generators buses are represented in Table2 and Table 3 respectively. Generators buses are: PV buses 2,5,8,11,13 and slack bus is 1.the others are PQ-buses.

The PSO population size is taken equal to 10, the maximum number of generations is 100, acceleration factors $C_1=C_2=2$, maximum and minimum inertia factors are $\omega_{max}=0.9, \omega_{min}=0.1$,

both penalty factors in (16) are chosen, $\lambda_P = \lambda_v = 1000$ and weighting factor in (17) is $\lambda = 1000$.

The complete algorithm has been implemented in Delphi oriented object programming, 10 runs have been performed for each type of objective function and the results which follow are the best solution of these 10 runs.

TABLE 2
Generators power Limits in Mw and Mvar

Bus N°	P_{gmin}	P_{gmax}	Q_{gmin}	Q_{gmax}
1	50	200	-20	200
2	20	80	-20	100
5	15	50	-15	80
8	10	35	-15	60
11	10	30	-10	50
13	12	40	-15	60

TABLE 3
Cost coefficients of the IEEE 30-bus system

Bus N°	$a(\$/h)$	$b(\$/Mwh)$	$c(\$/(Mw)^2h)$
1	0	2.00	0.00375
2	0	1.75	0.01750
5	0	1.00	0.06250
8	0	3.25	0.00834
11	0	3.00	0.02500
13	0	3.00	0.02500

Case A: simple quadratic fuel cost function

The optimal settings of the control variables are given in table 4 case A. The total fuel cost was initially 901.88 \$/H and it has been reduced by the proposed PSO to 799.374 \$/H, the active power losses is 8.758 Mw.

This solution is improved than the optimal fuel cost obtained by the other heuristic methods reported in the literature with both continuous and discrete control variables such improved genetic algorithm IGA[13] (see table 5) with 800.805 \$/h of fuel cost .

System of voltage profile for all bus in this case as shown in Figure 1. We notice that all control variables are within their limits.

Case B: Fuel cost with voltage profile optimization

The optimal setting of control variables are shown in table 4 case B, the total fuel cost is 802.738 \$/h, active power loss is 9.616 Mw. The system voltage profile of this case is compared to that of case A as shown in figure 1, it is clear that the voltage of load

buses profile is greatly improved compared to that of case A, and their values are near to 1 PU.

TABLE 4
Simulation results of different cases

	Case A	Case B	Case C	Case D
P_{g1}(mw)	177.726	176.810	176.658	176.231
P_{g2}	48.599	48.729	49.444	49.187
P_{g5}	21.077	21.578	21.287	21.39
P_{g8}	20.846	21.165	21.117	21.703
P_{g11}	11.91	12.734	11.565	12.455
P_{g13}	12	12	12	12
V_1(pu)	1.1	1.055	1.1	1.055
V_2	1.088	1.036	1.088	1.034
V_5	1.06	1.004	1.064	1.000
V_8	1.069	1.005	1.071	1.005
V_{11}	1.1	1.032	1.094	1.013
V_{13}	1.1	1.024	1.100	1.038
$T_{4,12}$	0.98	0.98	1.021	1.015
$T_{6,9}$	1.02	1.01	1.072	0.984
$T_{6,10}$	0.90	0.92	0.900	0.953
$T_{28,27}$	0.97	0.96	0.988	0.965
Q_{10}(pu)	0.00	0.02	0.027	0.047
Q_{12}	0.03	0.00	0.00	0.015
Q_{15}	0.03	0.03	0.05	0.039
Q_{17}	0.02	0.03	0.018	0.024
Q_{20}	0.05	0.02	0.016	0.045
Q_{21}	0.05	0.03	0.05	0.02
Q_{23}	0.01	0.02	0.05	0.009
Q_{24}	0.01	0.03	0.039	0.04
Q_{29}	0.04	0.03	0.018	0.034
Cost ($/H)	799.374	802.738	799.25	799.63
loss(mw)	8.758	9.616	8.67	9.566

Case C: Fuel costs with continuous variables

In this case the control variables are all continuous. The optimal settings of the control variables are given in table 4 case C, The total fuel cost has been reduced by the proposed PSO to 799.25 $/H, the active power losses is 8.67 Mw.

A comparison between the results of fuel cost obtained by the proposed PSO approach and those reported in the literature; with the same control variable limits, initial conditions, and other data, the problem was solved using gradient method [11], particle swarm optimization algorithm [6] and differential evolution [16] with optimal fuel cost respectively of 804.583 $/h , 800.41$/h,and 799.2891 $/h respectively. The results of this comparison are given in Table 5.
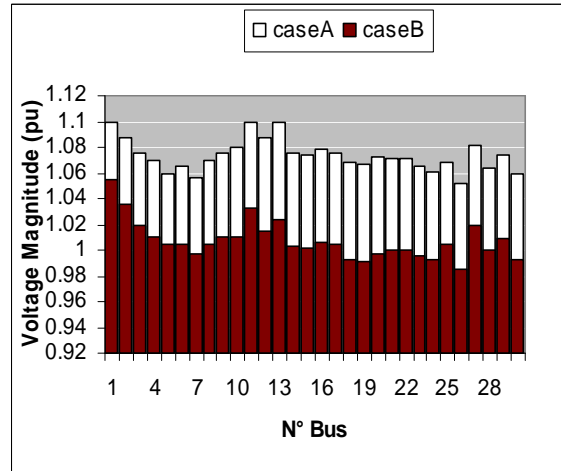

Fig. 1. Voltage Profile Solution (case A and B)

TABLE 5
Comparison of fuel cost for different methods

Method	Cost($/h)
Gradient based approach(11)	804.853
Improved genetic algorithm(13)	800.805
Particle swarm optimization(6)	800.41
Differential Evolution (16)	799.2891
Efficient Particle swarm optimization	799.25

These results show that the optimal power flow solutions determined by EPSO lead to good optimum fuel cost, which confirms that the EPSO is well capable of determining the global or near-global optimum dispatch solutions.

Case D: fuel cost with voltage profile improvement and continuous variables

In this case the results of simulation is given in table 4 case D with optimal control variables, the total fuel cost is greater then case C ; that is 799.63$/h
In PSO [6] the total fuel for this case is 806.38 $/h with all control variables are continuous.
The system of voltage profile of this case is compared to that of case c as shown in figure 2, it is clear that the voltage profile of load buses is greatly improved compared of case C, and their values are near to 1 PU.
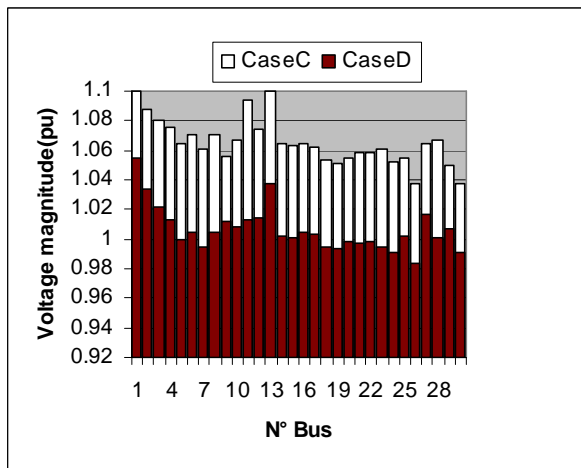
Fig. 2. Voltage profile solution (Case C and D)

VI- Conclusion

In this paper, a PSO solution to the OPF problem has been presented. The main advantages of the PSO to the OPF problem are optimization of convex or non-convex objective function, real coded of both continuous and discrete control variables, and easily handling nonlinear constraints. The proposed algorithm has been tested on the IEEE 30-bus system to minimize the total fuel cost with different type of objective functions. The optimal setting of control variables are obtained in both continuous and discrete values. The results were compared with the other heuristic methods such as GA, DE and PSO algorithm reported in the literature. These results demonstrate that EPSO converges to the global optimum or near global optimum and succeeds in keeping the variables within their limits.

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