

Application of the technique Frequency-Hopped to the localization of sources by an array of sensors and application to the algorithm MUSIC

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Abstract

In this paper, we present a different approach to the problem of estimating the angle of arrivals (AOA's) of D targets in Frequency-Hopped signaling sensors array for active systems, with D smaller than the number of sensors element, L . This method is based on the application of a new proposed model of received data available in different channels. The simulations results show that this approach improves the resolution in the estimation of the angle of arrivals compared with Monotone-Frequency signaling case.

Key words: targets, sources, array of sensors, frequency-hopped, estimation, angles of arrivals.

I. Introduction

The automatic determination of the number of beaming or reflective sources, their positions and other characteristic parameters of the signals from an appropriated processing of the data received through an array of sensors, witch has arbitrary position and directional characteristic, find its application in different domains such as radar/sonar, communication, astronomy, geophysical, oceanography and seismology.

Several algorithms of estimation of the angles of arrival of the sources have been developed as those of Schmith [1,2], Bienvenu [3] that present the algorithm MUSIC (MUltiple SIgnal Characterisation, algorithm), based on the approach of the eigndecomposition of the data autocorrelation matrix. In this algorithm a $P(\theta)$ function, called the orthogonality measure function, is constructed :

$$P(\theta) = \frac{1}{d^H(\theta) \cdot U_N \cdot U_N^H \cdot d(\theta)} \quad (1)$$

Based on the orthogonality property between the noise sub-space generated by the noise eign vectors, U_N , and the signal sub-space generated by the vectors of directions $d(\theta)$. The peaks of the function, $P(\theta)$, represent the angles of arrivals of the sources.

The application of the maximum Likelihood approach (Maximum Likelihood Estimator, MLE) to estimate the angles of arrivals of narrow-band sources was done by several researchers. Let's mention works of Ziskind [4], Wax [5] and Boehme [6] who propose an alternative projection algorithm where a logarithmic Likelihood function according to the angles of arrival, θ_k , is defined as follows:

$$g(\theta) = \sum_{m=1}^M |P_A(\theta_k) \cdot x(m)|^2 \quad (2)$$

Where $P_A(\theta_k)$ is the projection operator onto the space generated by the columns of the matrix of direction, $A(\theta_k)$. M is the number of samples. The angles of arrivals are those that maximize $g(\theta_k)$. Miller [7] propose the EM algorithm (Expectation Maximization Algorithm), generalized for the numerical resolution of the ML estimator (Maximum Likelihood) of the angles of arrivals of narrow-band sources. In [8] Weiss proposed an algorithm of least square minimizations for the calculation of the ML estimator is also proposed. However, the majority of these iterative procedures don't guarantee the convergence toward the global optimum. Besides, if the number of sources is big, these iterative procedures become very complicated in calculation load view. To surmount these problems, Huang [9] suggest the DP algorithm (Dynamic Programming Algorithm), to calculate the ML estimator of the number of sources and their angles of arrival, simultaneously. In [10] an active system for radar/sonar with a variator of impulse frequency is proposed in the goal to improve the resolution in tonality and the estimation of the angles of arrivals of the non-fluctuating targets. Huang and Barkat [11], of their part, applied the technical Frequency-Hopped to an active system for the estimation of the number of mobile targets by the AIC criteria (Akaike Information Criteria) and MDL criteria (Minimum Description Lenght), where the survey of the probability of false alarm-1 (to detect $D+1$ sources) and the probability of miss-1 (to detect $D-1$ sources, D being the number of sources) and the probability of the correct detection revealed an improvement compared to the case of monotonous frequency. Li [12] used the technical Frequency-Hopped to the algorithm MUSIC in the goal to detect a number of targets superior than the number of receipt channels

for active systems. In [13] Barkat and Aissous applied the technical Frequency-Hopped to improve the estimation of the positions of the sources by the maximum likelihood estimator. In [14] a MUSIC-based approach, Time-Space-Time MUSIC (TST-MUSIC), is proposed to jointly estimate the directions of arrivals and the delay's of a wireless multipath channel. A novel extension of the MUSIC algorithm for the frequency estimation problem is proposed in [11].

In this article, we present an approach of estimation of the angles of arrivals and the number of narrow-band sources for active systems based on the use of the technical Frequency-Hopped. Instead of using the vector observation, separately, for every channel as in [11], our vector of observation is constructed from all data received on all channels wholes. The formulation of the problem is made in the Section 2. The results of the illustrative simulations of the performances of the proposed method are presented in the Section 3. The Section 4, is dedicated to the conclusion.

II. Formulation of the problem

In communication and radar systems, the technique of coding said Frequency-Hopped is used to improve the performances of the systems. In our work this technique is used for the improvement of the estimation of the number and positions (directions of arrivals) of narrow-band sources, by an active system which has an array of sensors. Unlike the classical transmission, where the impulse is emitted as it by the system, the technical Frequency-Hopped consists in subdividing the emitted impulse in several equal sub-impulses. Each sub-impulse is characterized by a different carrier sub-frequency, f_p . these carriers sub-frequencies are uniformly spaced by a band of frequency Δf as indicated in figure 1.

Be $1/\Delta T$ and τ , respectively, the frequency of repetition of the impulses and the width of the impulse of the transmitted signal $s(t)$:

$$s(t) = \begin{cases} \sum_{p=1}^P c_p \exp \left[j2\pi \left[f_0 + \left(p - \frac{(P-1)}{2} \right) \Delta f \right] t \right] & \text{for } \tau \leq t \leq \Delta T \\ 0 & \text{for } 0 \leq t \leq \tau \end{cases} \quad (3)$$

P is number of sub-impulse; C_p is the complex amplitude of the p^{rd} sub-impulse. f_0 is the central frequency of the band $[f_0 - (p-1)\Delta_f/2, f_0 + (p-1)\Delta_f/2]$. Δf is the uniform width of frequency that separates two adjacent sub-frequencies carriers. To simplify the writing we will note in what follows the expression $\left[f_0 - \left(p - \frac{(p-1)}{2} \right) \Delta f \right]$ by f_p .

Let's consider a sequence of M impulses each of length τ , emitted by an active system, with straight array of L sensors. Every impulse consists in P sub-impulses, of $\tau'=\tau/P$ length each, and of frequency carrier, f_p , different. While using the reception configuration that consists in P Input/Output channels granted to the carriers frequencies, f_p , $p=1,2,\dots,P$, respectively, followed by P filters, [14,15], where each channel only receives the corresponding data to the frequency okay f_p . The sources are supposed faraway of the array of sensors what permits to use the model of plane wave for the incidental signals. The response of the l^{rd} sensor to the sub-frequency, f_p , can be expressed as follows:

$$x_l^p = \sum_{k=1}^D \eta_{lk} a_l^p(\theta_k) \exp[-j2\pi f_p \tau_k] + n_l^p(m) \quad (4)$$

$i=1,2,\dots,L ; m=1,2,\dots,M; p=1,2,\dots,P$

Where M is the total number of samples that is the same that the one of the impulses emitted by the system. $n_l^p(m)$ is the component of the additive noise associated to the l^{rd} sensor to the sub-frequency, f_p . τ_k is the necessary time to the signal to cross the distance between the k^{rd} source and the l^{rd} sensor. While taking the origin of phase change the first sensor, the delay is given by:

$$\tau_{lk} = (l-1)(\Delta/c) \cdot \sin(\theta_k) \quad (5)$$

Where c is the speed of propagation and Δ is the uniform spatial spacing between two adjacent sensors. $a_l^p(\theta_k)$ is the gain of the l^{rd} sensor for an incidental signal following the direction θ_k , $k=1,2,\dots,D$, under the sub-frequency f_p . θ_k are the angles of arrivals of the sources. η_{lk} is a complex random variable translating the effects of attenuations due to the propagation and to the reflection of the signal on the k^{rd} source and attaining the l^{rd} sensor. Under the hypothesis that the sources are faraway and the sensors are perfectly identical and omnidirectionnels, we can put respectively $\eta_k = \eta_{1k} = \eta_{2k} = \dots = \eta_{Lk}$ where η_k , $k=1, 2, \dots, D$ are supposed to be complex Gaussian, and $a^p(\theta_k) = a_1^p(\theta_k) = a_2^p(\theta_k) = \dots = a_L^p(\theta_k) = 1$. The observed data, corresponding to the sub-frequency, f_p , can be expressed under matrix shape, as:

$$x^p(m) = A^p(\theta) \cdot S(m) + N^p(m) \quad (6)$$

Where $x^p(m) = [x_1^p(m) x_2^p(m) \dots x_L^p(m)]^T$ is the vector of data corresponding to the sub-frequency, f_p , of dimension $(L \times 1)$. $S(m) = [\eta_1 \ \eta_2 \ \dots \ \eta_D]^T$, is the vector of sources of dimension $(D \times 1)$. $N^p(m) = [n_1^p(m) n_2^p(m) \dots n_L^p(m)]^T$, is the vector of noise corresponding to the sub-frequency, f_p , of dimension $(L \times 1)$. T denotes the transposed operation. $A^p(\theta)$ is the matrix of direction corresponding to the sub-frequency, f_p , of dimension $(L \times D)$ as:

$$A^p(\theta) = \begin{bmatrix} a^p(\theta_1) \exp(-j2\pi f_p \tau_{11}) \cdots a^p(\theta_D) \exp(-j2\pi \tau_{1D}) \\ a^p(\theta_1) \exp(-j2\pi f_p \tau_{21}) \cdots a^p(\theta_D) \exp(-j2\pi \tau_{2D}) \\ \vdots \\ a^p(\theta_1) \exp(-j2\pi f_p \tau_{L1}) \cdots a^p(\theta_D) \exp(-j2\pi \tau_{LD}) \end{bmatrix} \quad (7)$$

we notice, of the structure of $A^p(\theta)$, that every column, $d(\theta_k)$ contains information on the direction of the k^{rd} source from where the appellation of $A^p(\theta)$ matrix of direction and $d(\theta_k)$ vector of direction.

We suppose the following hypothesis:

- (i) for every sub-frequency carrier, f_p , M samples are taken. Every sample is taken from the m^{rd} corresponding impulse. The period of the impulses is taken big so that the samples of every sub-frequency are decorrelate and statistically independent.
- (ii) $S(m)$ is a process random stationary ergodique of correlation matrix (covariance), R_s , definite positive.
- (iii) the number of sources (targets) D is lower to the number of sensors L
- (iv) the noise, $n^p(m)$, is a process random gaussian of zero mean and variance, σ_n^2 , stationary, ergodique and decorrelate of the signals, $S(m)$.

After use of the Frequency Hopped technical we notice that we have, P , vectors of observations that form the spatial observation domain. What offers us a field, large enough, of use of these vectors, as in [11] where each vector has been used separately. In our work we arrange the set of the vectors of observations in only one vector of the following manner:

$$X(m) = [x^1(m)^T x^2(m)^T \dots x^P(m)^T]^T \quad (8)$$

While replacing every vector of observation, $x^p(m)$, by its matrix shape, given in (6), the new vector of observation, $X(m \text{ total})$, can be put under matrix shape as:

$$X(m) = A(\theta)S(m) + N(m) \quad (9)$$

Where $A(\theta) = [A^1(\theta)^T A^2(\theta)^T \dots A^P(\theta)^T]^T$ is the matrix of direction, for the P sub-impulses, of dimension $(LP \times D)$. $S(m) = [\eta_1(m) \ \eta_2(m) \dots \eta_D(m)]$ is the vector of sources of dimension $(D \times 1)$. $N(m) = [N^1(m)^T N^2(m)^T \dots N^P(m)^T]^T$ is the vector of noise for the P sub-impulses of dimension $(LP \times 1)$. The matrix of correlation of the total vector of observation, given by (9), is gotten while multiplying the equation (9) by his transposed conjugated complex and as taking its mathematical expectation, us will have:

$$R_x = A(\theta).R_s.A^H(\theta) + \sigma_n^2.I \quad (10)$$

where R_x is of dimension $(LP \times LP)$. R_s is the matrix of correlation of the sources vector, $S(m)$, of dimension $(D \times D)$. $\sigma_n^2.I$ is the matrix of covariance of the noise vector, $N(m)$, of dimension $(LP \times LP)$. Under the hypothesis (i), the matrix of correlation, R_x , can be estimated by:

$$\hat{R}_x = \frac{1}{M} \sum_{m=1}^M X(m).X^H(m) \quad (11)$$

The analysis of the dimensions of the total vector of observation, $X(m)$, and the new matrix of correlation, R_x , reveals that the proposed manner to arrange the set of observations vectors

corresponding to the different sub-frequencies carriers, f_p , $p=1,2,\dots,P$, give a fictional increase of the number of sensors, what is going to contribute to the improvement of the results of estimations.

The estimation of the angles of arrivals is made by the algorithm MUSIC [1, 2], that is based on the exploitation of the eigenstructure of the correlation matrix, R_x , (eigenstructure methods). The algorithm MUSIC is one of the algorithms the more used in the estimations of the sources since it presents the advantages to be able to estimate simultaneously the number and the positions of the sources, to be very simple to use and less expensive in time and load of calculations. However, the precision of the estimations is less precise compared to other more complex algorithms, as, the algorithms based on the estimation of the maximum of likelihood (Maximum Likelihood Estimator, MLE) and other variety of the MUSIC algorithm [7,18,19]. Many works are proposed to improve its performances, like [14,15,20]. So the new arrangement, proposed in our work, of the received data through the use of the technique of coding said "frequency hopped", give a fictional increase of the number of sensors used. what is going to improve the precision of evaluation of the algorithm MUSIC while keeping its advantages, above-stated.

The algorithm MUSIC [1, 2] is based on the exploitation of the two following properties:

- (i) The smallest eigen value, of the correlation matrix, R_x , is equal to the variance of the additive noise, σ_n^2 , with a multiplicity $(L - D)$, where L is the number of sensors and D the number of sources: $\lambda_{D+1} = \lambda_{D+2} = \dots = \lambda_L = \sigma_n^2$
- (ii) The orthogonality between the noise space generated by the corresponding eigen vectors to the smallest eigen values of the matrix of correlation, R_x , and the signal space generated by the columns of the matrix of direction $A(\theta)$:
 $[V_{D+1} V_{D+2} \dots V_L] \perp [a(\theta_1) a(\theta_2) \dots a(\theta_D)]$

A function, $P(\theta)$, called the function of measure of the orthogonality is constructed as:

$$P(\theta) = \frac{1}{d^H(\theta) \cdot U_N \cdot U_N^H \cdot d(\theta)} \quad (12)$$

Under the constraint that the number of samples, M , is finished, we can estimate the angles of arrivals of the incidents signals while finding the values of θ for which The function, $P(\theta)$ is maximal. The tracing of $P(\theta)$, according to θ , present the peaks for which their numbers are equal to the number of sources and their abscissas correspond to the angles of arrivals of the sources.

III. Results of simulations

The survey of the performances of the proposed method consists in analyzing its performances according to three criterias that we judged évaluatifs:

- (i) the precision of the estimation.
- (ii) the field of vision of the algorithm
- (iii) the angle of separation between the sources.

The analysis, in our case, is made while valuing the degree of improvement brought.

Let's define the root mean square error, RMSE, by:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{\text{Run}} \left(\theta_k - \hat{\theta}_{ki} \right)^2}{\text{Run}}} \quad (13)$$

θ_k : The angle of arrival of the k^{rd} source.

$\hat{\theta}_{ki}$: The estimated value, of the angle of arrival of the k^{rd} source.

Run: The number of executions of the calculation algorithm.

Let's consider a rectilinear array of 10 sensors, equidistant of a half length of wave $\Delta = \lambda_0/2$, where λ_0 correspond to the frequency, f_0 . The sensors are supposed omnidirectionnels, perfectly identical and of unit gain. The transmitted signal, used in our simulation, is given by:

$$s(t) = \begin{cases} \sum_{p=1}^P \exp \left[j2\pi \left[f_0 + \left(p - \frac{(P-1)}{2} \right) \Delta f \right] t \right] & \text{pour } 0 \leq t \leq \tau \\ 0 & \text{pour } \tau \leq t \leq \Delta t \end{cases} \quad (14)$$

P is the number of sub-impulses. Let's put $f_p = f_0 + (p-(P-1)/2) \Delta f$. The length of the impulse is taken equal to 1s, the central frequency to $(100/\tau) = 10^9 \text{Hz}$. The uniform separation interval between two adjacent sub-frequencies carriers are, $\Delta f = P/\tau = P \cdot 10^6 \text{Hz}$. The impulses repetition period is $\Delta T = 1000 \cdot \tau$. The total time of observation, T , is equal to $M \cdot \Delta T$, where M is the numbers of emitted impulses. The transmitted signal is reflexive by D targets (sources), assimilated to points distant of the same distance of the sensors array, independently according to the angles $\theta_1, \theta_2, \dots, \theta_D$. The signal to noise ratio, SNR_k , $k=1, 2, \dots, D$, is defined by:

$$\text{SNR}_k = 10 \log \left[\text{var}(\eta_k) / \sigma_n^2 \right] \quad (15)$$

We consider $\text{SNR}_1 = \text{SNR}_2 = \dots = \text{SNR}_D = \text{SNR}$.

The η_k , $k=1, 2, \dots, D$ are taken identical random variables, with zero mean and variance of α .

The figures 2 and 3, represent the tracing of the function of measure of the orthogonality for the estimation of three sources localized at $\theta_1=16^\circ$, $\theta_2=36^\circ$, $\theta_3=55^\circ$, for an unfavorable signal to noise ratio, $\text{SNR}=-5\text{dB}$, respectively without and with the use of the technical Frequency-Hopped with only two sub-frequency ($P=2$). By comparison between the two figures, one clearly notices, that picks of the orthogonality function become distinctly sharper, therefore a considerable improvement of the precision of estimation. The figures 4 and 5, represent the tracing of the same function for the estimation of the same sources but with a favorable signal to noise ratio, $\text{SNR}=+5\text{db}$, respectively without and with the use of the technical Frequency-Hopped, with only two-sub frequency ($P=2$). Where the same remarks, that those made for the figures 2 and 3, can be easily deduced.

The figure 6 is representing the mean of the committed error, RMSE, in the localization of the three sources above mentioned. It puts in evidence the degree of improvement brought in the estimation of the positions of the sources by the application of the technical Frequency-Hopped with only two sub-frequencies ($P=2$). One notices a considerable reduction of the root mean square error (of 0.448° to 0.154°).

To evaluate the degree of improvement brought according to different values of signal to noise ratio, SNR, one draws the differences of errors committed between the case where one uses the technical Frequency-Hopped ($P=2$) and the contrary case, as shown on the figure 7. One notices that the improvement in the estimation is much more important for the weak SNR (SNR = -5dB). This is extremely important since it is the case, weak SNR, where the quality of estimation deteriorates and requires to be corrected.

To determine the link between the number of sub-frequencies, used, and the degree of improvement brought, in the estimation of the positions of the sources, we calculate the root mean square error (RMSE) for different values of sub-frequencies. The results gotten during this simulation are represented in the figure 8 that draws the evolution of the RMSE, of the three previous sources, according to the number of sub-frequencies, P . We notice that the error decreases quickly, with the increase of the number of the sub-frequencies. Beyond four sub-frequencies, one reaches a saturation where the error becomes quasi-constant (RMSE $\approx 0.1^\circ$).

Therefore all increases of the number of under-frequency beyond this threshold, become useless and to increase the load of calculation.

The survey of the variation of RMSE committed according to the angle of arrival (AOA), allows us to define the field of vision of the algorithm MUSIC. This field of vision is defined by the region of the space where the mistake of estimation is the weakest. All the others parameters, are taken favorable.

The figures 9 and 10 draw the root mean square error committed during the estimation of only one source which its angle of arrival varies from 1° to 90° with a step of 1° . Under the hypothesis that the RMSE must not overtake the threshold of 0.1° , the figure 9, show an overtaking of the threshold of 0.1° for the values of the angle of arrival $\theta \geq 71^\circ$. The figure 10, watch that after use of the technical Frequency-Hopped, with only two sub-frequencies ($P=2$), that the threshold of 0.1° is overtake only, for the values of $\theta \geq 80^\circ$. Of the comparison between the figures 9 and 10, we notice that the field of vision of the algorithm is enlarged to pass of $[0^\circ, 71^\circ]$ (without Frequency-Hopped) to $[0^\circ, 80^\circ]$ (with Frequency-Hopped).

The table1, recapitulate the ability of the algorithm MUSIC to separate two sources according to the angle, $\Delta\theta$, that separates their incidental waves, for low signal to noise ratio, SNR=-5dB. One notes that the algorithm MUSIC makes a misjudgment of the number of sources (to detect only one source, whereas two sources exist) if the angle that separates the two sources is $\Delta\theta \leq 4^\circ$. While applying the technical Frequency-Hopped, with only two sub-frequencies, the algorithm MUSIC makes a misjudgment if the angle that separates the two sources is $\Delta\theta \leq 2^\circ$, therefore to improve the power of separation of the algorithm.

IV. Conclusion

We presented in this work a method for the resolution of the problem of estimation of the angles of arrivals of D targets by an array of L sensor. Our new method of arrangement of the data permitted:

- Picks of the orthogonality function become distinctly sharper so offer a better precision
- A considerable reduction of the root mean square error of estimation of the angles of arrivals it comes down from 0.448° to 0.154° for 02 sub-frequencies only.
- the improvement in the estimation is especially more important for the lower values of signal to noise ratio, SNR, where the performances of the algorithms of estimation deteriorate and require to be corrected.
- More the number of sub-frequencies used increases the precision more is better.
- Beyond a certain number of sub-frequencies $P=4$ the root mean square error has the tendency to stabilize to a stationary value
- An important augmentation of the field of vision of the algorithm. It passes $[0^\circ, 71^\circ]$ to attain $[0^\circ, 80^\circ]$
- The ability separation of the algorithm improves and pass of 4 degrees of separation to 2 degrees between two sources, for low signal to noise ratio.
- However the inconvenience of the proposed method is the increase of the calculation load with the number of sub-frequencies used

V. References

- [1] R. O. SCHMIDT and R.E FRANKS " Multiple sources of signal processing: an experimental system ", *IEEE Transaction on antenna and propagation*, Vol. pp. 281-289, 1986.
- [2] R.O. SCHMIDT, "Multiple emitter location and signal parameter estimation", *IEEE Transaction on antenna and propagation*, Vol. AP-34, pp.276-280, 1986
- [3] G.BIENVENU and L.KOPP. "Adaptativity to Background Noise Spatial Coherence for high resolution passive method", *In proceeding of ICASSP-80*, 1980, pp307-310.
- [4] I.ZISKIND and M.WAX, "Maximum likelihood localization of multiples sources by alternating projection", *IEEE transactions on Acoustics, Speech, and Signal processing*. Vol ASSSP-36, pp.1553-1560, 1988.
- [5] M.WAX, "Detection and estimaton of superimposed signals", Ph.D, dissertation Stanford University, Stanford, CA, 1985.
- [6] J.BOEHME, "Estimating the source parameters by maximum likelihood estimator and nonlinear regression", *in processing of ICASSP-84*, 1984, pp.731-734.
- [7] M.I.MILLER and D.R.FUHRMANN, "Maximum-likelihood narrow-band direction finding and the EM Algorithm", *IEEE transactions on acoustics, speech, and signal processing*., Vol. ASSSP-38, pp,1560-1577, 1990.

- [8] A.J.WEISS, A.S. WILLSKY and B.C.LEVY, "Maximum likelihood array processing for the estimation of superimposed signals" *proceeding of the IEEE*, Vol.76, no 2 February 1988.
- [9] Y.D. HUANG and M.BARKAT, "An Efficient Multiple Source Localisation Approach:Dynamic programming" *Digest of 1990 IEEE international AP-S symposium*, Vol. IV, pp.1450-1453
- [10] H. WANG J.X.ZHU "On performance improvement of tone frequency estimation in active radar/sonar systems with non fluctuating targets", *IEEE transactions on Acoustics, Speech, signal processing*. Vol ASSP-36, pp.1580-1591, 1988.
- [11] Y.D. HUANG and M.BARKAT "On estimating the number of sources with a frequency-hopped signaling sensor array", *IEEE, transactions of antennas and propagation*, Vol.39, pp.1384-1390, Sep.1991.
- [12] C.C.LI and H. WANG "a channel efficient method for high-resolution active direction finding of multiple targets". *Proceeding of the Asilomar conference on signals, systems and computers*, Monterey, CA, pp.114-118,1988.
- [13] M.BARKAT, K.AISSOUS, " Application de la technique Frequency Hopped à la localisation des sources par réseau de capteurs", ts, *Traitement du Signal*, Vol. 15, N° 2, (1998) pp 72-78.
- [14] Y.Y. Wang, W.H. Fang " TST-MUSIC for DOA-Delay Joint Estimation", IEEE 0-7803-6293-4/00/©, pp 2565-2568, 2000.
- [15] K. Mahata" Spectrum Estimation, Notch Filters, and MUSIC", IEEE 1053-587/©, pp 3727-3737, 2005.
- [16] H.L. VAN TREES. "*Detection Estimation and Modulation Theory*" Part1.New-york: Wiley 1968.
- [17] M.I. SKOLNIK, "*An introduction to radar systems*" 2nd ed., New York Mc Graw-Hill. 1980.
- [18] C. C. Teague" Root-MUSIC Direction Finding Applied to Multifrequency Coastal Radar", IEEE 0-7803-7536-x/©, pp 1896-1898, 2002.
- [19] J.C. .Mosher, R.M. Leahy" Source Localisation Using Recursively Applied and Projected (RAP)MUSIC", IEEE 1058-6393/©, pp 1302-1306, 1998.
- [20] F. Taga, H. Shitahira " Fast Kernel MUSIC Algorithm " 0-7803-3336/96-x/(c) pp 2028-2033, 1996 IEEE.

$\Delta\theta(\text{degré})$	1	2	3	4	5	6	7	8	9	10
N_1	1	1	1	1	<u>2</u>	2	2	2	2	2
N_2	1	1	<u>2</u>	2	2	2	2	2	2	2

Table 1: Evolution of the number of sources estimated by MUSIC according to the step $\Delta\theta$, separating two incidental angles with and without Frequency-Hopped ($P=2$), $L=10$, $\text{SNR} = -5\text{dB}$, $M=100$, $\text{RUN}=150$. (N_1 = without Frequency-Hopped, N_2 = with Frequency-Hopped)

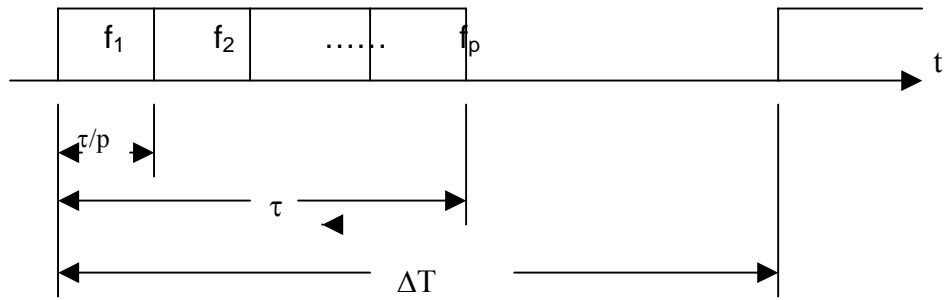


Figure1. Impulse coded in Frequency-Hopped

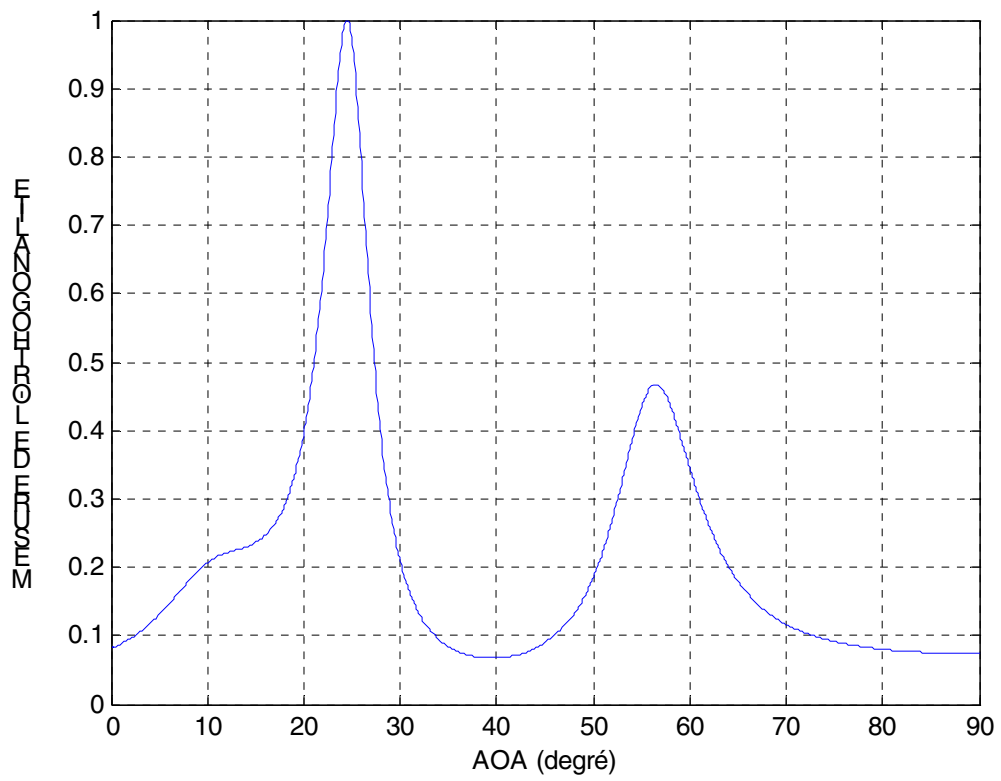


Figure 2. The function of measure of orthogonality of estimation of 03 sources, without Frequency-Hopped. SNR=-5dB, M=100, L=10, Run=150

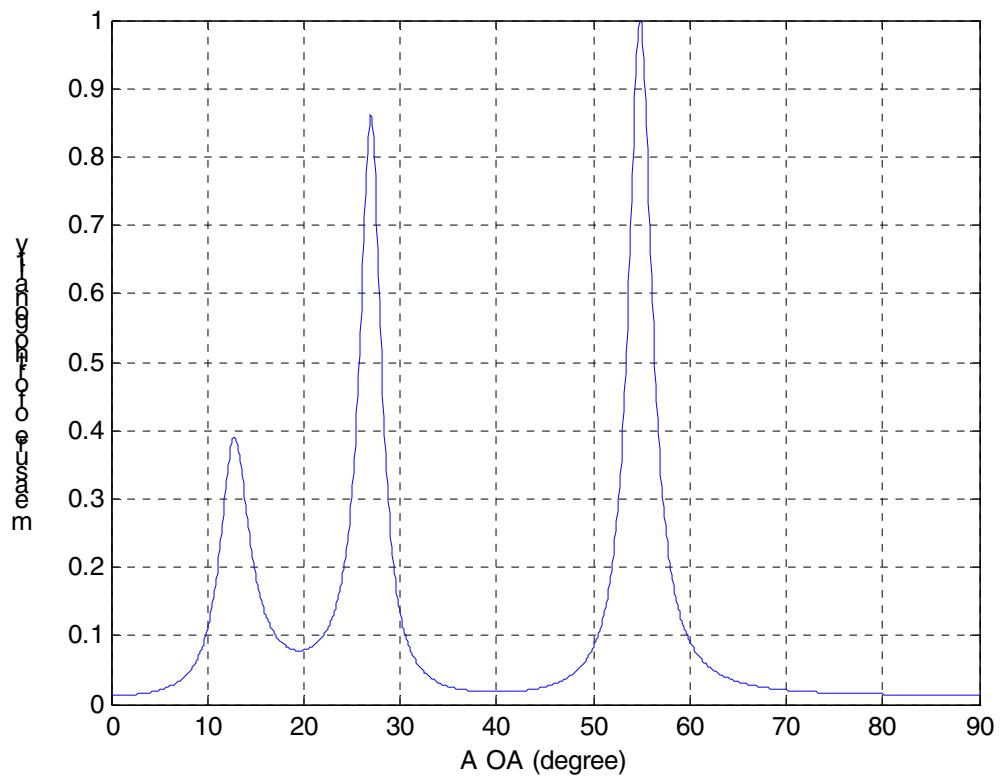


Figure 3. The function of measure of orthogonality of estimation of 03 sources, with Frequency-Hopped ($P=2$). SNR=-5dB, $M=100$, $L=10$, Run=150

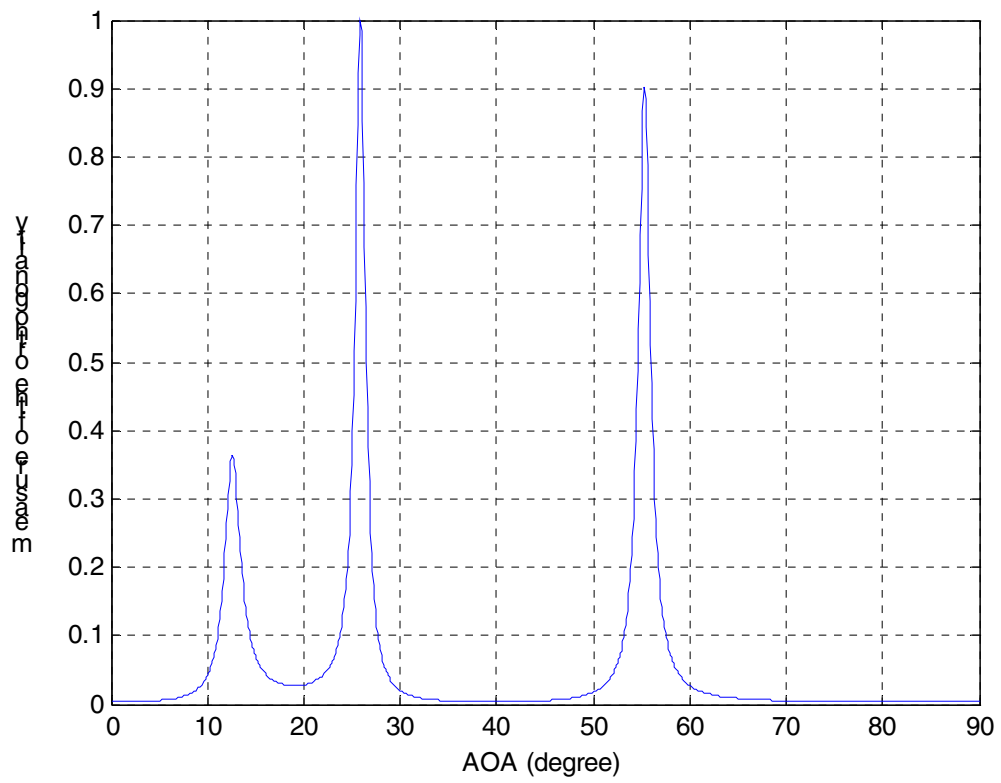


Figure 4. The function of measure of orthogonality of estimation of 03 sources, without Frequency-Hopped. SNR=+5dB, M=100, L=10, Run=150

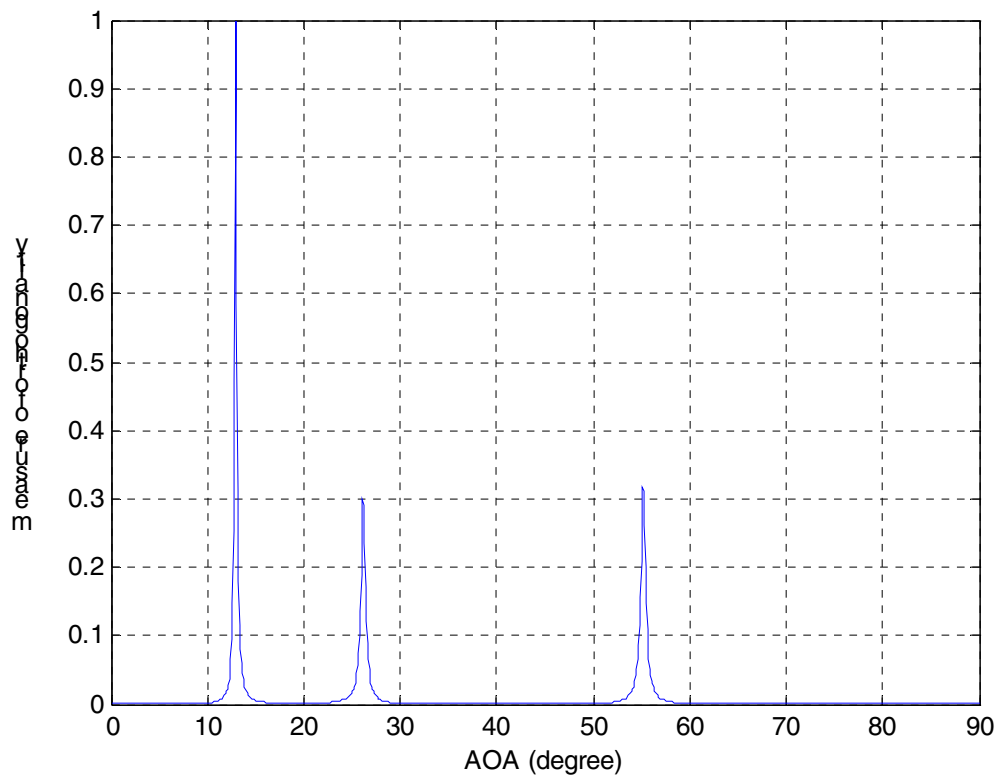


Figure 5. The function of measure of orthogonality of estimation of 03 sources, with Frequency-Hopped (P=2). SNR=+5dB, M=100, L=10, Run=150

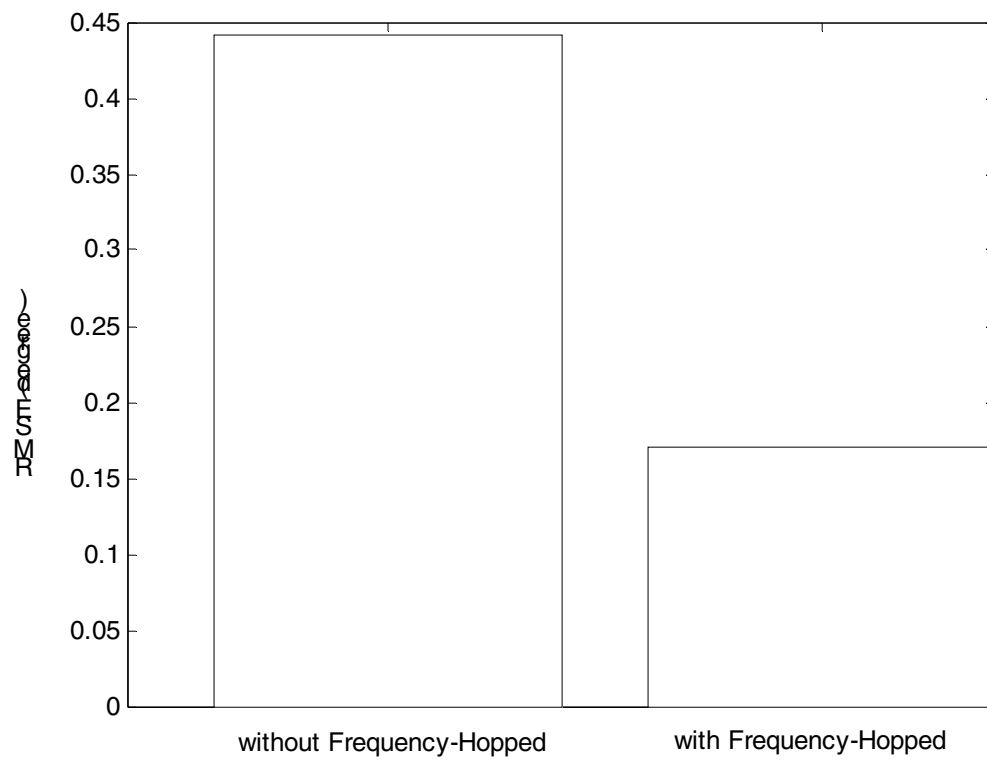


Figure 6. The root mean square error of estimation of 03 independent sources with and without application of Frequency-Hopped (P=2). For L=10, SNR=+5dB and Run=150

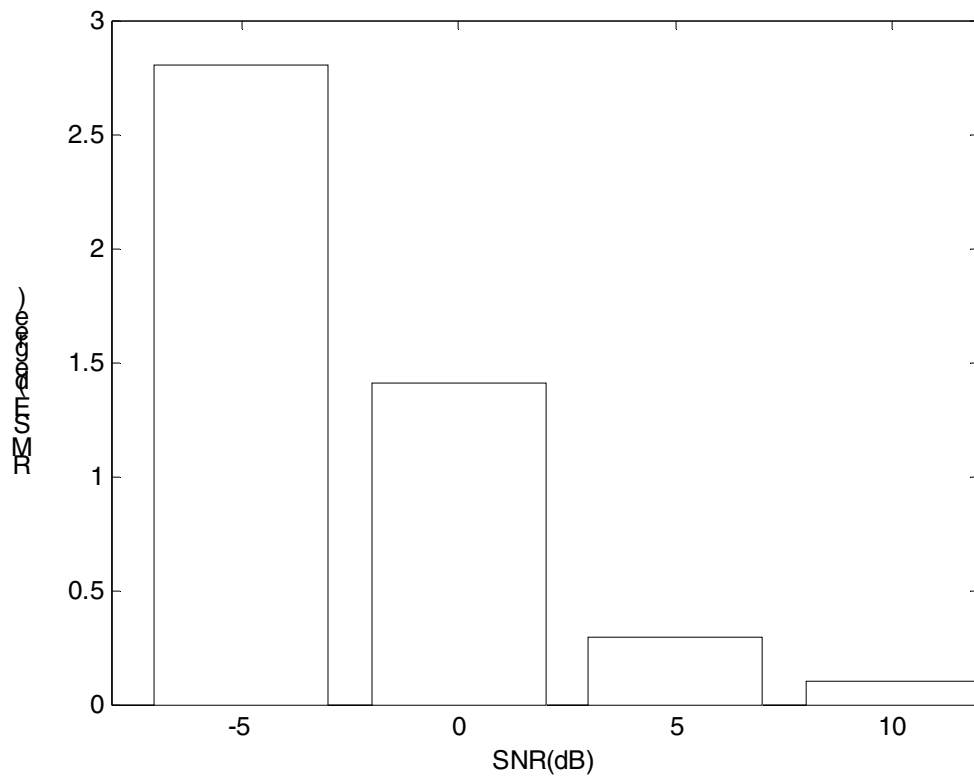


Figure 7. differences of errors committed between the case where one uses the technical Frequency-Hopped and the contrary case according to the signal to noise ratio, SNR. $P=2$, $L=10$, $M=100$, $\text{Run}=150$.

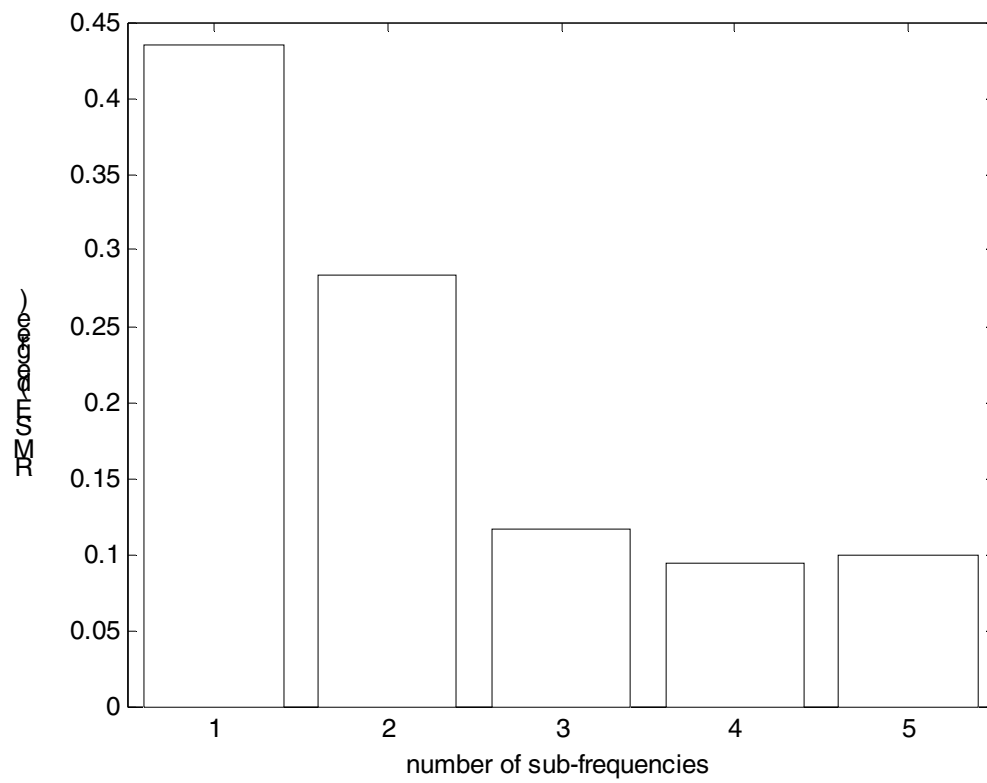


Figure 8. Evolution of the root mean square error of estimation of 03 sources according to the numbers of sub-frequencies, P, with $L=10$, $SNR=5dB$, $M=100$, $Run=150$.

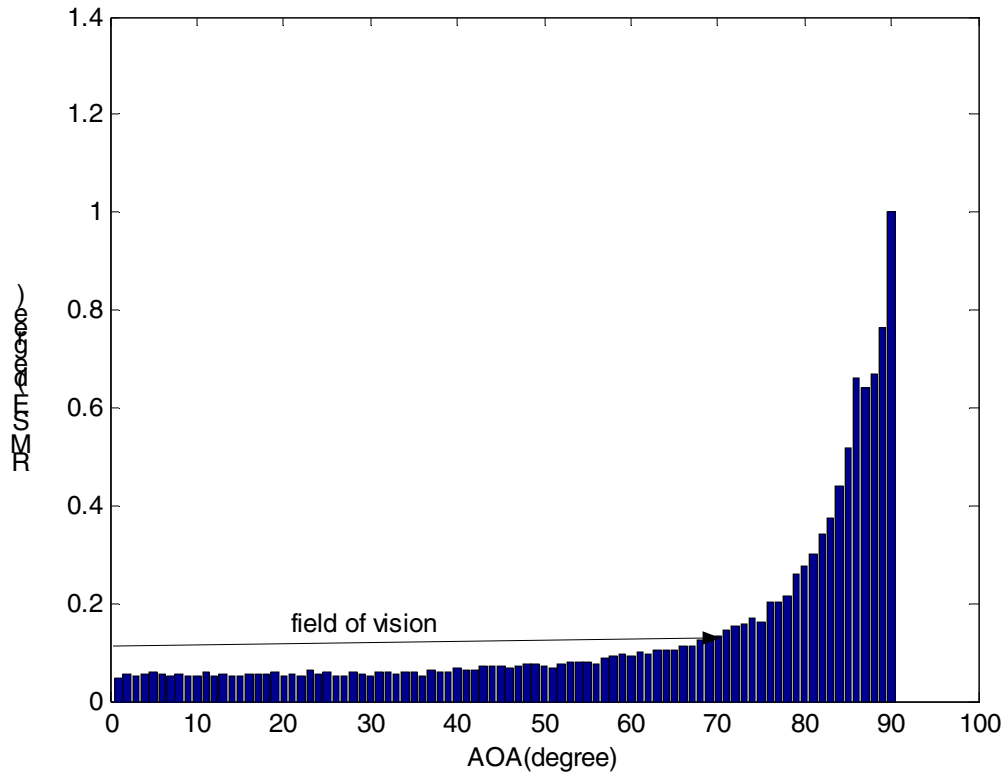


Figure 9. Evolution of the root mean square error of estimation of only one source according to its angle of impact, without frequency-hopped. , $L=10$, $\text{SNR}=5$ dB, $M=100$, $\text{Run}=150$.

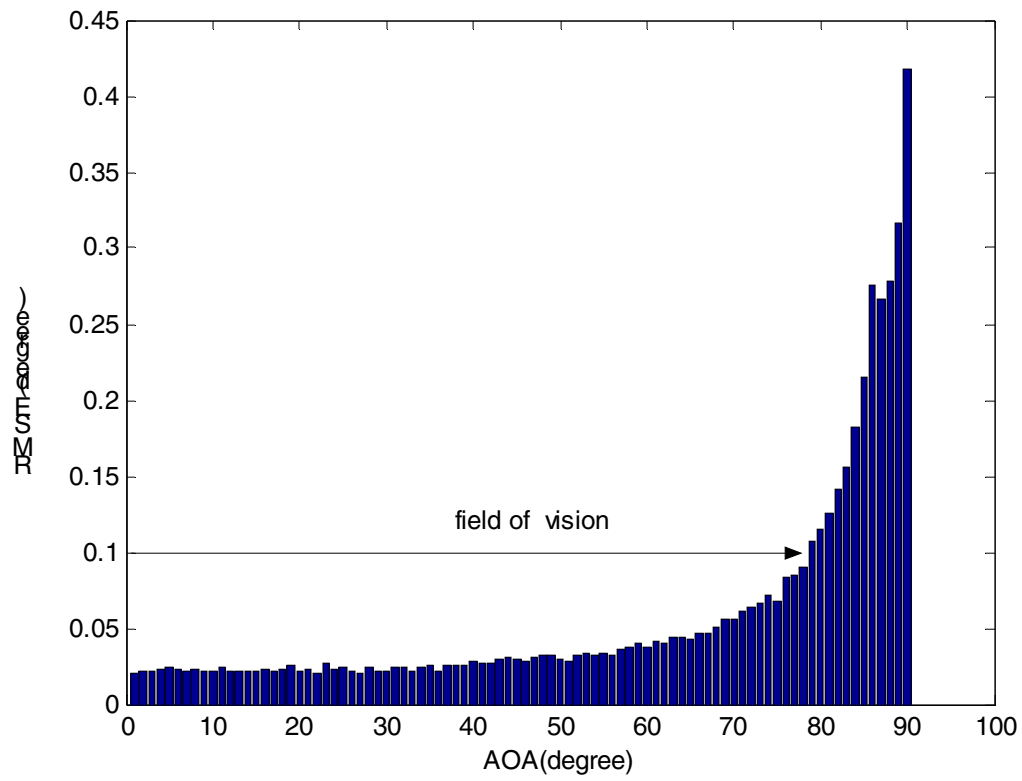


Figure 10. Evolution of the root mean square error of estimation of only one source according to its angle of impact, with frequency-hopped ($P=2$). , $L=10$, $\text{SNR}=5$ dB, $M=100$, $\text{Run}=150$.