CHAOS SUPPRESSION IN FRACTIONAL ORDER PERMANENT MAGNET SYNCHRONOUS MOTOR BY ROBUST ADAPTIVE CONTROL

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Abstract: *In this paper we investigate the control of three-dimensional non-autonomous fractional-order uncertain model of a permanent magnet synchronous motor (PMSM) via a one input control technique. We derive a dimensionless fractional order model of the PMSM from the integer order presented in the literatures. Various dynamic properties of the fractional order model like eigen values, Lyapunov exponents, bifurcation and bicoherence are investigated. The system chaotic behavior for various orders of fractional calculus are presented. A robust adaptive one input controller is derived to suppress the chaotic oscillations of the fractional order model. As the direct Lyapunov stability analysis of the robust controller is difficult for a fractional order first derivative, we have derived a new lemma to analyze the stability of the system. Numerical simulations of the proposed chaos suppression methodology are given to prove the analytical results derived through which we show that for the derived robust adaptive controller and the parameter update law, the origin of the system for any bounded initial conditions is asymptotically stable.*

Keywords: *Permanent Magnet Synchronous Motor; Fractional order systems; Chaos suppression; One input control; Adaptive control;*

I. Introduction

Permanent magnet synchronous motor (PMSM) is increasingly used in efficient AC servo driving control system due to its simple dynamics, high efficiency, high power density and high torque-current ratio. The investigation of chaos in PMSM is a field of active research due to its direct applications in many areas especially for industrial applications in low-medium power range. However, the performance of the PMSM is sensitive to system parameter and external load disturbance in the plant. Some investigations, for example, by Li et al. [1] and Jing et al. [2] show that with certain parameter values, the PMSM displays chaotic behavior. This chaotic behavior of PMSM can lead to performance degradation by causing torque ripples, low frequency oscillations and low performance to speed control. Ataei et al. [3] characterized the complex dynamics of the permanent-magnet synchronous motor (PMSM) model with a non-smooth-air-gap. Harb and Zaher [3] studied chaotic behaviors in Permanent Magnet Synchronous Motor (PMSM) for a certain range of its parameters, and it was eliminated by using optimal Lyapunov exponent methodology. Zribi et al [3] proposed to use a Lyapunov exponent control algorithm to control the Permanent Magnet Synchronous Motor (PMSM). Dynamical equations of three time scale brushless DC motor system were presented by Ge and Cheng [3].

In the recent years, the research on fractional order dynamical systems has been receiving increasing attention. It is found that with the help of fractional derivatives, many systems in interdisciplinary fields can be elegantly described. [7−9] Furthermore many integer order chaotic systems of fractional order have been studied widely.[10−14]. All the physical phenomena in nature exist in the form of fractional order, [15] integer order (classical) differential equation is just a special case of fractional differential equation. The importance of fractional-order models is that they can yield a more accurate description and give a deeper insight into the physical processes underlying a long range memory behavior.

Chaos modelling have applications in many areas in science and engineering [15-17]. Some of the common applications of chaotic systems in science and engineering are chemical reactors, Brusselators, Dynamos, Tokamak systems, biology models, neurology, ecology models, memristive devices, etc. An analysis of saddle-node and Hopf bifurcations in indirect field-oriented control (IFOC) drives due to errors in the estimate of the rotor time constant provides a guideline for setting the gains of PI speed controller in order to avoid Hopf bifurcation [18]. An appropriate setting of the PI speed loop controller permits to keep the bifurcations far enough from the operating conditions in the parameter space [8]. It has been proven the occurrence of either codimension one bifurcation such as saddle node bifurcationand Hopf bifurcation and codimension two such as Bogdanov-Takens or zero-Hopf bifurcation in IFOC induction motors [19-21].

II. Problem formulation and Preliminaries:

The Non-Linear dynamical dimensionless model of the Permanent Magnet Synchronous Motor (PMSM) is given in [2, 3].

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) + x_2(t)x_3(t) \\ \dot{x}_2(t) &= -x_2(t) - x_1(t)x_3(t) + \gamma x_3(t) \\ \dot{x}_3(t) &= \sigma \cdot \left[x_2(t) - x_3(t) \right] \end{aligned} \qquad (1)$$

The system shown in (1) shows chaotic behavior when the parameters are $\gamma = 20; \sigma = 5.46$.

The Fractional order model of the PMSM dimension less model shown in (1) can be defined as

$$\begin{aligned} {}_a^cD_t^{q1} x_1(t) &= -x_1(t) + x_2(t)x_3(t) \\ {}_a^cD_t^{q2} x_2(t) &= -x_2(t) - x_1(t)x_3(t) + \gamma x_3(t) \\ {}_a^cD_t^{q3} x_3(t) &= \sigma \cdot \left[x_2(t) - x_3(t) \right] \end{aligned} \qquad (2)$$

Where q_1, q_2 *and* q_3 are the fractional orders of the respective states. For studying the state portraits of the fractional order system (2), the system parameters are chosen as $\gamma = 20$ & $\sigma = 5.46$.

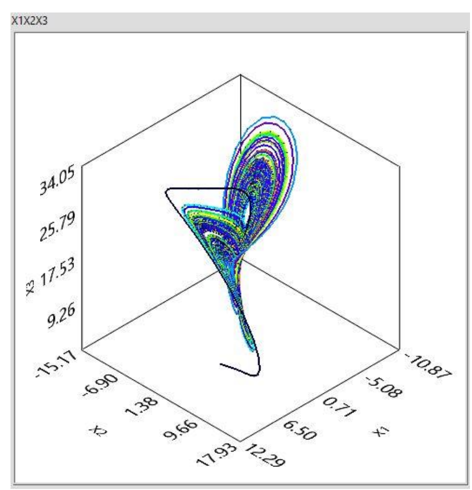

Fig 1: 3D State portrait of the Fractional order System

III. Dynamical analysis of the Fractional order system:

In this section we analyze the fractional order system for various properties of chaotic behavior like equilibria points, Lyapunov exponents, bifurcation and bicoherence.

a. Equilibria Points and Lyapunov Exponents:

The equilibria of the system (2) can be found by solving (3).

$$\begin{aligned} 0 &= -x_1(t) + x_2(t)x_3(t) \\ 0 &= -x_2(t) - x_1(t)x_3(t) + \gamma x_3(t) \\ 0 &= \sigma \cdot \left[x_2(t) - x_3(t)\right] \end{aligned} \quad (3)$$

The three equilibria points of the system (2) are $E_1 = (0,0,0)$ and $E_{2,3} = (\gamma - 1, \pm\sqrt{\gamma - 1}, \pm\sqrt{\gamma - 1})$.

And the Jacobian matrix of the system (2) is defined as,

$$J = \begin{pmatrix} -1 & x_3(t) & x_2(t) \\ -x_3(t) & -1 & \gamma - x_1(t) \\ 0 & \sigma & -\sigma \end{pmatrix} \quad (4)$$

Where $x_1(t), x_2(t) \& x_3(t)$ denotes the equilibrium points.

The Initial conditions are chosen as $x_1(t)=1, x_2(t)=2$ & $x_3(t)=4$ and the parameter values are chosen as $\gamma=20$ & $\sigma=5.46$. The Lyapunov exponents of the system (2) are $L1 = 0.452023$ $L2 = -0.009746$ $L3 = -7.902219$ .The Numerical results of the simulation are shown in Figure 2.

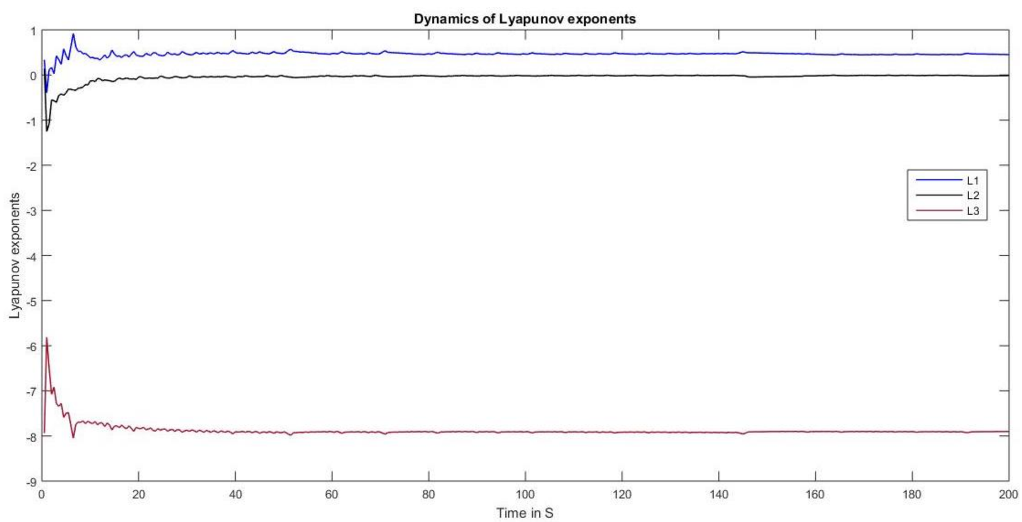

Fig 2: Lyapunov Exponents of the Fractional order System (2)

b. Bifurcation and Bicoherence:

By fixing $\sigma = 5.46$, γ is varied and the behavior of the fractional order system (2) is observed in figure 3. By fixing $\gamma = 20$, σ is varied and the system (2) performance is observed in figure 4.Generally speaking, when the system's biggest Lyapunov exponents is large than zero, and the points in the corresponding bifurcation diagram are dense, the chaotic attractor will be found to exit in this system. Therefore, From the Lyapunov exponents and bifurcation diagrams in figure 3 and 4 a conclusion can be obtained that chaos exit in the fractional order PMSM system (2) when selecting a certain range of parameters. Next the individual state responses are studied in detail by varying the parameters. Figure 5 shows the behavior of states $x_1(t), x_2(t) \& x_3(t)$ with reference to γ when $\sigma = 5.46$. Figure 6 shows the behavior of states $x_1(t), x_2(t) \& x_3(t)$ with reference to σ when $\gamma = 20$.

The bifurcation plots of the system (2) with the change in the order of the system and the parameters fixed at $\gamma = 20$ & $\sigma = 5.46$. Figure 7 shows the order q_1, q_2 and q_3 varied and the attractor bifurcation responses are investigated. As seen from the bifurcation plots, the system chaotic dynamics changes drastically with the fractional order. By comparing the eigen values and the Lyapunov exponents with the fractional order bifurcation graphs, it can be commented that as the order of the fractional equation lies between $0.6 \le q \le 0.9$, the systems chaotic behavior is showing larger Lyapunov exponents. Hence the chaos suppression with fractional order controls are efficient than the integer order control algorithms.

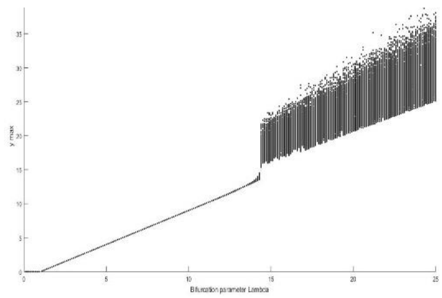

Figure 3: Bifurcation plot versus γ

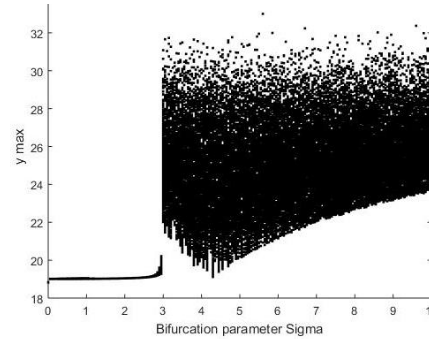

Figure 4: Bifurcation plot versus σ

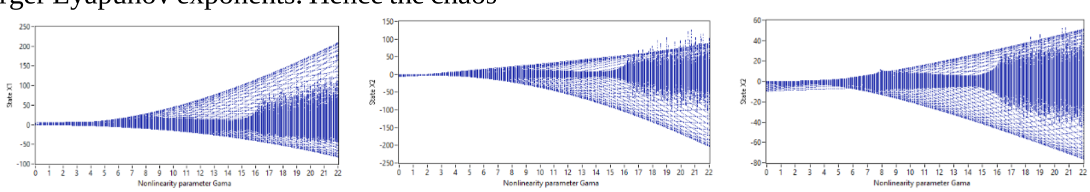

Figure 5: Bifurcation plot state $x_1(t), x_2(t) \& x_3(t)$ versus γ

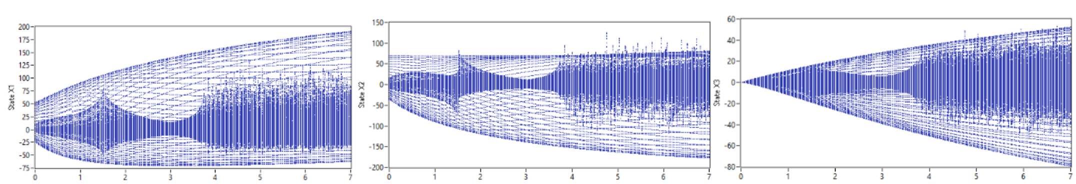

Figure 6: Bifurcation plot state $x_1(t), x_2(t) \& x_3(t)$ versus σ

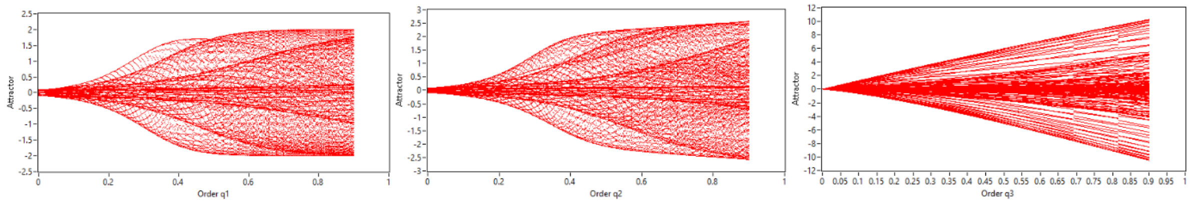

Figure 7: Bifurcation plot versus q_1, q_2, q_3

The bicoherence or the normalized bispectrum is a measure of the amount of phase coupling that occurs in a signal or between two signals. Both bicoherence and bispectrum are used to find the influence of a nonlinear system on the joint probability distribution of the system input. Phase coupling is the estimate of the proportion of energy in every possible pair of frequency components $f_1, f_2, f_3, \ldots, f_n$. Bicoherence analysis is able to detect coherent signals in extremely noisy data, provided that the coherency remains constant for sufficiently long times, since the noise contribution falls off rapidly with increasing N.

The auto bispectrum of a chaotic system is given by Pezeshki [21]. He derived the auto bispectrum with the Fourier coefficients.

$$B(\omega_1, \omega_2) = E[A(\omega_1)A(\omega_2)A^*(\omega_1 + \omega_2)] \tag{5}$$

where ω_n is the radian frequency and A is the Fourier coefficients of the time series. The normalized magnitude spectrum of the bispectrum known as the squared bicoherence is given by

$$b(\omega_1, \omega_2) = |B(\omega_1, \omega_2)|^2 / P(\omega_1)P(\omega_2)P(\omega_1 + \omega_2) \tag{6}$$

where $P(\omega_1)$ and $P(\omega_2)$ are the power spectrums at f_1 and f_2.

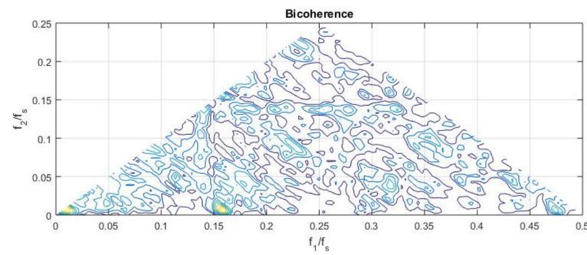

Fig 7: Bicoherence of the state $x_1(t)$

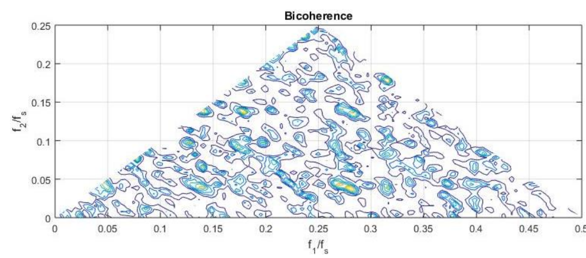

Fig 8: Bicoherence of the state $x_2(t)$

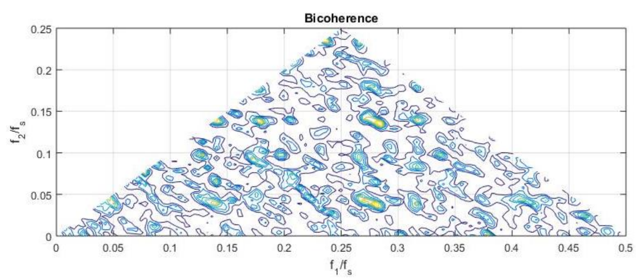

Fig 9: Bicoherence of the state $x_3(t)$

IV. Chaos suppression of the Fractional order system using Robust adaptive Controller:

The control goal of this paper is to design a suitable robust controller for suppression of chaotic oscillations in the fractional order Permanent Magnet Synchronous Motor (2)around zero.

For deriving the robust adaptive controller for the system (2) let us assume $x_2(t)=0$ and the system (2) modifies to a two dimensional system (7).

$$\begin{aligned} {}_a^cD_t^{q1}x_1(t) &= -x_1(t) \\ {}_a^cD_t^{q3}x_3(t) &= -\sigma x_3(t) \end{aligned} \qquad (7)$$

For the fractional-order system (2), if this system is controlled by the single active controller (8), then the system trajectories will converge to zero which will be asymptotically stable about the origin $x_1(t)=0$ & $x_2(t)=0$.

$$u(t) = \lambda\gamma sign(y) + \rho + f_2(X,y,t) \qquad (8)$$

where $\lambda = h(y)$, λ, h are positive constants.

The fractional order system (2) with the robust controller $u(t)$ is given by (9). Here we assume that the parameters of the system are uncertain and hence introduce a parameter estimates $\hat{\gamma}(t)$ & $\hat{\sigma}(t)$.

$$\begin{aligned} {}_a^cD_t^{q1}x_1(t) &= -x_1(t) + y(t)x_2(t) \\ {}_a^cD_t^{q2}y(t) &= -y(t) - x_1(t)x_2(t) \\ &\quad + \hat{\gamma}(t)x_2(t) + \partial f(X,y,t) - u(t) \\ {}_a^cD_t^{q3}x_2(t) &= \hat{\sigma}(t)\cdot\left[y(t) - x_2(t)\right] \end{aligned} \qquad (9)$$

where $\partial f(X,y,t)$ is a non-linear control part to be introduced for the robust control and is assumed to be always $\partial f(X,y,t) < \rho$ and where ρ is a known positive constant.

V. Stability analysis of the controller

In order to analyze the stability of the designed control algorithm we use Lyapunov stability theory.

The Lyapunov function for the controller (8) and system (9) can be given by (10)

$$V = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_\gamma^2 + e_\sigma^2\right) \qquad (10)$$

where e_γ & e_σ are the parameter estimation errors given by (11).

$$\begin{aligned} e_\gamma &= \gamma - \hat{\gamma} \text{ \& } \dot{e}_\gamma = -{}_a^cD_t^{q2}\hat{\gamma} \\ e_\sigma &= \sigma - \hat{\sigma} \text{ \& } \dot{e}_\sigma = -{}_a^cD_t^{q2}\hat{\sigma} \end{aligned} \qquad (11)$$

Differentiating (10) along the trajectories of (9) we will get the Lyapunov first derivative (12).

$$\begin{aligned} \frac{dV}{dt} &= \frac{1}{2}[x_1(t)\dot{x}_1(t) + x_2(t)\dot{x}_2(t) \\ &\quad + x_3(t)\dot{x}_3(t) + e_\gamma\dot{e}_\gamma + e_\sigma\dot{e}_\sigma] \end{aligned} \qquad (12)$$

By definition of fractional calculus [22, 23],

$$\dot{x}(t) = D_t^{1-q}\cdot D_t^q x(t) \qquad (13)$$

By solving (12) with respect to (13) and (2), we get (14).

$$\begin{aligned} \frac{dV}{dt} = {} & {}_a^cD_t^{1-q1}\,{}_a^cD_t^{q1}[-x_1(t) + y(t)x_2(t)] + \\ & {}_a^cD_t^{1-q2}\,{}_a^cD_t^{q2}[-y(t) \\ & - x_1(t)x_2(t) + \gamma x_2(t)] + \\ & {}_a^cD_t^{1-q3}\,{}_a^cD_t^{q3}[\sigma\cdot(y(t) \\ & - x_2(t))] + e_\gamma[-D_t^{q_1}\hat{\gamma}] + e_\sigma[-D_t^{q_1}\hat{\sigma}] \end{aligned} \qquad (14)$$

From (14) it is clear that the calculation of the sign of the first Lyapunov derivative is very difficult. Hence we derive a new lemma to find the sign of the Lyapunov first derivative.

a. Lemma-1:

As defined by if $e(t)$ be a time continuous and derivable function. Then for any time instant $t \geq t_0$,

$$\mathbf{\frac{1}{2}D_t^\alpha e^2(t) \leq e(t)\times D_t^\alpha e(t) \quad \forall\alpha\in(0,1)} \qquad (15)$$

Proof: To prove expression (15) is true we start with,

$$e(t)D_t^{\alpha}e(t)-\frac{1}{2}D_t^{\alpha}e^2(t)\geq 0\ \forall\alpha\in(0,1) \tag{16}$$

By Definition

$$D_t^{\alpha}e(t)=\frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{\dot{e}(\tau)}{(t-\tau)^{\alpha}}d\tau \tag{17}$$

$$\frac{1}{2}D_t^{\alpha}e^2(t)=\frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{e(\tau)\cdot\dot{e}(\tau)}{(t-\tau)^{\alpha}}d\tau \tag{18}$$

Modifying (30),

$$\frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{e(t)\cdot\dot{e}(\tau)-e(\tau)\dot{e}(\tau)}{(t-\tau)^{\alpha}}d\tau\geq 0 \tag{19}$$

Let us assume,

$$E(\tau)=e(t)-e(\tau)\ \&\ \dot{E}(\tau)=-\dot{e}(\tau) \tag{20}$$

Substitute (20) in (19)

$$\frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{E(\tau)\dot{E}(\tau)}{(t-\tau)^{\alpha}}d\tau\geq 0 \tag{21}$$

Integration (21) by parts

$$\frac{1}{\Gamma(1-\alpha)}(t-\tau)^{-\alpha}\cdot\frac{1}{2}E^2(\tau)-\int_{t_0}^{t}\frac{1}{2}E^2(\tau)\cdot\left(\frac{\alpha(t-\tau)^{-\alpha-1}}{\Gamma(1-\alpha)}\right)\leq 0 \tag{22}$$

$$\left[\frac{E^2(\tau)}{2\Gamma(1-\alpha)(t-\tau)^{\alpha}}\right]_{\tau=t}-\left[\frac{E^2(t_0)}{2\Gamma(1-\alpha)(t-t_0)^{\alpha}}\right]-\frac{1}{2}\frac{\alpha}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{E^2(\tau)}{(t-\tau)^{\alpha+1}}d\tau\leq 0 \tag{23}$$

Solving first term of (23) for $\tau=t$

$$\lim_{\tau\to t}\frac{E^2(\tau)}{2\Gamma(1-\alpha)(t-\tau)^{\alpha}}=\frac{1}{2\Gamma(1-\alpha)}\lim_{\tau\to t}\frac{\begin{bmatrix}e^2(t)+e^2(\tau)\\-2e(t)\cdot e(\tau)\end{bmatrix}^2}{\left(t-\tau\right)^{\alpha}} \tag{24}$$

$$=\frac{1}{2\Gamma(1-\alpha)}\lim_{\tau\to t}\left[\frac{-2e(t)\dot{e}(\tau)+2e(\tau)\cdot\dot{e}(\tau)}{-\alpha\left(t-\tau\right)^{\alpha-1}}\right]=0$$

Equation (24) can be rewritten as

$$\frac{E^2(t_0)}{2\Gamma(1-\alpha)(t-t_0)^{\alpha}}+\frac{\alpha}{2\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{E^2(\tau)}{(t-\tau)^{\alpha+1}}d\tau\geq 0 \tag{25}$$

which clearly holds as α lies between $0\leq\tau\leq 1$, the r.h.s of the equation (25) will always be a positive value and hence Proved.

b. Lyapunov First Derivative using Lemma-1

Applying Lemma-1(15) in equation (10) we get,

$$V(x_1, x_2, y) = \frac{1}{2}x_1^2(t) + \frac{1}{2}x_2^2(t) + \frac{1}{2}y^2(t) \leq x_1(t)\left[-x_1(t) + y(t)x_2(t)\right] \\ +y(t)\left[-y(t) - x_1(t)x_2(t) + \gamma x_2(t) + \Delta f(X, y, t) - u(t)\right] \\ +x_2(t)\left[\sigma(y(t) - x_2(t))\right] \tag{26}$$

$$V(x_1, x_2, y, e_\gamma, e_\sigma) \leq -x_1^2(t) + x_1(t)y(t)x_2(t) - y^2(t) - x_1(t)y(t)x_2(t) \\ +\gamma x_2(t)y(t) + \Delta f(X, y, t) \cdot y(t) - u(t)y(t) + \sigma x_2(t)y(t) - \sigma x_2^2(t) \tag{27}$$

$$V(x_1, x_2, y, e_\gamma, e_\sigma) \leq -x_1^2(t) - y^2(t) - \sigma x_2^2(t) + \gamma x_2(t)y(t) \\ + \Delta f(X, y, t) \cdot y(t) - u(t)y(t) + \sigma x_2(t)y(t) \tag{28}$$

$$V(x_1, x_2, y, e_\gamma, e_\sigma) \leq -x_1^2(t) - y^2(t) - \sigma x_2^2(t) \\ +\gamma x_2(t)y(t) + \partial f(X, y, t) \cdot y(t) - \lambda\gamma sign(y).y(t) \\ -\rho y(t) - f_2(X, y, t) \cdot y(t) + \sigma x_2(t)y(t) \tag{29}$$

$$V(x_1, x_2, y, e_\gamma, e_\sigma) \leq -x_1^2(t) - y^2(t) - \sigma x_2^2(t) \\ +x_2(t)y(t)\left[\gamma + \sigma\right] + y(t)[\partial f(X, y, t) \\ - f_2(X, y, t) - \lambda\gamma sign(y) - \rho] \tag{30}$$

As per assumption

$$\partial f(X, y, t) < \rho \text{ and } f_2(X, y, t) = x_2(t)\left[\gamma + \sigma\right] \tag{31}$$

$$V(x_1, x_2, y, e_\gamma, e_\sigma) \leq -x_1^2(t) - y^2(t) - \sigma x_2^2(t) \\ +x_2(t)y(t)\left[\gamma + \sigma\right] + y(t)[\partial f(X, y, t) - x_2(t)\left[\gamma + \sigma\right] \\ -\lambda\gamma sign(y) - \rho] \tag{32}$$

$$V(x_1, x_2, y, e_\gamma, e_\sigma) \leq -x_1^2(t) - y^2(t) - \sigma x_2^2(t) \\ +y(t) \cdot [\partial f(X, y, t) - \rho] - y(t)\lambda\gamma sign(y) \tag{33}$$

As per the assumption made during the selection of the robust controller (9), the non-linearity of the robust controller is always $\partial f(X, y, t) < \rho$. Hence (33) is a negative definite function which infers that the system is stable and is valid for any bounded initial conditions.

VI. Numerical Simulations using LabVIEW

The Fractional order PMSM system (2) with the robust adaptive controller (8) is implemented in LabVIEW for numerical analysis and validation. The initial values of the fractional order system (2) are taken as $x_1(t)=1, x_2(t)=2$ & $x_3(t)=4$.The adaptive control parameters are selected as $\lambda=4$ and $h=4$. The state trajectories of the controlled fractional-order chaotic system (2) are shown Figure 10, where the controller is switched at t =120s. It can be clearly observed that the state trajectories converges to zero as soon as the controller is introduced which clearly shows that the fractional order system (2) is controlled by the robust controller.Fig. 10 also shows the evolution of the states of the system (2) with controller (8), with the fractional orders $q_1=0.8, q_2=0.9$ & $q_3=0.8$. As proved from the analytical analysis already presented, the origin of the system for any bounded initial conditions is asymptotically stable.

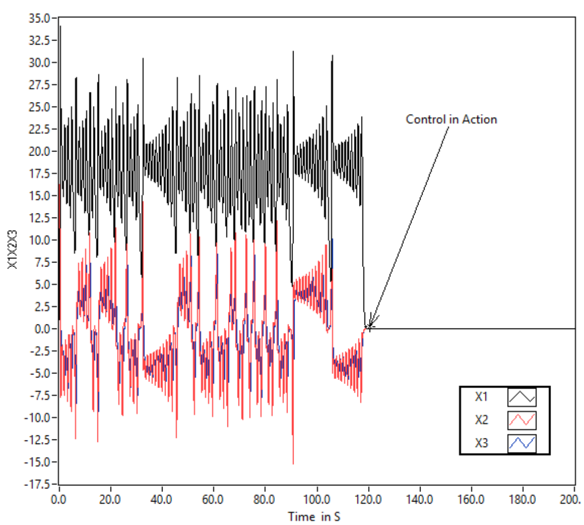

Fig.10: State trajectories with control in action at $t=120s$

VII. Conclusion and Discussions:

This paper investigates control of three-dimensional non-autonomous fractional-order uncertain model of a permanent magnet synchronous motor (PMSM) via a one input control technique. Firstly the dimensionless fractional order model of the PMSM is derived from the integer order model discussed I the literature using the Caputo fractional calculus. In order to study the effects of variation of parameters on the fractional order system's performance, we have investigated the bifurcation analysis of fractional order system. It is also shown that the fractional order PMSM are not only prone to instability due to Hopfbifurcation, it also exhibits limit cycles and chaos due to Bifurcation other than Hopf bifurcation which is shown by the bicoherence plots. This bispectrum analysis helps us in choosing the appropriate parameters for the proper working of the motor. AS understood from the dynamic analysis of the fractional order system, it is seen that chaos oscillations are exhibited for a particular selection of parameters. To suppress such chaotic oscillations, we have derived a robust one input control technique assuming that the operating parameters of the fractional order system (2) are unknown. The direct Lyapunov stability analysis of the robust controller is difficult and hence we have derived a new lemma to analyze the stability of the system. The proposed lemma is introduced in the Lyapunov first derivative and thus the parameter estimates are derived. We have also proved with numerical simulations that for the derived robust adaptive controller and the parameter update,the origin of the system for any bounded initial conditions is asymptotically stable.

References:

1. Li Z, Park J B, Joo Y H, Zhang B and Chen G "*Bifurcations and chaos in a permanent-magnet synchronous motor*“ IEEE Trans. Circuits and Systems I Theory and Applications, 49 383-387. 2002

2. Z. J. Jing, C. Yu and G. R. Chen, "*Complex dynamics in a permanent-magnet synchronous motor model*", Chaos, Solitons and Fractals, vol.22, no.4, pp.831-848, 2004.

3. M. Ataei, A. Kiyoumarsi and B. Ghorbani, "*Control of chaos in permanent magnet synchronous motor by using optimal Lyapunov exponents placement*", Physics Letters A, vol.374, no.41, pp.4226-4230,2010.

4. M. Harb and A. A. Zaher, "*Nonlinear control of permanent magnet stepper motors*", Communications in Nonlinear Science and Numerical Simulations, vol.9, no.4, pp.443-458, 2004.

5. M. Zribi, A. Oteafy and N. Smaoui, "*Controlling chaos in the permanent magnet synchronous motor*", Chaos, Solitons and Fractals, vol.41, no.3, pp.1266-1276, 2009.

6. Z. M. Ge and J. W. Cheng, "*Chaos synchronization and parameter identification of three time scales brushless DC motor system*", Chaos, Solitons and Fractals, vol.24, no.2, pp.597-616, 2005.

7. R. L. Bagley and R. A. Calico. "*Fractional order state equations for the control of viscoelastically damped structures*", Journal of Guidance, Control, and Dynamics, Vol. 14, No. 2, pp. 304-311.Koeller R C 1986 Acta Mech. 58 251, 1991

8. Sun H H, AbdelwahadAA and Onaval B "*Linear approximation of transfer function with a pole of fractional power*" IEEE transactions on automatic control CODEN IETAA9 1984, vol. 29, no 5, pp. 441-444 (12) .,1984

9. Heaviside O," *Electromagnetic Theory*" (New York: Chelsea), 1971

10. Yu Y, Li H X, Wang S and Yu "*Dynamic analysis of a fractional-order Lorenz chaotic system*" Chaos, Solutions & Fractals Volume 42, Issue 2, Pages 1181–1189, 30 October 2009

11. Li C G and Chen G "*Chaos and hyperchaos in the fractional-order Rössler equations*", Physica A: Statistical Mechanics and its Applications, Volume 341, Pages 55–61, 1 October 2004

12. L¨u J G and Chen G "*A note on the fractional-order Chen system Chaos, Solutions & Fractals*"Volume 27, Issue 3, Pages 685–688, February 2006

13. Deng W and Li C P "*Chaos synchronization of the fractional Lü system*" Physica A: Statistical Mechanics and its Applications Volume 353, Pages 61–72, 1 August 2005

14. Mandelbort B B "*The Fractal Geometry of Nature*" (New York: Freeman) 427, 2006.

15. Bazanella A.S. and R. Reginatto, "*Instability Mechanisms in Indirect Field Oriented Control Drives: Theory and Experimental Results*", IFAC 15th Triennial World Congress, Barcelona, Spain, 2002.

16. Bazanella A.S., R. Reginatto and R.Valiatil, "*Hopf bifurcations in indirect field oriented control of induction motors: Designing a robust PI controller*", Proceedings of the 38th Conference on Decision and Control, Phoenix, Arizona USA., 1999.

17. Reginatto .R, F. Salas, F. Gordillo and J.Aracil, "*Zero-Hopf Bifurcation in Indirect Field Oriented Control of Induction Motors*", First IFAC Conference on Analysis and Control of Chaotic Systems (CHAOS'06), 2006.

18. Salas .F, R. Reginatto, F. Gordillo and J.Aracil, Bogdanov-Takens "*Bifurcation in Indirect Field Oriented Control of Induction Motor Drives*", 43rd IEEE Conference on Decision and Control, Bahamas, 2004.

19. Salas .F, F. Gordillo, J. Aracil and R. Reginatto, "*Codimension-two Bifurcations In Indirect Field Oriented Control of Induction Motor Drives*", International Journal of Bifurcation and Chaos, 18(3), pp. 779-792, 2008.

20. C. Pezeshki, "*Bispectral analysis of systems possessing chaotic motions*", Journal of Sound and Vibration, vol. 137, no. 3, pp. 357-368, 1990.

21. Li R H and Chen W S "*Fractional order systems without equilibria*" Chin. Phys. B 2013

22. Petras, I."*A note on the fractional-order Chua's system*". Chaos Solitons and Fractals 38, 140– 147 ,2008.

23. Katugampola, U.N., "*A New Approach To Generalized Fractional Derivatives*", Bull. Math. Anal. App. Vol 6, Issue 4, pages 1–15, 15 October 2014