Comparison of Voltage Stability and Loss Reduction by Modal Analysis and Sensitivity Methods

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***Abstract*—Voltage instability has been given much attention in the power system in recent years, as it is being regarded as one of the major problems in power system insecurity. To maintain a stable and secure operation of a power system is therefore a very important and is becoming a challenging issue. Based on voltage stability analysis and on loss on reduction, the DG placement problem is solved. DG units may also participate in security as well as power generation based on their locations and size. In this paper Dg placement location is solved using the modal analysis and sensitivity analysis methods. The analysis is performed for IEEE 33 Bus system. The Modal Analysis method and sensitivity methods are used to observe the stability of the power system. Stability margin Prediction or distance to voltage collapse is based on the load demand. For each system the most critical mode is identified modal analysis and sensitivity factors are calculated by using loss sensitivity analysis and voltage sensitivity index. Comparison is made for the voltages for each methods.**

Keywords— Voltage instability; Loss sensitivity index; Voltage stability index; Modal analysis; Distributed Generation; Voltage collapse; Participation factor

I. Introduction

In recent years power systems are having tendency to fail due to load variation, ever increased load demands, etc. Due to these variations, the power systems are becoming diffident. The problem has taken with increased attention. The power systems voltage instability [1] is increasing the losses in power systems, and with increase in losses, the cost of power systems is becoming more and more. Reduction in voltage stability may develop severe problems in the power system, which might lead to system collapse in the form of sudden voltage dips at the load busses. Following any disturbance, regardless of its nature, voltage at a particular bus drops intensively thereby forcing the voltage at nearly load buses to drop gradually to a state of voltage collapse [2][3]. Voltage collapse problem has been one of the major problems facing by the electric power utilities. This problem is also considered as a main concern in power system operation and planning. It can be characterized by a continuous decrease in the system voltage. In the initial stage the decrease of the system voltage starts gradually and thereby decreases rapidly. To improve voltage stability distributed generation is used.

Distributed generation (DG) is going to play a major role in power systems worldwide. The importance of DGs in future smart grids increases considering the fact that DGs will have a role in system security, reliability, efficiency, and quality as well. To clear voltage stability problems, as a cause of the most recent blackouts the DGs capability can be used. Considering that most DGs are located at the distribution level, determination of the best locations for installing DGs to maximize their benefits is very important in system design and expansion. In this paper, a DG placement problem is solved by using voltage stability technique i.e., modal analysis, while the objective is to maximize the VSM and simultaneously minimize the losses. There are many advantages of installing DGs in distribution systems. Some of these advantages are listed below.

- Postponing the upgrade of an existing system.
- Peak shaving.
- Reduction of power losses.
- Low maintenance cost.
- High reliability.
- Power quality improvement in some cases.
- Possibility to exploit CHP generation
- Meeting the increasing demand without requirement of extravagant investment.
- Shorter construction schedules.

A. Problem Statement

Voltage stability is defined as the ability of power system to maintain steadily acceptable bus voltages at each node under normal operating conditions, after load increase following system configuration changes or when the system is being subjected to disturbances. Basically, voltage stability can be classified into large-disturbance voltage stability and small-disturbance voltage stability. The former is concerned with a system's ability to control voltages following large disturbances such as system faults, loss of generation, or circuit contingencies. The latter is concerned with a system's ability to control voltages following small perturbations such as incremental changes in system load. Thus, the progressive and uncontrollable drop in voltage as result of increase in load demand, or change in system operation conditions could result eventually in a wide spread voltage collapse. And the voltage stability is of 2 types i.e.; short term and long term voltage stability.

B. Power flow equations

Power flows in a distribution system are computed by the following set of simplified recursive equations [4][5] derived from the single-line diagram

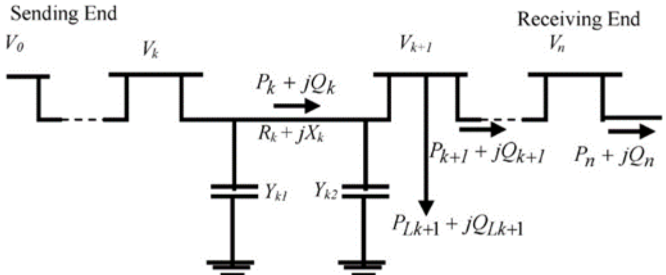


Fig. 1.Single-line diagram of a main feeder.

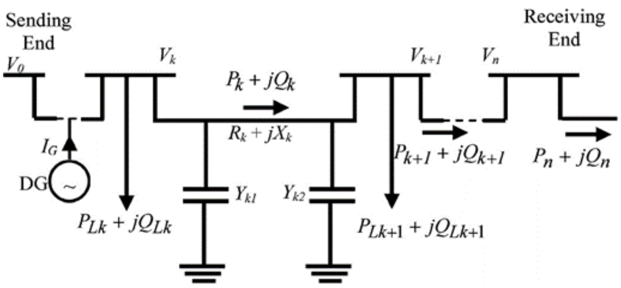
$$\begin{aligned} P_{k+1} &= P_k - P_{Loss,k} - P_{Lk+1} \\ &= P_k - \frac{R_k}{|V_k|^2}\left\{P_k^2 + \left(Q_k + Y_k|V_k|^2\right)^2\right\} - P_{Lk+1} \quad (1) \\ Q_{k+1} &= Q_k - Q_{Loss,k} - Q_{Lk+1} \\ &= Q_k - \frac{X_k}{|V_k|^2}\left\{P_k^2 + \left(Q_k + Y_{k1}|V_k|^2\right)^2\right\} - Y_{k1}|V_k|^2 \\ &\quad - Y_{k2}|V_{k+1}|^2 - Q_{Lk+1} \quad (2) \\ |V_{k+1}|^2 &= |V_k|^2 + \frac{R_k^2 + X_k^2}{|V_k|^2}\left(\mathrm{P}_k^2 + Q_k'^2\right) - 2(R_k P_k + X_k Q_k) \\ &= |V_k|^2 + \frac{R_k^2 + X_k^2}{|V_k|^2}\left(\mathrm{P}_k^2 + \left(Q_k + Y_k|V_k|^2\right)^2\right) \\ &\quad - 2\left(R_k P_k + X_k\left(Q_k + Y_k|V_k|^2\right)\right). \quad (3) \end{aligned}$$

The power loss in the line section connecting buses and may be computed as

$$P_{loss}(k, k+1) = R_k \frac{(P^2+Q^2)}{V_k{}^2} \quad (4)$$

The total power loss of the feeder PTloss may be determined by summing up the losses of all line sections of the feeder, which is given as

$$\mathrm{P}_{T,Loss} = {}_{k-1}^{\ n}\sum P_{Loss}(k, k+1) \quad (5)$$



II. Sensitivity Methods

A. Loss Sensitivity analysis for DG location

Loss Sensitivity analysis [6] is used to compute sensitivity factors of candidate bus locations to install DG units in the system. Estimation of these candidate buses helps in reduction of the search space for the optimization procedure. And a load of connected between k-1 and k buses as given

Active power loss in the kth line between k-1 and k buses is given by

$$P_{(line\ loss)} = ((P_{(Lk,eff)}{}^2 + Q_{(Lk,eff)}{}^2).R_k)/(V_k{}^2) \quad (6)$$

Now, the loss sensitivity factor (LSF) can be obtained with the equation

$$\frac{\partial P_{lineloss}}{\partial P_{Lk,eff}} = \frac{2 * P_{Lk,eff} * R_k}{V_k^2}. \quad (7)$$

Using (15), LSFs are computed from load flows [7] and values are arranged in descending order for all buses of the given system. It is worth to note that LSFs decide the sequence in which buses are to be considered for DG unit installation. The size of the DG is varied in multiples of 500MW from 500MW to 2500MW.

B. Voltage sensitivity index

In this case each DG is penetrated [8] at a time, by a DG of 20% size of the maximum feeder loading capacity. After putting a DG at each node its voltage sensitivity index can be calculated by using the equation.

$$\mathrm{VSI} = \sqrt{\frac{\sum_{k=1}^{n}(1-V_k)^2}{n}} \quad (8)$$

Where Vk is the voltage at the kth node and N is the number of nodes .The node with the least BVSI will be chosen for DG placement. The algorithm for DG location and sizing can be given as:

Step 1: run the load flow for the base case
Step 2: find the bus voltage sensitivity indices at each node using the equation by penetrating the 20% of DG value at respective node and rank the sensitivity of all nodes in ascending order to form priority list.
Step 3: select the bus with lowest priority and place DG at the bus.
Step 4: change the size of DG in small steps and calculate power loss for each by running load flow.
Step 5: store the size of DG that gives minimum loss.
Step 6: compare the loss with the previous solution. If loss is less than previous solution, then store this new solution and Discard previous solution.
Step 7: repeat Step 4 to Step 6 for all buses in the priority list.
Step 8: end

III. MODAL ANALYSIS

Modal analysis can predict voltage collapse [9][10] in power system networks. Since voltage stability is dynamic in nature but static tools are promising for predicting problem characteristics [11][12]. It involves mainly the computing of the smallest Eigen values and associated eigenvectors of the reduced Jacobian matrix obtained from the load flow solution. The Eigen values are associated with a mode of voltage and reactive power variation, which can provide a relative measure to voltage instability. Then, the participation factor can be used effectively to find out the weakest nodes or buses in the system. The weakest bus in the system can be found by using the participation factor values. The participation factor values can be found from the Eigen vectors of Eigen values. The procedure is explained as below.

The linearized steady state system power voltage equations are given by

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}J_{P\theta} & J_{PV}\\ J_{Q\theta} & J_{QV}\end{bmatrix}\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix}$$

Where,

ΔP = Incremental change in bus real power
ΔQ = Incremental change in bus reactive power
Δθ = Incremental change in bus voltage angle
ΔV = Incremental change in bus voltage

Considering, Δ P=0 the reduced Jacobian matrix is obtained as []

$$J_R = J_{QV} - J_{Q\theta}J_{P\theta}^{-1}J_{PV} \quad (9)$$

And

$$\Delta Q = J_R \Delta V \quad (10)$$

Or $\Delta V = J_R^{-1}\Delta Q$ (11)

Let $J_R = \xi \Lambda \eta$ (12)

ξ right eigenvector matrix of J_R
η left eigenvector matrix of J_R
Λ diagonal eigenvalue matrix of J_R

Then, inverting (5) yields

$$J_R^{-1} = \xi\Lambda^{-1}\eta \quad (13)$$

And substituting (6) in (4) results in

$$\Delta V = \xi^{-1}\Lambda\eta\Delta Q \quad (14)$$

$$\Delta V = \sum_i (\xi_i\eta_i / \Lambda_i)\Delta Q \quad (15)$$

Where η_i is the *i*th row of the left eigenvector of J_R , and ξ_i is the *i*th column of the right eigenvector. The *i*th mode of the Q-V response is defined by the *i*th eigenvalueΛ_i, and the corresponding right and left eigenvectors and Since, $\xi^{-1} = \eta_i$, (7) may be written as

$$\eta\Delta V = \Lambda^{-1}\eta\Delta Q \quad (16)$$

By defining $v= \eta\Delta V$ as the vector of modal voltage variation and $q= \eta\Delta Q$ as the vector of modal reactive power variation, one can write uncoupled first-order equations as

$$v = \Lambda^{-1}q \quad (17)$$

Thus for the *i*th mode, we have

$$v_i = (1/\Lambda_i)q_i \quad (18)$$

If Λ_i>0, the *i*th modal voltage and the *ith* modal reactive power variations move in the same direction, indicating voltage stabilitof the system; whereas $\Lambda_i < 0$ refers to instability of the system. The magnitude of Λ_i indicated a relative degree of instability of the *ith* modal voltage. The smaller the magnitude of a positive, the closer the ith modal voltage is to being unstable. The voltage collapses when$\Lambda_i = 0$, because any change in the modal reactive power causes an infinite change in the modal voltage.

A. Participation Factor

The relative contribution of the power at bus in mode is given by the bus participation factor[13][14]

$$P_{ki} = \xi_{ki}\eta_{ki} \quad (19)$$

Participation factors determine most critical areas which lead the system to instability. Usually, the higher magnitude of the participation factor of a bus in a specific mode, the better the remedial action on that bus in stabilizing the mode

B. Calculation Of Eigen Values And Eigen Vectors Of J_R

An algorithm for calculating the minimum singular value and the corresponding left and right singular vectors, for the reduced Jacobian matrix is as follows.

1. Firstly the Jacobian matrix for the given load flow is formed.
2. Next the reduced Jacobian matrix is obtained
3. The right and left Eigen vectors of a reduced Jacobian matrix are to be founded.
4. The Eigen values are obtained from the reduced Jacobian matrix.
5. For minimum Eigen value of the bus find the participation factors for the corresponding mode and bus.
6. The process is Repeated for all buses at the mode, bus with maximum participation factor, indicates the weakest bus.

C. Dg Placement Size In Modal Analysis

Let a Dg be placed at bus 'm' and 'β' be a set of branches connected between the source and Dg unit buses. If the Dg unit is placed at bus 'x' the 'β' consists of branches x1, x2,x n. The Dg unit supplies real current I real and for radial network it changes only the active component of current of branch set 'β'. The current of other branches is not affected by the Dg unit. The new active component of current.

$$I_{ai}\,(new) = I_{ai} + D_i * I_{DG} \quad (20)$$

D_i= 1 if branch iε β

= 0 otherwise.

I_{pi} is the active component of current of ith branch in the original system obtained from the load flow solution.

The loss $P_{La}^{com}$is associated with the active component of branch currents in the compensated system.

$$P_{La}^{com} = \Sigma_{i=1}^{b}(I_{ai} + D_i I_{DG})^2 R_I \quad (21)$$

Savings in Active power loss is:

$$S = P_{La} - P_{La}^{com} = -\Sigma_{i=1} (2D_i I_{ai} I_{DG} + D_i^2 I_{DG})^2 R_i \quad (22)$$

To minimize the loss Eq. is differentiated w.r.t I_{DG} and

$$\Sigma_{i=1}^{b} (D_i I_{ai} R_i) \quad (23)$$

$$I_{DG} = \Sigma (D_i R_i) \quad (24)$$

Distributed generator size

$$P_{DG} = V_m I_{DG} \quad (25)$$

Now the obtained DG size is placed at the position obtained from the modal analysis. And multiple DG's are placed for best results.

IV. Evaluation Indices

Index	Formula
DG penetration level	PL= $\frac{S_{DG}}{S_{load}} \times 100\%$
Active loss reduction	ALR= $\frac{Re\{losses_0\} - Re\{losses_1\}}{Relosses_0}$
Reactive loss reduction	RLR= $\frac{Im\{losses_0\} - Im\{losses_1\}}{Im\{losses_0\}}$
Voltage Index	VI= $\sum_{i=1}^{n} (V_{i,0} - V_{i,1})^2$

To observe the effect of DG on the performance of the System, some indices are used as shown in Table I. The penetration level of DG units is defined by PL, where S_{DG} and S_{load} are the apparent power of DG/DGs and the total apparent load of the network, respectively. ALR and RLR are active and reactive loss reduction after placing DG/DGs, where 0 represents the base case and 1, the case after DG installation [9]. Higher values of and ALR and RLR indicate better performance of DGs in loss reduction.

The VI index is a good tool to determine the deviation of bus voltage from targets, where $V_{i,0}$ is the desired voltage at bus (usually 1 p.u.) and $V_{i,1}$ is the bus voltage when DG is presented in the network, both in per unit the lower the VI, the better the performance of DG units

V. Case Study

An IEEE 33-bus test system is used for sensitivity analysis, voltage stability index and for modal analysis.

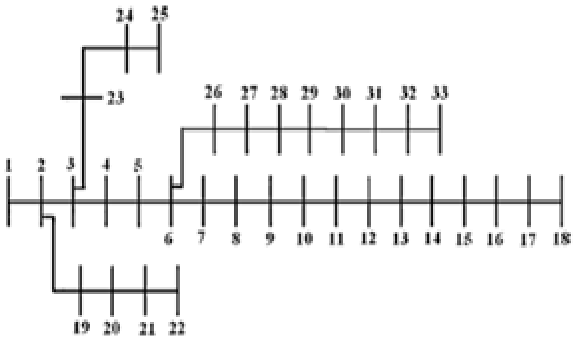


Fig. 3. Single-line diagram of the 33-bus radial distribution network.

Firstly sensitivity analysis method is used to calculate the sensitivity at every bus and the loss sensitivity is calculated at every bus. And the DG is placed at the minimum sensitive bus by varying the DG size from 500Kw to 2500Kw in multiples of 500. And the losses are calculated after every DG size change and the optimum size is considered by taking the minimum loss value. The DG is finally placed considered it as the optimum size. Again the sensitivity analysis is repeated for next DG placement. Here the most sensitive obtained is 6th bus and the first DG is placed at the 6th bus by varying the size of the DG. Loss is calculated at every DG size and optimal size is obtained by considering minimum loss. The second location obtained with least loss sensitivity at 11th bus and the third location is obtained is 24th bus with least sensitivity.

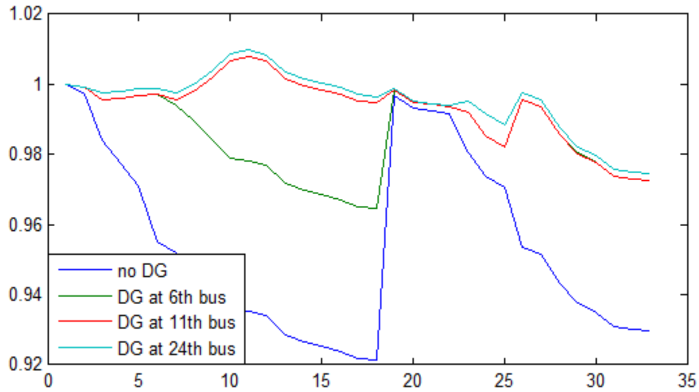


Fig 4. Voltage profile comparison before and after the placement of DG using Loss Sensitivity Analysis

Next voltage sensitivity analysis is performed and the minimum sensitive bus is chosen for DG location and the DG sizes are varied here also and the loss is taken into consideration for optimum size. The first location is obtained is 18th bus and a DG is placed at the 18th bus by varying the Dg size from 500kw to 2500kw in multiples of 500. And the process is repeated for the further two DG's placing. And loss is calculated at every DG size and optimal size is obtained by considering minimum loss.

Second and third locations are 16 and 33.

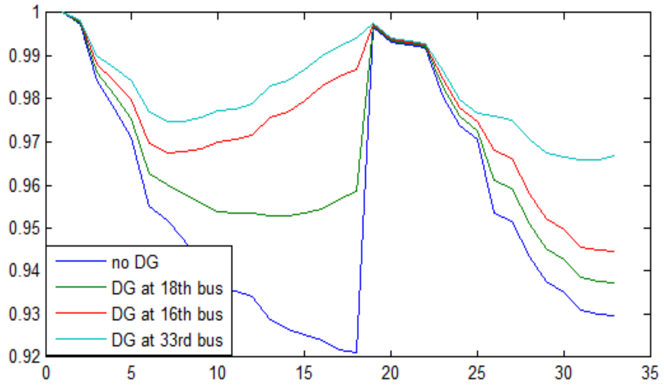


Fig 5. Voltage profile comparison before and after the placement of DG using Voltage Sensitivity Index.

Finally modal analysis is performed. The Eigen value analysis [16] is done for selected buses in order to identify the weakest bus. A power flow program based on Mat lab is developed to:

1. Calculate the load flow solution.
2. Analyse the voltage stability based on modal analysis

The modal analysis method is applied and the voltage profile of the buses is presented from the load flow simulation. Then, the minimum eigenvalue of the reduced Jacobian matrix is calculated. After that, the weakest load buses, which are subject to voltage collapse, are identified by computing the participating factors. . Note that all the eigenvalues are positive which means that the system voltage is stable. The buses 18, 33 and 22 have the highest participation factors for the critical mode. The largest participation factor value (0.0313) at bus 18 indicates the highest contribution of this bus to the voltage collapse. Therefore it is identified that the weakest bus on the IEEE 33 bus power system is bus no.18.Therefore a DG is placed at the bus no.18 and the voltage profile is observed similarly the DG's at the buses 33 and 22 are placed as they are the most participating buses.

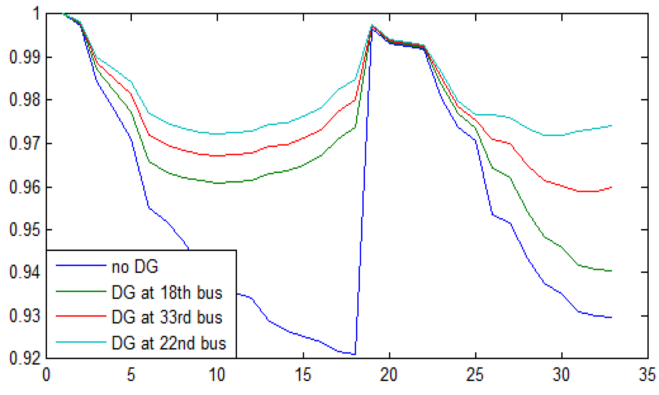


Fig 6. Voltage profile comparison before and after the placement of DG using Modal Analysis

Bus no	Before DG placement voltages	After DG placement the voltages using the three different methods		
		Modal analysis	VSI	LSA
1	1.0000	1.0000	1.0000	1.0000
2	0.9972	0.9982	0.9982	0.9993
3	0.9842	0.9901	0.9901	0.9974
4	0.9775	0.9871	0.9871	0.9978
5	0.9709	0.9844	0.9843	0.9986
6	0.9552	0.9771	0.9769	0.9989
7	0.9518	0.9747	0.9748	0.9974
8	0.9473	0.9735	0.9749	0.9998
9	0.9415	0.9727	0.9758	1.0039
10	0.9362	0.9723	0.9773	1.0086
11	0.9354	0.9724	0.9778	1.0097
12	0.9341	0.9729	0.9788	1.0085
13	0.9286	0.9743	0.9829	1.0034
14	0.9266	0.9748	0.9843	1.0015
15	0.9252	0.9762	0.9867	1.0003
16	0.9238	0.9783	0.9901	0.9990
17	0.9218	0.9823	0.9923	0.9971
18	0.9212	0.9851	0.9941	0.9966
19	0.9967	0.9976	0.9976	0.9988
20	0.9931	0.9941	0.9941	0.9952
21	0.9924	0.9934	0.9934	0.9945
22	0.9918	0.9927	0.9927	0.9939
23	0.9806	0.9866	0.9865	0.9953
24	0.9739	0.9800	0.9799	0.9916
25	0.9706	0.9767	0.9766	0.9883
26	0.9536	0.9766	0.9760	0.9974
27	0.9516	0.9760	0.9750	0.9954
28	0.9434	0.9735	0.9705	0.9877
29	0.9376	0.9719	0.9675	0.9822
30	0.9350	0.9719	0.9666	0.9796
31	0.9309	0.9729	0.9658	0.9757
32	0.9300	0.9736	0.9659	0.9749
33	0.9297	0.9743	0.9667	0.9746

Table 2. Comparison of voltages after DG placement in all methods

TABLE III

SUMMARY OF THE PLACEMENT ALGORITHM RESULTS FOR DIFFERENT PENETRATION LEVELS

DG NO	MODAL ANALYSIS		LSA		VSI		ALR	RLR	VI
	LOC	SIZE(kw)	LOC	SIZE(kw)	LOC	SIZE(kw)			
	……..		…….		……..		…..	……	0.1168
1	18	720.7777	6	1500	18	500	40.67	40.91	0.660
2	33	425.8272	11	1500	16	500	52.11	49.05	0.0363
3	22	356.8212	24	1500	33	500	50.11	44.23	0.0221

VI. CONCLUSION

In this paper comparison is made between the modal analysis and sensitivity analysis methods like loss sensitivity analysis and voltage sensitivity analysis methods. Three DGs are placed in each method and the voltage values are observed at each bus after the three DGs placed and the voltages are calculated. From the table2 and from the indices that are calculated it can be observed that among the methods used, loss sensitivity analysis produces the best results.

And the voltages values after the DG placement is within the permissible limits i.e. ±5% of the bus voltages

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