ROBUST SLIDING MODE CONTROL AND FLUX OBSERVER FOR INDUCTION MOTOR USING TWO-TIME-SCALE APPROACH

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Abstract: *This paper presents a new method for designing robust adaptive sliding mode flux observer for induction motor drive. The idea is to combine singular perturbation and sliding mode techniques, obtaining in this way, the so-called two-time-scale sliding mode observer. The method assumes that the rotor speed signal is available. The control algorithm is based on the indirect field oriented sliding mode control with an on-line adaptation of the rotor resistance to keep the machine field oriented. The control-observer scheme seeks to provide asymptotic tracking of speed and rotor flux in spite of the presence of an uncertain load torque and unknown value of the rotor resistance. The validity for practical implementation has been verified through computer simulations.*

Key words: *Induction motor, singular perturbation theory, sliding mode control, sliding mode observer, stability.*

1. Introduction

The control of the induction motor has attracted much attention in the past few decades. One of the most significant developments in this area has been the field oriented control. The orientation of the flux made possible to act independently on the rotor flux and the electromagnetic torque through the intermediary of the components of the stator voltage. There are two major types, called direct and indirect field orientation [1, 2].

In the last years, the sliding mode technique has been widely studied and developed for the control and state estimation problems since the works of Utkin [3]. This control technique allows good steady state and dynamic behavior in the presence of system parameters variation and disturbances [4, 5].

For induction motor, rotor speed and stator currents are easily measured but rotor fluxes are rather difficult to measure. In fact, different observer structures have been proposed to estimate those fluxes from rotor speed and stator currents [6, 7, 8, 9].

In other hand, singular perturbation theory provides the mean to decompose two-time-scale systems into slow and fast subsystems which greatly simplifies their structural analysis and control design [10].

So, the idea of combining singular perturbation theory and sliding mode technique constitutes a good possibility to achieve classical control objectives for systems having unmodeled or parasitic dynamics and parametric uncertainties [11, 12, 13, 14]. Therefore, the decomposition of the original multi-time-scale system into separate slow and fast subsystems permits a simple analysis and design.

In this paper, a sequential methodology is used to design a robust sliding mode observer in order to estimate the slow electromagnetic variables (rotor flux) under the assumption that only the fast variables (stator currents) and the motor speed are available for measurement.

This paper is organized as follows: We first recall the methodology of model reduction by the singular perturbation theory and present the design of a general two-time-scale sliding mode observer in Section 2 and Section 3, respectively. In Section 4, we briefly review the indirect field oriented sliding mode control of induction motors. In Section 5, the design of the proposed two-time-scale sliding-mode observer for induction motors is presented. In that section, a stability analysis study of this observer is made via singular perturbation and Lyapunov theories. In Section 6, and through simulation, the studied observer is associated to the indirect field oriented sliding-mode control where rotor fluxes are replaced by those delivered by the proposed observer. Finally, in Section 7, we give some comments and conclusions.

2. Two-Time-Scale Approach

The two-time-scale approach can be applied to systems where the state variables can be split into two sets, one having “fast” dynamics, the other having “slow” dynamics. The difference between the two sets of dynamics can be distinguished by the use of a small multiplying scalar ε. Generally, the scaling parameter ε is the speed ratio of the slow versus fast phenomena.

If the slow states are expressed in the t time-scale, then, the fast ones will be in the τ time-scale defined by

$$\tau = (t - t_0) / \varepsilon \qquad (1)$$

The reader is referred to [10] for a general theory on singular perturbation.

2.1. Nonlinear singularly perturbed systems

Let us consider the following class of nonlinear singularly perturbed systems described by the so-called standard singularly perturbed form

$$\begin{aligned} \frac{d}{dt}x &= f_x(x,z,u,t,\varepsilon) \quad x(t_0)=x_0 \\ \varepsilon\frac{d}{dt}z &= f_z(x,z,u,t,\varepsilon), \quad z(t_0)=z_0 \end{aligned} \tag{2}$$

Where $x\in\Re^n$ is the slow state, $z\in\Re^m$ is the fast state, $u\in\Re^p$ is the control input and ε is a small positive parameter such that $\varepsilon\in[0,1]$. f_x and f_z are assumed to be bounded and analytic real vector fields, and consider a vector of measurement that are linearly related to the fast state vector as

$$y = z\,, \qquad y\in\Re^m \tag{3}$$

2.2. Slow reduced subsystem

In the limiting case, as $\varepsilon\to 0$ in (2), the asymptotically stable fast transient decays 'instantaneously', leaving the reduced-order model in the *t* time-scale defined by the quasi-steady-states $x_s(t)$ and $z_s(t)$.

$$\frac{d}{dt}x_s = f_x(x_s,z_s,u_s,t,0) \tag{4}$$

$$0 = f_z(x_s,z_s,u_s,t,0) \tag{5}$$

and the substitution of a root of (5)

$$z_s = h(x_s,u_s,t) \tag{6}$$

into (4) yields a reduced model

$$\frac{d}{dt}x_s = f_x(x_s,h(x_s,u_s,t),u_s,t,0) \quad x_s(t_0)=x_0 \tag{7}$$

Where the index (s) indicates that the associated quantity belongs to the system without ε.

2.3. Fast reduced subsystem

The fast dynamic (also know as *boundary layer system)* denoted z_f, which represents the derivation of z_s from z is obtained by transforming the *slow* time scale *t* to the *fast* time scale $\tau=(t-t_0)/\varepsilon$. System of equation (2) becomes

$$\begin{aligned} \frac{d}{d\tau}x &= \varepsilon\, f_x(x,z,u,\varepsilon\tau+t_0) \\ \frac{d}{d\tau}z &= f_z(x,z,u,\varepsilon\tau+t_0) \end{aligned} \tag{8}$$

Introducing the derivation of z_s from z, i.e., $z_f=z-z_s$ and again examine the limit as $\varepsilon\to 0$. Then, it yields

$$\frac{d}{d\tau}z_f = f_z\big(x_0, z_s(0)+z_f(\tau), u_f(\tau), t_0\big) \tag{9}$$

With

$$z_f(0) = z_0 - z_s(0)$$

Where $u_f = u - u_s$ is the fast part of the input control.

2.4. Two-Time-Scale variables approximation

Fast and slow variables given by (7) and (9) can be combined into a composite structure in order to approximate the original states of (2) as given in [10]:

$$\begin{aligned} x &= x_s(t) + O(\varepsilon) \\ z &= z_s(t) + z_f(\tau) + O(\varepsilon) \end{aligned} \tag{10}$$

3. Two-Time-Scale Sliding Mode Observer

Consider the above continuous nonlinear singularly perturbed system of equation (2) which is given by

$$\begin{cases} \dot{x} = f_x(x,z,u,\varepsilon) \\ \varepsilon\,\dot{z} = f_z(x,z,u,\varepsilon) \end{cases} \tag{11}$$

It is also assumed that the above system is observable. Consequently, the observer design may be considered for the state observation of slowly variables from the measurement of fast variables [12, 13, 14].

3.1. Sliding mode observer design

By structure, observer based on sliding mode approach is very similar to the standard full order observer with replacement of the linear corrective terms by a discontinuous function [7, 12].

The corresponding sliding mode observer for the system of (11) can be written as a replica of the system with an additional nonlinear auxiliary input term as follows:

$$\begin{cases} \dot{\hat{x}} = f_x(\hat{x},z,u,\varepsilon) + G_x I_s \\ \varepsilon\,\dot{\hat{z}} = f_z(\hat{x},z,u,\varepsilon) + G_z I_s \end{cases} \tag{12}$$

Where $I_s = sign\big(S(y,\hat{y})\big)$ is the switching function. G_x and G_z are the gain matrices with $(n\times m)$ and $(m\times m)$ dimensions respectively, to be determined.

The sliding mode function S can be chosen as a linear function of $(y-\hat{y})$ as given in [7, 12], so

$$S(y,\hat{y}) = \Lambda(y-\hat{y}) \tag{13}$$

Where $(y-\hat{y})^T = \big[(y_1-\hat{y}_1)\ \ (y_2-\hat{y}_2)\ \ \ldots\ \ (y_m-\hat{y}_m)\big]$ and Λ is $(n\times m)$ gain matrix to be specified.

The error dynamics is calculated by subtracting (12) from (11):

$$\begin{cases} \dot{e}_x = f_x(x,z,u,\varepsilon) - f_x(\hat{x},z,u,\varepsilon) - G_x I_s \\ \varepsilon\,\dot{e}_z = f_z(x,z,u,\varepsilon) - f_z(\hat{x},z,u,\varepsilon) - G_z I_s \end{cases} \tag{14}$$

or

$$\begin{cases} \dot{e}_x = \Delta f_x - G_x I_s \\ \varepsilon\,\dot{e}_z = \Delta f_z - G_z I_s \end{cases} \tag{15}$$

where

$$e_x = x - \hat{x}\,,$$

$$e_z = z - \hat{z}\,,$$

$$\Delta f_x = f_x(x,z,u,\varepsilon) - f_x(\hat{x},z,u,\varepsilon)\,,$$

$$\Delta f_z = f_z(x,z,u,\varepsilon) - f_z(\hat{x},z,u,\varepsilon)\,.$$

The observer gains design can be based on sequential application of resulted subsystems of (15) by applying singular perturbation methodology. We first need to analyze the fast variables tracking properly using the so-called reaching condition, and, thereafter, the slow variables asymptotic-convergence.

3.2. Stability analysis in the fast time-scale

For fast error dynamic subsystem, the associated time scale is defined by $\tau = (t - t_0)/\varepsilon$, then (15) can be transformed into

$$\begin{cases} \dfrac{de_x}{d\tau} = \varepsilon(\Delta f_x - G_x I_s) \\ \dfrac{de_z}{d\tau} = \Delta f_z - G_z I_s \end{cases} \tag{16}$$

Setting $\varepsilon = 0$ in (15), it yields

$$\frac{de_x}{d\tau} = 0 \tag{17}$$

$$\frac{de_z}{d\tau} = \Delta f_z - G_z I_s \tag{18}$$

In this time-scale, the stability analysis consists of determining G_z so that in this time scale (τ), the surface $S(\tau) = 0$ is attractive.

It can be shown that when sliding mode occurs on $S(\tau)$, the equivalent value of the discontinuous observer maxillary input is found by solving the equation (18) for $G_z I_s$ after insuring zero for $de_z / d\tau$

$$G_z \tilde{I}_s = \Delta f_z$$

and the equivalent switching vector is obtained as

$$\tilde{I}_s = G_z^{-1} \Delta f_z \tag{19}$$

3.3. Stability analysis in the slow time-scale

Slowly error dynamic subsystem can be found by making $\varepsilon = 0$ in (15), so

$$\frac{de_x}{dt} = \Delta f_x - G_x I_s \tag{20}$$

$$0 = \Delta f_z - G_z I_s \tag{21}$$

From (21), the equivalent switching vector can be re-found

$$\tilde{I}_s = G_z^{-1} \Delta f_z$$

Therefore, by appropriate choice of G_x, the desired rate of convergence $e_x \to 0$ can be obtained.

4. Sliding Mode Control Review of I.M

Assuming that the induction model system is controllable and observable, the sliding mode control consists into two phases:

- Designing an equilibrium surface, called sliding surface, such that any state trajectory of the plant restricted to the sliding surface is characterized by the desired behavior;
- Designing a discontinuous control law to force the system to move on the sliding surface in a finite time.

4.1. Dynamic model of induction motor

Under the assumptions of linearity of the magnetic circuit and neglecting iron losses, the state space model of three-phase induction motor expressed in the synchronously rotating reference frame $(d-q)$ is

$$\begin{cases} \dfrac{d}{dt} i_{sd} = -\dfrac{R_\lambda}{\sigma L_s} i_{sd} + \omega_s i_{sq} + \dfrac{\mu}{\sigma L_s}\dfrac{1}{T_r}\phi_{rd} + \dfrac{\mu}{\sigma L_s}\omega\phi_{rq} + \dfrac{1}{\sigma L_s} v_{sd} \\ \dfrac{d}{dt} i_{sq} = -\omega_s i_{sd} - \dfrac{R_\lambda}{\sigma L_s} i_{sq} - \dfrac{\mu}{\sigma L_s}\omega\phi_{rd} + \dfrac{\mu}{\sigma L_s}\dfrac{1}{T_r}\phi_{rq} + \dfrac{1}{\sigma L_s} v_{sq} \\ \dfrac{d}{dt}\phi_{rd} = \dfrac{M}{T_r} i_{sd} - \dfrac{1}{T_r}\phi_{rd} + \omega_{sl}\phi_{rq} \\ \dfrac{d}{dt}\phi_{rq} = \dfrac{M}{T_r} i_{sq} - \omega_{sl}\phi_{rd} - \dfrac{1}{T_r}\phi_{rq} \\ \dfrac{d\omega}{dt} = \dfrac{p}{J}(T_{em} - T_L) - \dfrac{f}{J}\omega \end{cases} \tag{22}$$

With constants defined as follows

$$R_\lambda = R_s + \frac{M^2}{L_r^2} R_r, \quad \sigma = 1 - \frac{M^2}{L_s L_r}, \quad \mu = \frac{M}{L_r},$$

Where the state variables are the stator currents (i_{sd}, i_{sq}), the rotor fluxes (ϕ_{rd}, ϕ_{rq}) and the rotor speed ω. Stator voltages (v_{sd}, v_{sq}) and split frequency ω_{sl} ($\omega_{sl} = \omega_s - \omega$) are the control variables. The electromagnetic torque expressed in terms of the state variables is

$$T_{em} = \frac{pM}{L_r}(\phi_{rd} i_{sq} - \phi_{rq} i_{sd}) \tag{23}$$

4.2. Rotor field oriented induction motor model

Among the various sliding mode control solutions for induction motor proposed in the literature, the one based on indirect field orientation can be regarded as the simplest one. Is purpose is to directly control the inverter switching by use of two switching surfaces.

The induction motor equations in the synchronously rotating reference frame $(d-q)$, oriented in such a way that the rotor flux vector points into d-axis direction, are the following

$$\begin{cases} \dfrac{d}{dt}\omega = f_1 \\ \dfrac{d}{dt}\phi_{rd} = f_2 \\ \dfrac{d}{dt} i_{sd} = f_3 + \dfrac{1}{\sigma L_s} v_{sd} \\ \dfrac{d}{dt} i_{sq} = f_4 + \dfrac{1}{\sigma L_s} v_{sq} \end{cases} \tag{24}$$

with

$$\omega_{sl} = \frac{M}{T_r}\frac{i_{sq}}{\phi_{rd}} \tag{25}$$

Where

$$\begin{cases} f_1 = k_c \phi_{rd} i_{sq} - \frac{p}{J} T_L - \frac{f}{J}\omega \\ f_2 = \frac{M}{T_r} i_{sd} - \frac{1}{T_r}\phi_{rd} \\ f_3 = -\frac{R_\lambda}{\sigma L_s} i_{sd} + \omega_s i_{sq} + \frac{\mu}{\sigma L_s}\frac{1}{T_r}\phi_{rd} \\ f_4 = -\omega_s i_{sd} - \frac{R_\lambda}{\sigma L_s} i_{sq} - \frac{\mu}{\sigma L_s}\omega\phi_{rd} \end{cases} \tag{26}$$

and

$$k_c = \frac{p^2 M}{J L_r}$$

4.3. Speed and flux sliding mode controller

Using the reduced non-linear induction motor model of equation (24), it is possible to design both a speed and a flux sliding mode controllers. Let us defined the sliding surfaces

$$\begin{cases} S_{c1} = S_{c1}(\omega) = \lambda_\omega(\omega^* - \omega) + \frac{d}{dt}(\omega^* - \omega) \\ S_{c2} = S_{c1}(\phi_r) = \lambda_\phi(\phi_r^* - \phi_{dr}) + \frac{d}{dt}(\phi_r^* - \phi_{dr}) \end{cases} \tag{27}$$

Where $\lambda_\omega > 0$, $\lambda_\phi > 0$, ω^* and ϕ_r^* is the speed reference and the reference rotor flux, respectively.

To determine the control law that is expected *t* steer the sliding functions (27) to zero in finite time, one has to consider the dynamics of $S_c = (S_{c1}, S_{c2})^T$, described by

$$\dot{S}_c = F + D\,V_s \tag{28}$$

where

$$F = \begin{bmatrix} (\ddot{\omega}^* + \lambda_\omega \dot{\omega}^* + \frac{K_f}{J}\dot{T}_L) + (-\lambda_\omega + \frac{K_f}{J}) f_1 - k_c(i_{sq} f_2 + \phi_{rd} f_4) \\ (\ddot{\phi}^* + \lambda_\phi \dot{\phi}^*) + (-\lambda_\phi + \frac{1}{T_r}) f_2 - \frac{M}{T_r} f_3 \end{bmatrix}$$

$$D = \frac{1}{\sigma L_s}\begin{bmatrix} k_c \phi_{rd} & 0 \\ 0 & M/T_r \end{bmatrix}, V_s = \begin{bmatrix} v_{sq} \\ v_{sd} \end{bmatrix},$$

If the Lyapunov method of stability is used to ensure that S_c is attractive and invariant, the following condition has to be satisfied

$$S_c^T \cdot \dot{S}_c < 0 \tag{29}$$

So, it is possible to choose the switching control law for stator voltages as follows

$$\begin{bmatrix} v_{sq} \\ v_{sd} \end{bmatrix} = -D^{-1}F - D^{-1}\begin{bmatrix} K_\omega & 0 \\ 0 & K_\phi \end{bmatrix}\begin{bmatrix} sign(S_{c1}) \\ sign(S_{c2}) \end{bmatrix} \tag{30}$$

Where

$$K_\omega > 0,\ K_\phi > 0 \tag{31}$$

***Proof* (1)**: Substituting the resulted of (30) in (28), in (29) achieves the proof.

The sliding mode causes drastic changes of the control variable introducing high frequency disturbances. To reduce the chattering phenomenon a saturation function $sat(S_c)$ instead of the switching one $sign(S_c)$ has been introduced

$$sat(S_{ci}) = \begin{cases} \frac{S_{ci}}{\delta_i} & \text{if } |(S_{ci})| \le \delta_i \\ sign(S_{ci}) & \text{if } |(S_{ci})| > \delta_i \end{cases}, \tag{32}$$

Where $\delta_i > 0$ for $i = 1, 2$ with $\delta_1 = \delta_\omega$ and $\delta_2 = \delta_\phi$.

***Remarks*:**

- From the above control law of equation (30), it can be see that the implementation of these algorithms requires the estimation of torque load and rotor flux since stator currents, stator voltages and speed rotor are available by measures. In the next section, we are interested by a robust estimation of rotor flux. The estimated torque load can be easily obtained by using the mechanical equation of the induction motor model.
- In the following, we will assume to operate with constant speed reference, rotor flux reference and load torque, so that $\dot{\omega}^* = 0$, $\dot{\phi}_r^* = 0$ and $\dot{T}_L = 0$.

5. Two-Time-Scale Sliding Mode Observer Design for I.M

5.1. Dynamic model of induction motor

Using the model of equation (22), the state space model of induction motor expressed in the fixed stator reference frame $(\alpha - \beta)$ is

$$\begin{cases} \frac{d}{dt} i_{s\alpha} = -\frac{R_\lambda}{\sigma L_s} i_{s\alpha} + \frac{\mu}{\sigma L_s}\frac{1}{T_r}\phi_{r\alpha} + \frac{\mu}{\sigma L_s}\omega\phi_{r\beta} + \frac{1}{\sigma L_s} v_{s\alpha} \\ \frac{d}{dt} i_{s\beta} = -\frac{R_\lambda}{\sigma L_s} i_{s\beta} - \frac{\mu}{\sigma L_s}\omega\phi_{r\alpha} + \frac{\mu}{\sigma L_s}\frac{1}{T_r}\phi_{r\beta} + \frac{1}{\sigma L_s} v_{s\beta} \\ \frac{d}{dt}\phi_{r\alpha} = \frac{M}{T_r} i_{s\alpha} - \frac{1}{T_r}\phi_{r\alpha} - \omega\phi_{r\beta} \\ \frac{d}{dt}\phi_{r\beta} = \frac{M}{T_r} i_{s\beta} + \omega\phi_{r\alpha} - \frac{1}{T_r}\phi_{r\beta} \end{cases} \tag{33}$$

Voltage, current and flux transformation from the synchronously to the stationary reference frame and vice versa is made by the rotational transformation [1, 2]:

$$\begin{pmatrix} x_\alpha \\ x_\beta \end{pmatrix} = \Re(\theta_s)\begin{pmatrix} x_d \\ x_q \end{pmatrix} \tag{34}$$

with

$$\Re(\theta_s) = \begin{pmatrix} \cos(\theta_s) & -\sin(\theta_s) \\ \sin(\theta_s) & \cos(\theta_s) \end{pmatrix} \tag{35}$$

and

$$[\Re(\theta_s)]^{-1} = \Re(-\theta_s)$$

Where $x = v, i, \phi$, and θ_s is the angular displacement of the synchronously rotating reference frame.

5.2. Singularly perturbed model

Based on the well-know of induction machine model dynamics [11, 14], the slow variables are $(\phi_{r\alpha}, \phi_{r\beta})$ and the fast variables are $(i_{s\alpha}, i_{s\beta})$. Therefore, the corresponding standard singularly perturbed form, of (33), with $x = (\phi_{r\alpha}, \phi_{r\beta})^T$, $z = (i_{s\alpha}, i_{s\beta})^T$ and $\varepsilon = \sigma L_s$ is

$$\begin{cases} \varepsilon \dot{z}_1 = -R_\lambda z_1 + \mu\alpha_r x_1 + \mu\omega x_2 + v_{s\alpha} \\ \varepsilon \dot{z}_2 = -R_\lambda z_2 - \mu\omega x_1 + \mu\alpha_r x_2 + v_{s\beta} \\ \dot{x}_1 = M\alpha_r z_1 - \alpha_r x_1 - \omega x_2 \\ \dot{x}_2 = M\alpha_r z_2 + \omega x_1 - \alpha_r x_2 \end{cases} \tag{36}$$

Where $\alpha_r = \frac{1}{T_r} = \frac{R_r}{L_r}$ and $R_\lambda = R_s + M\,\mu\,\alpha_r$.

5.3. Singularly Perturbed Sliding Mode Observer

From Section 3, the observer equations, of the above model, based on the sliding mode concept are

$$\begin{cases} \varepsilon \dot{\hat{z}}_1 = -\hat{R}_\lambda z_1 + \mu\hat{\alpha}_r \hat{x}_1 + \mu\omega\hat{x}_2 + v_{s\alpha} + G_{z_1} I_s \\ \varepsilon \dot{\hat{z}}_2 = -\hat{R}_\lambda z_2 - \mu\omega\hat{x}_1 + \mu\hat{\alpha}_r \hat{x}_2 + v_{s\beta} + G_{z_2} I_s \\ \dot{\hat{x}}_1 = M\hat{\alpha}_r z_1 - \hat{\alpha}_r \hat{x}_1 - \omega\hat{x}_2 + G_{x_2} I_s \\ \dot{\hat{x}}_2 = M\hat{\alpha}_r z_2 + \omega\hat{x}_1 - \hat{\alpha}_r \hat{x}_2 + G_{x_2} I_s \end{cases} \tag{37}$$

With $\hat{\alpha}_r = \alpha_r + \Delta\alpha_r$ and $\hat{R}_\lambda = R_s + M\,\mu\,\hat{\alpha}_r$, in which

$$\hat{\alpha}_r = \frac{\hat{R}_r}{L_r} = \frac{R_r}{L_r} + \frac{\Delta R_r}{L_r} \tag{38}$$

Where $\hat{x}_i$ and $\hat{z}_j$ are the estimation of x_i and z_j for $i \in \{1,2\}$ and $j \in \{1,2\}$. G_{x1}, G_{x2}, G_{z1} and G_{z2} are the observer gain.

The switching vector I_s is chosen as

$$I_s = \begin{bmatrix} sign\,(s_1) \\ sign\,(s_2) \end{bmatrix} \tag{39}$$

with

$$S = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} z_1 - \hat{z}_1 \\ z_2 - \hat{z}_2 \end{bmatrix} = \begin{bmatrix} e_{z_1} \\ e_{z_2} \end{bmatrix} \tag{40}$$

From (36)-(37) and setting $e_{x_i} = x_i - \hat{x}_i$ and $e_{z_j} = z_j - \hat{z}_j$ for $i \in \{1,2\}$ and $j \in \{1,2\}$, the estimation error dynamics are

$$\begin{cases} \varepsilon\dot{e}_{z_1} = \mu\big[(+\alpha_r e_{x_1} + \omega e_{x_2}) + \Delta\alpha_r (Mz_1 - \hat{x}_1)\big] - G_{z_1} I_s \\ \varepsilon\dot{e}_{z_2} = \mu\big[(-\omega e_{x_1} + \alpha_r e_{x_2}) + \Delta\alpha_r (Mz_2 - \hat{x}_2)\big] - G_{z_2} I_s \\ \dot{e}_{x_1} = -\big[(+\alpha_r e_{x_1} + \omega e_{x_2}) + \Delta\alpha_r (Mz_1 - \hat{x}_1)\big] - G_{x_2} I_s \\ \dot{e}_{x_2} = -\big[(-\omega e_{x_1} + \alpha_r e_{x_2}) + \Delta\alpha_r (Mz_2 - \hat{x}_2)\big] - G_{x_2} I_s \end{cases} \tag{41}$$

Equation (41) can be expressed in the matrix form as

$$\begin{cases} \varepsilon\dot{e}_z = \mu\big[(\alpha_r I - \omega J)e_x + \Delta\alpha_r (Mz - \hat{x})\big] - G_z I_s \\ \dot{e}_x = -\big[(\alpha_r I - \omega J)e_x + \Delta\alpha_r (Mz - \hat{x})\big] - G_x I_s \end{cases} \tag{42}$$

Where I is the (2×2) identity matrix and J is the (2×2) skew symmetric matrix defined by

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Exploiting the time-properties of multi-time-scale systems of equations (36) and (37), (e_{z_1}, e_{z_2}) are fast variables and (e_{x_1}, e_{x_2}) are slow variables. So, the stability analysis of the above system consists of determining G_{z1} and G_{z2} to ensure the attractiveness of the sliding surface $S(\tau) = 0$ in the fast time scale. Thereafter G_{x1} and G_{x2} are determined, such that the reduced-order system obtained when $S(\tau) = \dot{S}(\tau) = 0$ is locally stable.

5.4. Fast reduced-order error dynamics

From singular perturbation theory, the fast reduced-order system of the observation errors can be obtained by introducing the fast time-scale $\tau = (t - t_0)/\varepsilon$. System of equations (42) gives

$$\begin{cases} \frac{d}{d\tau} e_z = \mu\big[(\alpha_r I - \omega J)e_x + \Delta\alpha_r (Mz - \hat{x})\big] - G_z I_s \\ \frac{d}{d\tau} e_x = -\varepsilon\big[(\alpha_r I - \omega J)e_x + \Delta\alpha_r (Mz - \hat{x})\big] - \varepsilon G_x I_s \end{cases} \tag{43}$$

Making $\varepsilon = 0$ in the above system, it yields

$$\frac{d}{d\tau} e_z = \mu\big[(\alpha_r I - \omega J)e_x + \Delta\alpha_r (Mz - \hat{x})\big] - G_z I_s \tag{44}$$

$$\frac{d}{d\tau} e_x = 0 \tag{45}$$

By appropriate choice of the observer gain terms G_{z_1} and G_{z_2}, sliding mode occurs in (44) along the manifold $S = e_z = 0$.

***Proposition* (1):** Assume that e_{x_1} and e_{x_2} are bounded in this time (practical assumption) and ω varies slowly, and consider (44) with the following observer gains matrix

$$G_z = \begin{bmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{bmatrix}; \tag{46}$$

The attractivity condition of the sliding surface $S(\tau) = 0$ is given by

$$S^T \left(\frac{dS}{d\tau}\right) < 0 \tag{47}$$

In this time-scale $\frac{dx}{d\tau} = 0$ and $\frac{de_x}{d\tau} = 0.$

So,

$$S^T \frac{d}{d\tau} S = S^T \big[\mu\big[(\alpha_r I - \omega J)e_x + \Delta\alpha_r (Mz - \hat{x})\big] - G_z I_s\big] \tag{48}$$

Or

$$\begin{aligned} S^T \frac{d}{d\tau} S = &-s_1\big[\eta_1 sign(s_1) - \mu\big[\alpha_r e_{x1} + \omega e_{x2} + \Delta\alpha_r (Mz_1 - \hat{x}_1)\big]\big] \\ &- s_2\big[\eta_2 sign(s_2) - \mu\big[\alpha_r e_{x2} - \omega e_{x1} + \Delta\alpha_r (Mz_2 - \hat{x}_2)\big]\big] \end{aligned} \tag{49}$$

Thus, (47) is verified with the set defined by the following inequalities:

$$\begin{aligned} \eta_1 &> \left| \mu\left[\alpha_r e_{x1} + \omega e_{x2} + \Delta\alpha_r (Mz_1 - \hat{x}_1)\right] \right| \\ \eta_2 &> \left| \mu\left[\alpha_r e_{x2} - \omega e_{x1} + \Delta\alpha_r (Mz_2 - \hat{x}_2)\right] \right| \end{aligned} \quad (50)$$

Once the trajectory reaches the sliding surface $S = e_z = 0$, the slowly state variables behave as if $G_z I_s$ is replaced by its equivalent value $(G_z I_s)_{eq}$, which can be calculated from the subsystem (44) assuming $e_z = 0$ and $\dot{e}_z = 0$.

5.5. Slow reduced-order error dynamics

For slow error dynamics, we use the system (42) and setting $\varepsilon = 0$. So, we can write

$$0 = \mu\left[(\alpha_r I - \omega J)e_x + \Delta\alpha_r (Mz - \hat{x})\right] - G_z I_s \quad (51)$$

$$\dot{e}_x = -\left[(\alpha_r I - \omega J)e_x + \Delta\alpha_r (Mz - \hat{x})\right] - G_x I_s \quad (52)$$

From equation (51), we can get the equivalent switching vector $\tilde{I}_s$ as

$$\tilde{I}_s = \mu G_z^{-1}\left[(\alpha_r I - \omega J)e_x + \Delta\alpha_r (Mz - \hat{x})\right] \quad (53)$$

In this time-scale, we can replace I_s by $\tilde{I}_s$ in equation (52). So, subsystem (52) can be written as follows:

$$\dot{e}_x = H\left[(\alpha_r I - \omega J)e_x + \Delta\alpha_r (Mz - \hat{x})\right] \quad (54)$$

with

$$H = -(I + \mu G_x G_z^{-1}) \quad (55)$$

5.6. Stability analysis of the slow reduced-order error observer

To derive adaptive law of rotor resistance, we define the positive-definite candidate Lyapunov function

$$W = \frac{1}{2}(e_x)^T e_x + \frac{q_1}{q_2}\frac{1}{2}(\Delta\alpha_r)^2 \quad (56)$$

where

$$q_1 \cdot q_2 > 0 \quad (57)$$

The *t*-time-derivative of W is

$$\dot{W} = \frac{1}{2}\left\{(e_x)^T \dot{e}_x + (\dot{e}_x)^T e_x\right\} + \frac{q_1}{q_2}\Delta\alpha_r \frac{d}{dt}\Delta\alpha_r \quad (58)$$

Unfortunately, the fluxes errors (e_x) are not available. In the following, we consider how to avoid this problem. So, by defining the function

$$Q = A e_x + \Delta\alpha_r B \quad (59)$$

where

$$A = (\alpha_r I - \omega J)$$
$$B = (Mz - \hat{x})$$

The rotor flux error dynamics of (54) are rewriting, using (59), as

$$\dot{e}_x = H Q \quad (60)$$

Now, replacing (60) in (58), it yields

$$\dot{W} = \frac{1}{2}\left\{(e_x)^T H\, Q + Q^T H^T e_x\right\} + \frac{q_1}{q_2}\Delta\alpha_r \frac{d}{dt}\Delta\alpha_r \quad (61)$$

Proposition (2): With the following choice

$$H = -q_1 A^T \quad (62)$$

$\dot{W}$ will to be negative-definite if $\frac{d}{dt}\Delta\alpha_r = -\frac{q_2}{\mu}(G_z I_s)^T B$ and $q_1 > 0$.

***Proof* (2):** Using (62), equation (61) becomes

$$\dot{W} = -q_1 Q^T Q + q_1 \Delta\alpha_r (Q^T B + \frac{1}{q_2}\frac{d}{dt}\Delta\alpha_r) \quad (63)$$

Therefore, condition for (63) to be negative-definite will be satisfied if

$$q_1 > 0 \quad (64)$$

and

$$\frac{d}{dt}\Delta\alpha_r = -q_2 Q^T B \quad (65)$$

Using (51) and (59), the adaptive law of (65) becomes feasible:

$$\frac{d}{dt}\Delta\alpha_r = -\frac{q_2}{\mu}(G_z I_s)^T B \quad (66)$$

With the assumptions of (57) and (64), it yields

$$q_2 > 0 . \quad (67)$$

6. Simulation Results

The proposed estimation algorithm has been simulated for the induction motor whose data are given in appendix 2. As a controller, the indirect field oriented sliding mode control is used. It is assumed that the load torque is unknown and all the parameters are known and constant except for the rotor resistance which will change during the operating motor. For this closed loop system, rotor flux feedback signal and rotor resistance are replaced with the estimate corresponding values of equation (37) and (66), respectively. With the assumption that all states including rotor flux and all parameters are known, rotor flux and rotor resistance estimated by the proposed method are compared to their actual values.

1,5 kW	220/380 V	3,68/6,31 A
$N = 1420$ rpm	$R_s = 4.85\Omega$	$R_r = 3.805\Omega$
$L_s = 0{,}274$ H	$L_r = 0{,}274$ H	$M = 0{,}258$ H
$p = 2$	$J = 0{,}031$ Kg.m^2	$f = 0.00113$ N.m.s/rd

Table 01: Parameters of induction motor.

The sliding mode control and observer parameters were chosen as $\lambda_\omega = 120$, $\lambda_\phi = 120$, $K_\omega = 80$, $K_\phi = 80$, $\delta_\omega = 0.5$, $\delta_\phi = 0.5$, $q_1 = 25$, $q_2 = 0.03$ and $G_z = diag(3{,}3)$.

The results are summarized in this section:

6.1. Rotor resistance variation effect

This test consists in increasing the rotor resistance. As shown in Fig. 1(a), the motor is started with its nominal rotor resistance value $R_{rn} = 3.805\Omega$. Then, the rotor resistance of the motor model is suddenly set to $1.5R_{rn}$ at $t = 1s$, and to $2R_{rn}$ at $t = 2s$. The reference speed and reference rotor flux are maintained at 1400 rpm and 1.0 Wb, respectively. Fig. 1(b) shows the speed response of the motor; a very good speed regulation is obtained. In Fig. 1(c) and 1(d), are shown the estimated rotor fluxes and the error between them and the actual values. It can be noticed the high flux tracking and the good rotor flux orientation. Fig. 1(e) compares the estimated and actual rotor resistance. After a short convergence time, the estimated rotor resistance reaches the actual value. Fig. 1(f) shows the stator currents error. These results show that the sliding mode control with the proposed observer can track the reference command accurately and quickly.

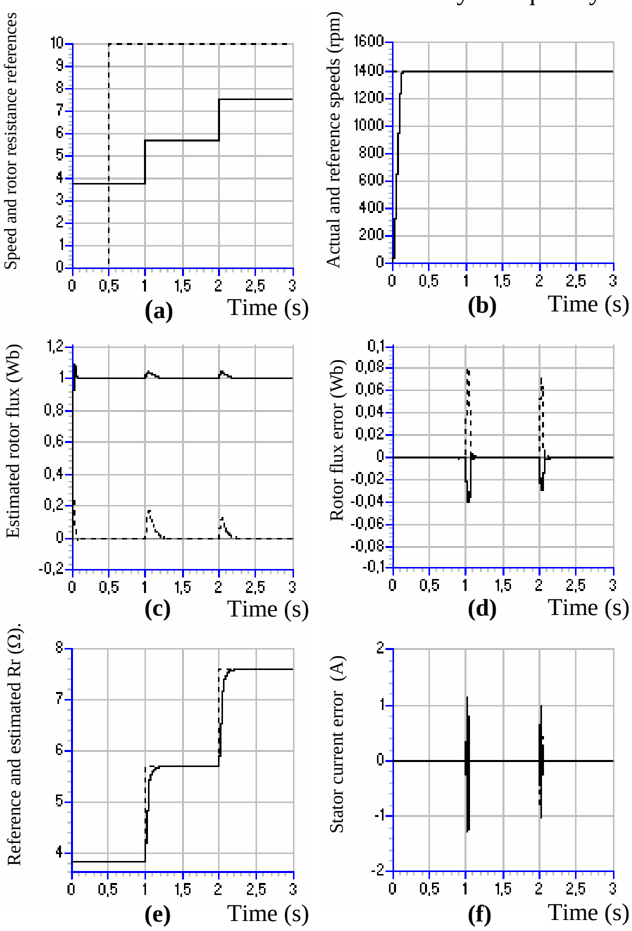

Fig. 1. Sensitivity of the system performance to changes on the rotor resistance by 50% and after 100% with $\phi_{rd}^* = 1.0Wb$:

(a) Reference signals of R_r (solid) and load torque (dotted),

(b) Reference (dotted) and actual (solid) speed,

(c) Rotor flux estimation: $\hat{\phi}_{rd}$ (solid) and $\hat{\phi}_{rq}$ (dotted),

(d) Rotor flux error: $e_{\phi_{rd}}$ (solid) and $e_{\phi_{rq}}$ (dotted),

(e) Reference (dotted) and estimated (solid) rotor resistance,

(f) Stator current error: $e_{i_{s\alpha}}$ (solid) and $e_{i_{s\beta}}$ (dotted).

6.2. Performance under external load disturbances

The observer sensitivity to external load disturbances is also investigated in this study. The objective is to follow the speed and rotor flux references in spite of disturbances in load torque with a constant error (of +25%) in the rotor resistance value. This practical error is made to test the efficacy of the adaptive law of equation (66). Fig. 2(a) shows the actual and the estimated applied load torque. Due to the rotor inertia, the estimated load torque presents negative values in the start-up motor and after, it follows exactly the actual signal. Fig. 2(b) presents very good performance for speed regulation. Fig. 2(c) shows the motor and the real load torque. Fig. 2(d) presents the actual and estimated rotor resistance. Fig. 2(e) and 2(f) show that the completely decoupled control of rotor flux and torque is obtained and the observer is very robust to external load disturbances.

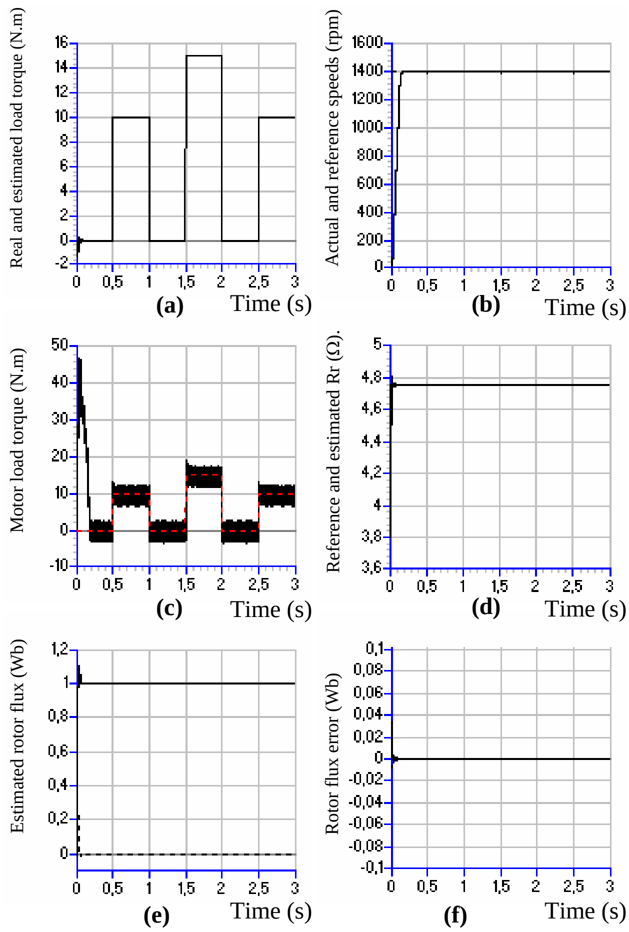

Fig. 2. Sensitivity of the system performance to changes in the external load with $R_r = 1.25R_{rn}$ and $\phi_{rd}^* = 1.0Wb$:

(a) Real (dotted) and estimated (solid) load torque,

(b) Reference (dotted) and actual (solid) speed,

(c) Load torque (dotted) and motor torque (solid),

(d) Reference (dotted) and estimated (solid) rotor resistance,

(e) Rotor flux estimation: $\hat{\phi}_{rd}$ (solid) and $\hat{\phi}_{rq}$ (dotted),

(f) Rotor flux error: $e_{\phi_{rd}}$ (solid) and $e_{\phi_{rq}}$ (dotted).

6.3. Performance over wide speed range

In this case, we consider the speed tracking performances for wide variation range of reference speed. The rotor flux reference is kept at its rated value of 1.0 Wb and the motor is operating without external load disturbances. The observer performance for speed tracking is presented in Fig. 3(a). The actual and the estimated rotor resistance are shown in Fig. 3(b). Fig. 3(d) and 3(c) show the estimation of the rotor fluxes and the error between the estimated rotor fluxes and the actual rotor fluxes, respectively. These results prove that the speed tracking is quite good and the rotor-field is always well oriented.

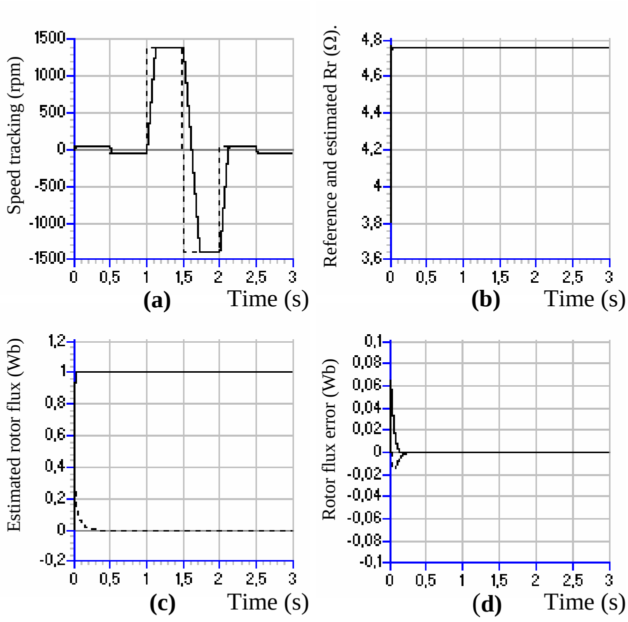

Fig. 3. Sensitivity of the system performance to change in the reference speed with $R_r = 1.5 R_{rn}$ and $\phi_{rd}^* = 1.0 Wb$:

(a) Reference (dotted) and actual (solid) speed,
(b) Reference (dotted) and estimated (solid) rotor resistance,
(c) Rotor flux estimation: $\hat{\phi}_{rd}$ (solid) and $\hat{\phi}_{rq}$ (dotted),
(d) Rotor flux error: $e_{\phi_{rd}}$ (solid) and $e_{\phi_{rq}}$ (dotted).

7. Conclusion

A robust sliding mode rotor flux observer with on-line adaptation of the rotor resistance estimation for induction motor has bee derived using singular perturbation and Lyapunov theories. The accurate rotor flux obtained by the proposed algorithm has been applied to indirect field oriented sliding mode control. The efficiency of the control-observer structure scheme with rotor resistance estimation has been successfully verified by simulation. The proposed sliding mode observer-based control demonstrated good performance, especially; it is robust under rotor resistance variation, external load disturbances and speed tracking.

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Appendix: In this paper, the following notations are used:

ω , ω_{ref}	Angular motor speed and reference speed,
v_{sd}, v_{sq}	Stator voltages in synchronously rotating reference frame,
i_{sd} , i_{sq}	Stator currents in synchronously rotating reference frame,
ϕ_{rd}, ϕ_{rq}	Rotor fluxes in synchronously rotating reference frame,
$v_{s\alpha}, v_{s\beta}$	Stator voltages in stationary reference frame,
$i_{s\alpha}, i_{s\beta}$	Stator currents in stationary reference frame,
$\phi_{r\alpha}, \phi_{r\beta}$	Rotor fluxes in stationary reference frame,
ω_s , ω_{sl}	Synchronous frequency and slip frequency,
T_e, T_L	Electromagnetic and load torques,
L_s, L_r	Stator and rotor inductances,
R_s, R_r	Stator and rotor resistances,
T_s, T_r	Stator and rotor time constants,
M , σ	Mutual inductance and leakage factor,
J, p	Moment of inertia of the rotor and numbers of pole pairs,
f	Coefficient of viscous friction.