

# IDENTIFICATION OF POWER TRANSFORMER LEAKAGE INDUCTANCE WHEN ENERGIZATION

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**Abstract:** The algorithm to calculate transformer leakage inductance, which is based on the loop equation of transformer, needs to have the values of voltages and currents at both sides of transformer. However, in existing substations, the voltage at no-load side of transformer cannot be obtained during its no-load closing, so a new method to calculate transformer leakage inductance that is suitable to the condition of no-load closing of transformer is proposed: so long as knowing the resistance and leakage inductance of transformer under normal condition and utilizing recurrence method in difference equation, transformer leakage inductance under its no-load closing can be rapidly calculated. On this basis, the leakage inductance is normalized to make the setting of protection device more universal. Dynamic simulation results show that the proposed method is not affected by the installation site of transformer in the substation; it can be ensured that the transformer can be reliably and rapidly switched off while transformer internal fault occurs, and the proposed method is enough sensitive to slight fault.

**Key words:** Transformer protection, Loop circuit equation, Normalized leakage inductance, Magnetizing inrush current, Internal fault current

## 1. Introduction

With large-capacity transformers put into operation, the rapidity and reliability of Transformer differential protection are confronted with increasingly serious challenges. Magnetizing inrush current is a main factor which results in disoperation of differential protection and how to distinguish between inrush current and fault current has been one of the most important research topics in transformer differential protection. The theories of second harmonic restraint, waveform matching and dead angle are adopted generally in present engineering. Among them, the theory of second harmonic restraint is the most widely used, and has accumulated a wealth of operating experience. In the situation of stable operation, the theory has adequate security to meet operational requirements. However, with the increase of the capacity of transformer and the improvement of the working point position, the problems of slow action and poor reliability of differential protection based on second harmonic restraint principle are becoming increasingly acute.

The second harmonic restrain ratio is difficult to choose; it can be set or corrected through the experimental ways according to real situation in application, and the hidden trouble of misjudgment lurks in results.

The micro-processor based transformer primary protection, based on transformer loop equation, get rid of the shackles of the principle of differential protection; it avoids the problem of magnetizing inrush current from the beginning and eliminates nonlinear term which reflects the transformer core magnetic flux directly. As a result, the problems of magnetizing inrush current and over-excitation current can be get rid of completely. On this basis, literatures [12-15] also consider that changes in the distribution of the leakage flux will lead to corresponding change in value of leakage inductance of faulted winding in the case of transformer failures, and a new criterion of transformer protection is constructed based on the value of leakage inductance. However, the principle of transformer protection, whether based on the loop equation or leakage inductance parameter, needs to have the values of voltages and currents at both sides. But in existing substation, the voltage at no-load side of transformer cannot be obtained during its no-load closing for voltage transformers in substation are installed on bus. Therefore, voltage transformer (TV) has to be installed at no-load side if the principle above is used in analysis, which will definitely cause a serious of problems such as complex circuit, overlapping investment and high cost.

## 2. Leakage Inductance of Two Winding Three-Phase Transformer During Its No-Load Closing

For the Y/ $\Delta$  connected transformer in figure 1, setting:  $i_a, i_b, i_c$  are currents of  $\Delta$ -windings,  $i_A, i_B, i_C$  are currents of Y-windings,  $u_a, u_b, u_c$  are voltages of  $\Delta$ -windings,  $u_A, u_B, u_C$  are voltages of Y-windings,  $i_{La}, i_{Lb}, i_{Lc}$  are phase currents of A,B,C out of  $\Delta$

-windings,  $L_A, L_B, L_C$  are leakage inductance in Y-windings,  $L_a, L_b, L_c$  are leakage inductance in  $\Delta$ -windings,  $R$  is the resistance of Y-windings,  $r$  is the resistance of  $\Delta$ -windings. Impute parameters of  $\Delta$ -side to Y-side and eliminate main flux:

$$\begin{cases} u'_a - u'_b - u_A + u_B = i'_{La}r' - (i_A - i_B)R + \\ L'_1 \frac{di'_{La}}{dt} - L_2 \frac{d(i_A - i_B)}{dt} \\ u'_b - u'_c - u_B + u_C = i'_{Lb}r' - (i_B - i_C)R + \\ L'_1 \frac{di'_{Lb}}{dt} - L_2 \frac{d(i_B - i_C)}{dt} \\ u'_c - u'_a - u_C + u_A = i'_{Lc}r' - (i_C - i_A)R + \\ L'_1 \frac{di'_{Lc}}{dt} - L_2 \frac{d(i_C - i_A)}{dt} \end{cases} \quad (1)$$

where  $u'_a, u'_b, u'_c, i'_{La}, i'_{Lb}, i'_{Lc}, r', L'_1$  are values imputed to Y-side. In normal situation, three phase leakage inductance of  $\Delta$ -windings have the same value  $L'_1$  while those of Y-side have the same value  $L_2$ .

Set that transformer's  $\Delta$ -side is of no-load while  $i'_{La}, i'_{Lb}, i'_{Lc}$  are all zero. Take the first equation in (1) for example; convert continuous differential equations into discrete difference-equations according to trapezoidal method and choose  $T$  as sample interval.

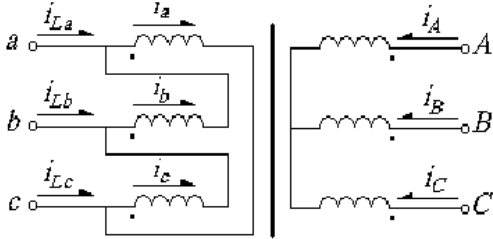


Fig. 1 Three-phase transformer with Y/ $\Delta$  connection

Then, the differential equation at time  $kT$  and  $(k+1)T$  has the following digital realization form:

$$\begin{cases} u'_{ab}(k) = u'_a(k) - u'_b(k) \\ u_{AB}(k) = u_A(k) - u_B(k) \\ u_{12}(k) = u'_{ab}(k) - u_{AB}(k) \\ i_{AB}(k) = i_A(k) - i_B(k) \end{cases} \quad (2)$$

$$u_{12}(k) = -i_{AB}(k)R - L_{2AB}(k+2) \frac{i_{AB}(k+1) - i_{AB}(k-1)}{2T} \quad (3)$$

$$u_{12}(k+1) = -i_{AB}(k+1)R - L_{2AB}(k+2) \frac{i_{AB}(k+2) - i_{AB}(k)}{2T} \quad (4)$$

where  $L_{2AB}$  is a calculated value of A-phase leakage inductance in Y-windings,  $u_{12}$  is the voltage

difference between A-phase and B-phase of both sides of transformer. The formula of leakage inductance at time  $(k+2)T$  can be formed combining (3) and (4):

$$L_{2AB}(k+2) = 2T[u_{12}(k)i_{AB}(k+1) - u_{12}(k+1)i_{AB}(k)] \cdot [i_{AB}(k+1)i_{AB}(k-1) + i_{AB}(k)i_{AB}(k+2) - i_{AB}^2(k) - i_{AB}^2(k+1)]^{-1} \quad (5)$$

The equations of leakage inductance and resistance at time  $(k+3)T$  can be obtained through the equations at time  $(k+2)T$ :

$$L_{2AB}(k+3) = 2T[u_{12}(k+1)i_{AB}(k+2) - u_{12}(k+2)i_{AB}(k+1)] \cdot [i_{AB}(k+2)i_{AB}(k) + i_{AB}(k+1)i_{AB}(k+3) - i_{AB}^2(k+1) - i_{AB}^2(k+2)]^{-1} \quad (6)$$

If the voltage at no-load side is given, the leakage inductance parameters at any time can be calculated through equation (5). However, in existing sustains, the no-load side voltages  $u'_a, u'_b$  and  $u'_c$  cannot be obtained, so this method still has problems in practical application. As a result, recurrence method in difference equation is adopted to calculate transformer leakage inductance under its no-load closing. Firstly, the resistance and leakage inductance of transformer under normal condition are utilized to calculate three voltage differences of adjacent time  $u_{12}(1), u_{12}(2)$  and  $u_{12}(3)$  at the initial instant of no-load closing.

$$\begin{cases} u_{12}(1) = -i_{AB}(1)R - L_2(i_{AB}(2) - i_{AB}(0))/2T \\ u_{12}(2) = -i_{AB}(2)R - L_2(i_{AB}(3) - i_{AB}(1))/2T \\ u_{12}(3) = -i_{AB}(3)R - L_2(i_{AB}(4) - i_{AB}(2))/2T \end{cases} \quad (7)$$

Plug three initial values  $u_{12}(1), u_{12}(2)$  and  $u_{12}(3)$  into equations (5) and (6) to calculate  $L_{2AB}(3)$  and  $L_{2AB}(4)$ , and calculate  $u_{12}(3)$  and  $u_{12}(4)$  through equation (7), and then calculate  $L_{2AB}(5)$  by plugging  $u_{12}(3)$  and  $u_{12}(4)$  into (5). After that,  $u_{12}(5)$  can be calculated by formula (7) and then  $L_{2AB}(6)$  can be figured out through  $u_{12}(4)$  and  $u_{12}(5)$ . So, the value of  $L_{2AB}$  at any time can be calculated by repeating the recursive process above. The leakage inductance  $L_{2AB}, L_{2BC}$  and  $L_{2CA}$  can be figured out by dealing with the other equations of (1) in the same way. The performance of no-load closing of transformer can be monitored on-line utilizing these three groups of leakage inductance. When winding deformation or internal short circuit faults occur to transformers, three groups of leakage inductance will differ from each other, which can be used to set protection criteria of transformer. The leakage inductance parameters of three winding three phase transformer

can be deduced according to the solving process of leakage inductance of two winding three phase transformer (and not explained here).

Since leakage inductance of different transformers vary greatly, the threshold is difficult to choose if the difference of leakage inductance is the only criterion to judge whether transformer break down inside, which bring troubles in practical application. For that reason, the author discovers that leakage inductance can be normalized after calculating leakage inductance of transformers of various types and parameters, which can round out the principle of transformer protection based on identifying magnetizing inrush current and internal fault current through leakage inductance.

Specific methods of normalization: calculate three groups of leakage inductance  $L_{2AB}$ ,  $L_{2BC}$  and  $L_{2CA}$  in real time, and choose the minimum as reference value 1, and then make three groups of leakage inductance do division with the minimum. The calculation formula is as follows:

$$\begin{cases} L_{\min} = \min(L_{2AB}, L_{2BC}, L_{2CA}) \\ L'_{2ij} = L_{2ij} / L_{\min} \end{cases} \quad (8)$$

where  $L_{2ij}$  denotes three groups of leakage inductance  $L_{2AB}$ ,  $L_{2BC}$ ,  $L_{2CA}$ .

### 3. Design of Protection Scheme

According to the analysis above, take two winding three-phase transformer as the example to set protection criteria under the condition of no-load closing of transformer. After the transformer's no-load closing, three groups of normalized leakage inductance  $L_{2AB}$ ,  $L_{2BC}$  and  $L_{2CA}$  can be estimated on-line by equations (5)-(8), and then the average values of leakage inductance  $L_{2ABE}$ ,  $L_{2BCE}$  and  $L_{2CAE}$  are calculated during 1/4 cycle after no-loading. Plug these three groups of average values into equation (9) to calculate  $\sigma^2$ .

$$\sigma^2 = \frac{100}{3} [(L_{2ABE} - L_{2BCE})^2 + (L_{2BCE} - L_{2CAE})^2 + (L_{2CAE} - L_{2ABE})^2] \quad (9)$$

Since leakage inductance have been normalized, 3 groups of average values  $L_{2ABE}$ ,  $L_{2BCE}$  and  $L_{2CAE}$  should be close to 1 and  $\sigma^2$  should be close to 0 under the condition of no-load closing of transformer of any type and parameter. When no-load closing on the condition of internal faults, the leakage inductance of fault phase and healthy phase differ from each other and  $\sigma^2$  should be greater than the value under normal circumstances. If  $\sigma^2$  is greater than  $\sigma_{zd}^2$ , decide that

transformer internal faults occur and protection act.  $\sigma_{zd}^2$  is set as 1 in this paper.

### 4. Testing Results and Analysis

The connection scheme of the dynamic testing system is as Fig.2 shown [4-6]; the test transformer in this system is three single-phase transformers adopting Y-d 11 connection.

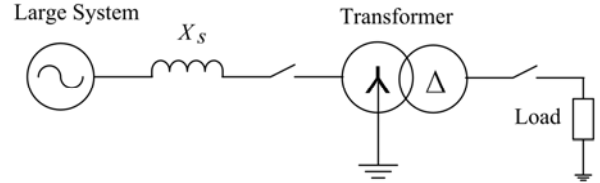


Fig. 2 Connection scheme of the dynamic testing system

Parameters of single-phase transformer is as follows: rated capacity is 10kVA; rated voltage of low voltage side is 380V and rated current is 25.3A; rated voltage of high voltage side is 1kV and rated current is 10A; no-load current is 1.45%; no-load loss is 1%; short-circuit loss is 0.35%; short circuit voltage is 9.0%~15.0%. 20 points are adopted at each power frequency cycle.

In Fig.3, the waveform of three-phase magnetizing inrush current is shown. Besides, the instantaneous leakage inductance and the normalized leakage inductance obtained by utilizing recurrence method in difference equation are also displayed in the figure.

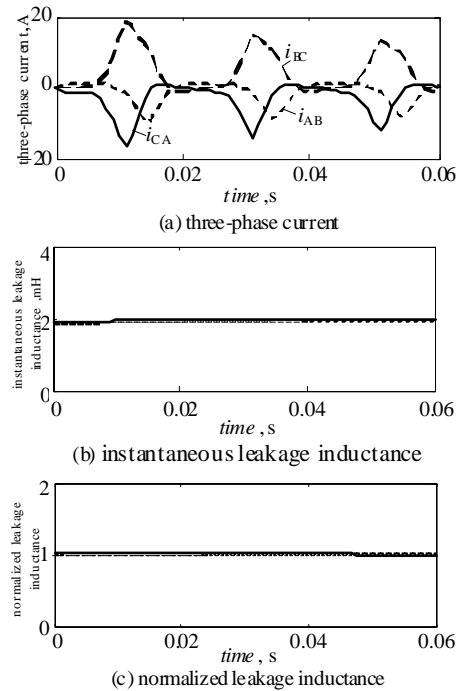


Fig. 3 The normalized leakage inductance under inrush condition

As Fig.3(b) shown, three-phase leakage inductance values are all close to the leakage inductance of transformer under normal condition and the value is 1.9mH. The normalized leakage inductance is shown in Fig.3(c). According to the protection scheme, some values can be obtained, that is  $L_{2ABE}=1.0272$ ,  $L_{2BCE}=1.0000$  and  $L_{2CAE}=1.0224$ . Plug these values into (12) to calculate  $\sigma^2$ , that is  $\sigma^2=0.0422$  and it is much less than setting value 1, so protections do not act reliably. By analyzing the other data of no-load closing of transformer, it can be found that three groups of normalized leakage inductance are mainly distributed near vertical coordinates 1 under inrush condition, and the difference is small. So, magnetizing inrush current can be identified by protection criteria effectively.

Three-phase differential current waveform of no-load closing of transformer under the condition of 9% turn-to-turn fault of A-phase is shown in Fig.4 (a). The curve of three-phase leakage inductance change over time, shown as Fig.4 (b), can be calculated by utilizing recurrence method in difference equation. At the initial moment of no-load closing, the curve has some fluctuations, but it can tend towards stability in a short time. The main reason is that the physical dimensions of windings do not change any more after the failure and leakage inductance tends towards stability.

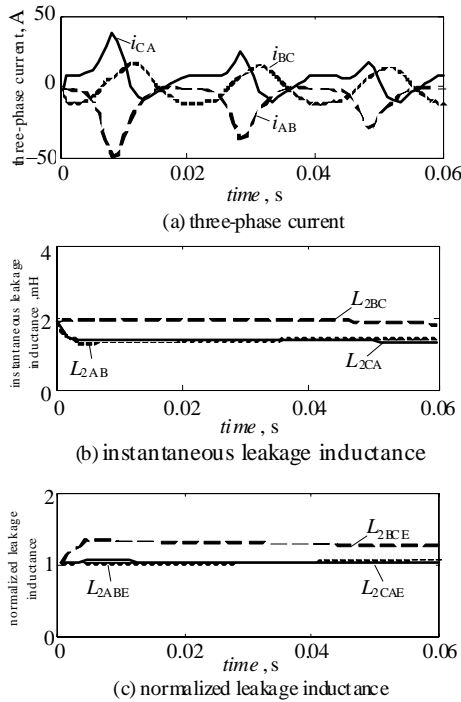


Fig. 4 The normalized leakage inductance under light internal fault condition

As shown in Fig.4 (b), normal phase leakage inductance value  $L_{2BC}$  is also close to the leakage inductance of transformer under normal condition and the value is 1.9mH, but the leakage inductances of faulted phases  $L_{2AB}$  and  $L_{2CA}$  are all less than normal leakage inductance. The normalized leakage inductance is shown in Fig.4(c). According to the protection scheme, some values can be obtained, that is  $L_{2ABE}=1.0000$ ,  $L_{2BCE}=1.3943$  and  $L_{2CAE}=1.0521$ . Plug these values into (12) to calculate  $\sigma^2$ , that is  $\sigma^2=9.1763$  and it is much larger than the setting value, so protections do act correctly. By analyzing the other data of no-load closing of transformer, it can also be found that three groups of normalized leakage inductance have a great difference. So, magnetizing inrush current can also be identified by protection criteria effectively.

Calculated results of  $\sigma^2$  when the transformer is switching on with no load are given in Tab. I. In Tab. I: A%9 indicates that transformer A-phase turn-to-turn fault occurs and faulted turns account for 9 percent of all A-phase turns; B%18 indicates that transformer B-phase turn-to-turn fault occurs and faulted turns account for 18 percent of all A-phase turns; C%18 indicates that transformer C-phase turn-to-turn fault occurs and faulted turns account for 18 percent of all C-phase turns. As Tab. I shows, the value of  $\sigma^2$  under internal fault condition when the transformer is switching on with no load is  $10^2$ - $10^3$  times larger than the value under magnetizing inrush current condition and the margin is big. If the setting  $\sigma_{zd}^2$  is set to 1, the protection can act reliably.

TABLE I

Calculated results of  $L_{MIN}$  and  $\sigma^2$  when the transformer is switched on with no load

operating conditions	$L_{2ABE}$	$L_{2BCE}$	$L_{2CAE}$	$\sigma^2$	$\sigma_1^2$	
normal	1	1.000	1.028	1.019	0.047	0.042
	2	1.018	1.027	1.000	0.038	0.041
	3	1.000	1.023	1.012	0.027	0.025
	4	1.014	1.000	1.019	0.020	0.023
	5	1.017	1.028	1.000	0.041	0.039
fault	A9%	1.000	1.397	1.058	9.229	9.587
	B18%	1.012	1.000	1.467	14.191	14.659
	C18%	1.470	1.000	1.009	14.435	14.133
	A	1.000	1.692	1.007	31.596	31.278
	C	1.665	1.000	1.004	29.318	29.126

The last column of Tab. I is the calculated results of  $\sigma_1^2$  by utilizing voltages and currents of both sides on condition that the voltages at no-load side are

given. Compare  $\sigma_1^2$  with calculated results of  $\sigma^2$  in this paper, it can be found that results of these two methods are very close under internal fault or normal condition when the transformer is switching on with no load. As a result, the method that utilizing recurrence method in difference equation to calculate transformer leakage inductance is correct and effective.

## 5. Conclusion

In this paper, for the disadvantages of transformer protection principle based on the loop equation in practical application, recurrence method in difference equation is proposed to calculate leakage inductance. On this basis, a new algorithm is proposed to identify magnetizing inrush current based on normalized leakage inductance. The method makes the setting of protection device, which has nothing to do with specific parameters and types of transformer, more universal. In addition, the method avoids influence of feed-in of ring current at  $\Delta$ -side and also has high sensitivity for slight fault. Dynamic simulation data identifies the correctness and effectiveness of the proposed method.

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