

OPTIMIZATION OF ELECTRICITY TARIFFS FOR PEAK LOAD BALANCING AT THE DOMESTIC LEVEL"

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Abstract: Domestic load balancing is a major concern for several countries, particularly where the demand is close to the available generation capacity. This is represented in the deregulated market by higher pricing during peak periods. Moreover, households face a vast assortment of increasing electricity prices and increased awareness for environmental sustainability.

The optimization of power electricity tariffs offered to consumers by their utilities is invariably the primary reason given for introducing competitive electricity markets. In This paper develops optimization model for residential electricity tariffs for demand response (DR) in order to maximize the consumers' gain by shifting to Economy7 (TOU) tariff rather than standard tariff. A linear programming, a detailed development of the Simplex method, and the problem of minimizing a linear cost function subject to linear equality and inequality constraints have been introduced. Results have been presented, and discussed.

Key words: Domestic load, Tariff, Demand Response, linear programming, load profile.

1. Introduction

Peak demand is a key issue in power supply system when demand exceeds the available capacity. Continuous growth in peak demand increases the risk of power failures, and increases the marginal cost of supply. In the UK the domestic sector accounts for almost one third of the total electricity consumption. It contributes the largest peak demand, particularly in the winter season, which has consequences on the power infrastructure [1]. The domestic electricity usage pattern varies depending on many determinants, such as weather, household composition, income, behaviour patterns of occupants; etc. The load of a typical appliance connected to the grid is fluctuating over time and depending on time of usage.

The vast variation among electricity consumption levels at peak hours and off peak hours not only has a large impact on energy cost and blackouts, but also on environment due to over prerequisite of the power grid and the resulting energy waste [14]. Electric Utilities are interested in reducing the peak demand by providing financial incentives to consumers to enable them to shift consumption away from peak demand hours. Under competitive electricity markets condition, if reasonable electricity tariff to consumers is determined, consumers could be encouraged to modify style of their consumption in response of financial incentives.

Demand response (DR), is defined as the changes in electricity usage by end use consumers from their usual usage patterns in response to changes of the electricity price over time. DR relates to the fact that the behaviour of occupants in their own homes result in significant changes in electricity load that are often highly correlated and thus have a considerable impact on the electricity supply system both locally and system wide.

Economy 7 tariff which is a type of Time of Use tariffs (TOU) is one of the significant tools of demand side management (DSM) to encourage consumers to adjust their usage during the high demand periods [2]. It is a cheaper night time electricity tariff which normally operates from around midnight where seven hours of low tariff electricity at night but slightly more expensive tariff throughout the day. In the UK, this is most effective for those consumers that use electric heating. This is because of the high load of heating. However, the majority of houses in the UK now have gas central heating systems. Currently in the UK, most domestic consumers, who have no electric heating, have slight or no incentive to shift their usage away from peak periods as they are charged at standard electricity tariff for their consumption regardless of time of use.

Dynamic energy pricing research could be grouped into two categories: profit maximization for utility companies or cost minimization for consumers. There have been numerous investigations dealing with domestic tariffs pricing and demand response (DR) [5-15].

This paper aims to develop optimization model for domestic electricity tariffs in the presence of load shifting, the results previously developed by the authors in [3, 4] is used.

2. Tariffs Structures

Today in UK, most of domestic consumers have been charged at standard electricity tariff for their electricity usage regardless of time of use. Standard tariff consists of a Standing charge tariff, a Tier 1 charge for agreed amount of usage, and a Tier 2 charge for the remainder. On the other hand, many UK suppliers have provided time of use (TOU) tariffs such as Economy 7 and Economy, where consumers are charged different rates depending on when they use electricity. To shift more energy consumption into the night, some main appliances such as, washing machines, tumble dryers, kettles or dishwasher cycle, electric boilers and heater might be configured to run during the night period tariff i.e. early morning. The results previously developed by the authors in [3, 4] is used to develop optimization model for electricity tariffs at domestic level.

The consumer's behavior has been examined in response to the tariff changes by calculating consumer's electricity bill under different electricity tariffs schemes which offered by different suppliers [16]. The different tariffs plan is shown in Tables 1 and 2.

Table 1 Economy 7 electricity tariffs rates (including VAT)

Brand	Supplier	
	X	Y
Tier 1 rate	23.80p/kWh for the first 1000 kWh	22.134p/kWh for the first 720 kWh
Tier 2 rate	11.21p/kWh for the remainder	13.288p/kWh for the remainder
Night Rate	5.03p per kWh	4.63p per kWh

Table 2. Standard tariffs (including VAT) offered by different suppliers

Supplier	1	2	3	4	5
Standin g Charge	0	8.421 p/day	0	0	13.301 p/ day
Tier 1	14.91 p/kWh First 720 kWh	11.88 5p/kWh First 728 kWh	19.91 p/kWh First 900 kWh	23.5380 p/kWh First 500 kWh	10.2250 p/kWh
Tier 2 for the rest	13.90 p/kWh	0	9.84 p/kWh	9.172 p/kWh	0

3. Household Profiles

A tool previously developed by the authors in [3] is used to obtain a better understanding of the effect of different tariff schemes on consumers' behaviour in the domestic sector. The tool which based on information, which is available in public reports and statistics was developed to generate a realistic half hourly electricity load profiles for different types of households for small local communities excluding the heating and hot water. Eight scenarios which present the most common occupancy pattern in the UK have been assumed according to household type (Table 3). The generated electricity load profile for an assumed particular community of 400 households giving the maximum, average and minimum daily possible values is shown in figure 1.

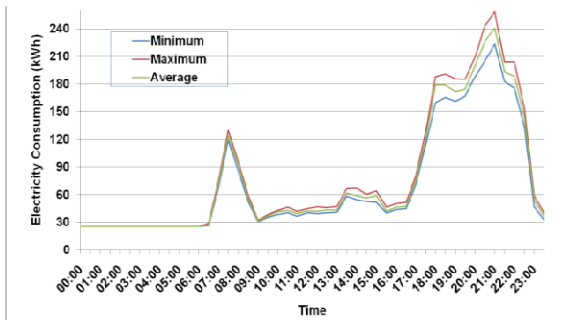


Fig. 1 Total energy consumption profile of the 400 household community [3]

Table 3 UK households type scenarios

Scenario	Household Type
1	Single working adult
2	two retired
3	Single adult with children
4	two working adults
5	two adults with children
6	two retired
7	three adults or more
8	three adults or more with children

4. Linear Programming

Linear programming is a powerful technique for dealing with the problem of allocating limited resources among competing activities. A linear programming problem could be defined as the problem of maximizing or minimizing a linear function subject to linear constraints. The constraints may be equalities or inequalities. Any linear optimization model can be written as to maximize or minimize the objective function subject to (abbreviated s.t.) a set of constraints. In general, the standard form of an LP-problem can be written in more efficient notation as:

$$\text{Maximize or Minimize } Z = \sum_{j=1}^n c_j x_j \quad (1)$$

Subject to:

$$\sum_{j=1}^n a_{ij} x_j \leq \text{or } \geq \text{or } = b_i \quad (i = 1, 2, \dots, m). \quad (2)$$

$$x_j \geq 0 \quad (j = 1, 2, \dots, n). \quad (3)$$

This can be written in the following matrix form:

$$\text{Min /Max } \mathbf{C}^T \mathbf{X}$$

$$\text{S.t. } \mathbf{A} \mathbf{X} = \mathbf{b}$$

$$\mathbf{X} \geq \mathbf{0}$$

Where: $\mathbf{X} = (x_1, \dots, x_n)^T$, $\mathbf{C} = (c_1, \dots, c_n)^T$, $\mathbf{b} = (b_1, \dots, b_m)^T$, and $\mathbf{A} = [a_{ij}]$ is an $(m \times n)$ matrix, of real numbers

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

There are many methods to solve the linear programming problem, such as graphical method, trial and error method, vector method and simplex method. Though we use graphical method for solution when we have two problem variables, the other method can be used when there are more than two decision variables in the problem.

5. Formulating a Mathematical Model for the Consumer's Electricity Bill

Consider the problem of setting tariff, where setting of economy 7 electricity tariff needs to be established to

offers financial incentives to domestic consumers who agree to reduce their energy usage when energy demand is high. The form of the electricity tariff structure is one of the first considerations in any optimization problem which involves minimizing electricity costs.

In our case, a mathematical model was developed to calculate the electricity bills for each household type under both standard tariff (S) and Economy 7 tariff (E). Then, the difference between the bills under tiered rates (standard tariff) and the Economy 7 tariff with load shifting was calculated to assist decision making in resetting the economy 7 tariffs.

The key decision is to minimize the lose by shifting to Economy 7 by finding out how much the electricity rates should be paid by the consumers to save in their electricity bills under economy7 tariff compared to standard tariff.

5.1. Consumer's Electricity Bill under Standard Tariff scheme

If

Q_F = Tier 1 consumption limit (KWh)

Q_2 = Tier 2 extra usage above tier 1 (KWh)

x_1 = Tier 1 rate (p/KWh),

x_2 = Tier 2 rate (p/KWh)

Then the total monthly bill under standard tariff (S) will be:

$$S = Q_F \cdot x_1 + Q_2 \cdot x_2 \quad (4)$$

And the total electricity consumption (Q) will be

$$Q = Q_F + Q_2 \quad (5)$$

5.2. Consumer's Electricity Bill under Economy 7 Tariff scheme

(a) Without load shifting

If

Q_N = elec. consumed during night period (KWh)

Q_4 = Tier 2 extra usage above tier 1 (KWh)

x_3 = Tier 1 rate (p/KWh),

x_4 = Tier 2 rate (p/KWh),

x_5 = Night rate (p/KWh),

then the total monthly bill without load shifting under Economy 7 will be

$$E = Q_F \cdot x_3 + Q_4 \cdot x_4 + Q_N \cdot x_5 \quad (6)$$

And the total electricity consumption will be

$$Q = Q_F + Q_4 + Q_N \quad (7)$$

(b) With load shifting

If

Q_{sh} = part of tier2 load shifted to night period(KWh),
then the new electricity bill under Economy 7 with load shifting will be

$$E_{sh} = Q_F \cdot x_3 + (Q_4 - Q_{sh}) \cdot x_4 + (Q_N + Q_{sh}) \cdot x_5 \quad (8)$$

If

$Q_4 = KQ_2$ Then $Q_N = (1 - K)Q_2$

And if $Q_{sh} = yQ_4 = KyQ_2$

Then

$$E_{sh} = Q_F \cdot x_3 + (1 - y)kQ_2 \cdot x_4 + (1 - k + ky)Q_2 \cdot x_5 \quad (9)$$

The difference between the consumer's bill under both Economy 7 tariff with load shifting and standard tariff is obtained in equation 10.

$$E_{sh} - S = Q_F \cdot (x_3 - x_1) - Q_2 x_2 + (1 - y)kQ_2 \cdot x_4 + (1 - k + ky)Q_2 \cdot x_5 \quad (10)$$

Simplifying, we obtain:

$$E_{sh} - S = Q_F \cdot (x_{31}) + Q_2(x_{52} + kx_{45} - kyx_{45}) \quad (11)$$

Where, $x_{31} = x_3 - x_1$, $x_{45} = x_4 - x_5$, $x_{52} = x_5 - x_2$

Using the tariff schemes offered by different suppliers shown in tables1 and 2, the range of variables are:

$$0 \leq x_{31} \leq 7$$

$$-9 \leq x_{52} \leq 0$$

$$0 \leq x_{45} \leq 10$$

$$0 \leq ky \leq 0.25$$

Where k is known for different scenarios, and ky is fixed from a range [0 - 0.25].

The Aim is to maximize the gain (Equation 11) by shifting to Economy 7 plan.

Also since change to Economy should result in no change in bill when ky is zero, so we have:

$$Q_F \cdot (x_{31}) + Q_2(x_{52} + kx_{45}) = 0 \quad (12)$$

The values of Q_F , Q_2 and K for the eight scenarios are summarized in Table 4.

Table 4. Values of Q , Q_F , Q_2 and K

	Q(KWh)	Q_F (KWh)	Q_2 (KWh)	K
Scenario. 1	145	60	85	0.59
Scenario. 2	147	60	87	0.62
Scenario. 3	383	60	323	0.63
Scenario. 4	246	60	186	0.57
Scenario. 5	397	60	337	0.62
Scenario. 6	251	60	191	0.66
Scenario. 7	369	60	309	0.62
Scenario. 8	418	60	358	0.66

Since the two adults, and the two adults with children (scenario 5) households represent about half of the energy shifted. As such, the utility could focus marketing literature, and any incentives to such consumers. To summarize, we have the following problem formulation for the fifth scenario:

$$E_{sh} - S = 75x_{31} - 329.7x_{25} + 182.547x_{45} \quad (13)$$

6. Optimization Model

The objective is to minimize the function represented in equation (13). The basic problem is to find a rate

design that promotes consumers to take actions to conserve, without undermining supplier's ability to recover its legitimate costs of operation (to ensure the supplier will not lose): $S = E$.

This gives us the complete model of this problem:

$$\begin{aligned} &\text{Minimize} \quad 75x_{31} - 329.7x_{25} + 182.547x_{45} \\ &\text{Subject to:} \\ &1x_{31} + 0x_{25} + 0x_{45} \leq 7 \\ &0x_{31} + 1x_{25} + 0x_{45} \leq 9 \\ &0x_{31} + 0x_{25} + 1x_{45} \leq 10 \\ &75x_{31} - 329.7x_{25} + 264.7x_{45} = 0 \\ &75x_{31} - 171x_{25} + 126.5x_{45} = 0 \\ &x_{31}, x_{25}, x_{45} \geq 0 \end{aligned}$$

The resulting minimization problem is formulated as a linear function of parameters; it can be solved by linear programming (LP) methods.

In this study, the computer package named LINDO (Linear Interactive and Discrete Optimizer) has been used to get the solution for the decision problems. The LP objective function value and the outputs from LINDO Optimization Software are reported in Table 5.

From Table 5, it can be seen that the LINDO package found the optimum solution after 3 iterations (pivots) and an optimum solution has been arrived at with $x_{31} = 2.988$, $x_{25} = 8.708$, $x_{45} = 10$ and Minimize $Z = 821.2498$.

Table 5. Optimal solution of the model

LP OPTIMUM FOUND AT STEP 3		
OBJECTIVE FUNCTION VALUE		
1) -821.2498		
VARIABLE	VALUE	REDUCED COST
X31	2.988094	0.000000
X25	8.708228	0.000000
X45	10.000000	0.000000
ROW	SLACK OR SURPLUS	DUAL PRICES
2)	4.011906	0.000000
3)	0.291772	0.000000
4)	0.000000	82.124985
5)	0.000000	-1.000000
6)	0.000000	0.000000
NO. ITERATIONS= 3		

7. Optimization of Electricity Tariffs

For electricity tariffs to be effective, it is essential that the tariffs offered is designed in a way as to adequately motivate consumers to change their electricity usage behaviour.

In order to propose a new rates design, the dataset which was derived from the optimum solution achieved in the last section and standard tariff rates used by the

authors in [4] is used for the calculation. Based on the optimum solution shown in table 5 and the value of Standard tariff tier 1 rate (x_1), the results of the proposed rates are shown below:

For: x_1 (Standard tariff Tier 1 rate) = 19.91 p/KWh, the remaining rates are calculated as follows:

x_3 (Economy 7 tariff Tier 1 rate) = $19.91 + 2.98815 = 22.89809$ p/KWh

x_5 (Night rate) = $13.15 - 8.708 = 4.441$ p/KWh

x_2 (Stand. tariff Tier 2 rate) = $8.708 + 4.441 = 13.15$ p/KWh

x_4 (Econ. 7 tariff Tier 2 rate) = $10 + 4.441 = 14.441$ p/KWh

8. Consumer's Electricity Bill under a New Tariff Scheme

The calculation performed was intended to determine if households would benefit naturally from the adoption of the new economy 7 tariffs scheme. To answer this question, a comparison between consumer's bills on the electricity consumption data from each household type under the standard tiered rates and new designed economy 7 rates was performed as shown in table 6.

Positive difference indicates that the expected economy 7 tariff bill would be lower than the actual standard tariff and thus indicates that the household is expected to benefit under the new economy 7 tariff. Similarly, negative differences indicate that the expected bill under economy 7 tariff would be higher than the actual tiered costs, and thus the household would not be expected to benefit naturally under the economy 7. From table 4 it can be seen that, with no load shifting, the single adult, single retired and two retired households benefit naturally from the new rate of economy 7 tariff while the electricity bills under both tariff schemes for two adults, two adults with children and three adults or more with children households are the same. Also from table 6, it is clear that the single adult with children and three adults or more households would be expected to benefit naturally from the new rate of economy 7 tariff.

If the load is now shifted from Tier 2 to night time, there could be possible savings. Figures 2 shows that, for each scenario, the savings compared with standard tariff (negative indicates loss). The results indicate that, a minimum of 20% load shift is required for single adult and single retired households to benefit from adoption of economy 7 tariff. However, a load shift of about 5% is sufficient for scenario 6 (two retired household) to benefit.

Table 6 Overall bill savings with daily load shifting

% of Tier 2 load shifted to night	Scenarios							
	1	2	3	4	5	6	7	8
	Percentage Savings							
0	-4	-4	1	0	0	-2	1	0
10	-2	-2	5	3	5	2	5	4
15	-1	-1	7	5	7	4	7	7
20	0	0	10	7	9	6	9	9
25	1	1	12	8	11	7	12	11
30	2	2	14	10	14	9	14	14
35	3	3	16	12	16	11	16	16
40	4	4	18	14	18	13	18	18
45	5	5	21	15	20	15	20	21
50	6	6	23	17	23	16	22	23

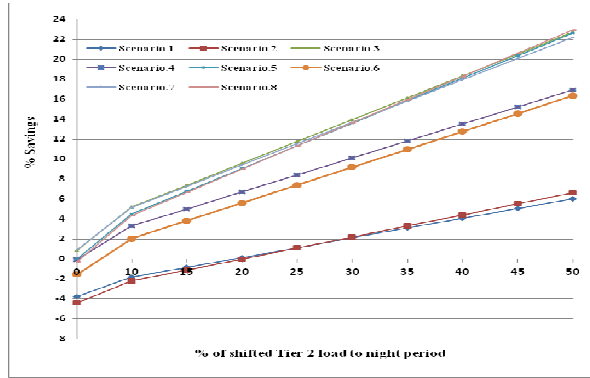


Fig 2. Percentage savings in consumer's bill using a new Economy 7 tariff compared to Standard tariff

The previous calculations were all done by numerically shifting the load. However, in practice, it is the usage of appliances that has to be shifted by behaviour. In this section we examine shifting the usage of appliances such as washing machines, tumble dryers, kettles and irons to run a bit earlier in the morning (before 8am) than what was previously understood of the community simulated using the proposed rates. Table 7 shows the shifts and gains made with various changes in appliance usage. The table shows that significant amount of change in behaviour from the consumer, the total amount of load shift was only around 23%. At this level all consumers get a benefit ranges from around 2 to about 12 percent.

Table 7. Percentage savings in consumer's bill using economy 7 tariff compared to standard tariff for load shifting of (i) Washing Machine (W/M) (ii)W/M + Dryer (iii)W/M + Dryer + Iron (iv)W/M + Dryer + Iron + 0.3 kettle (v)W/M + Dryer + Iron + 0.3 kettle + Vac. Cleaner

Load shift	I	ii	iii	iv	v
% of Tier 2 load shifted to night	9	17	20	21	23
Scenarios	% Savings				
1	-1.4	0.4	1.3	1.6	2.1
2	-2.5	-0.6	0.7	1.4	1.9
3	4.6	9.0	10.4	10.8	11.6
4	2.5	4.9	6.3	6.6	7.2
5	3.5	7.8	9.1	9.5	10.2
6	1.7	3.7	5.3	5.2	5.5
7	5.1	8.8	10.1	11.0	11.8
8	3.3	7.6	8.7	9.6	10.2

The benefit to the utility would be dependent on what rates they actually purchase the electricity. If the cost is significant, then there would be an opportunity to invest in trying to change the behaviour of the consumers. However, the focus of any attempt should be more focused on the type of households where the benefit would be significant. The two (sc. 4) adults household and the two adults with children household (sc. 5) represent about half the energy shifted [4]. As such, the utility could focus

marketing literature, and any incentives to such consumers.

If the load is now shifted from Tier 2 to night time for scenarios 4 and 5, there could be possible savings. Using a standard tariffs scheme as baseline, the new proposed tariff and old tariff are compared. Figure 3 shows for scenarios 4 and 5, the savings compared with standard tariff (negative indicates loss). The graph indicates that the new designed rates for electricity tariffs seem to be effective, in old rates; a minimum of 25% load shift is required. However, new rates, the households would benefit naturally from the adoption of new rates.

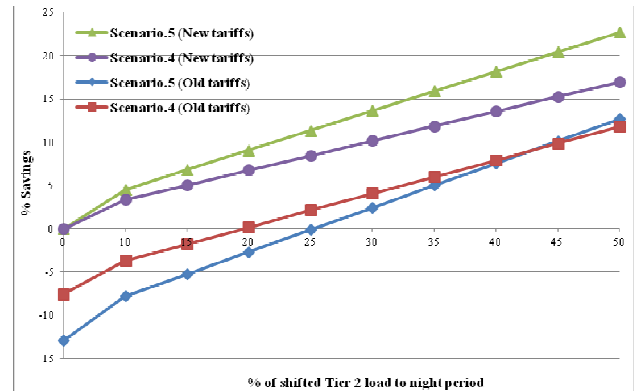


Fig 3. Percentage savings in consumer's bill using new proposed tariffs and old tariffs using a standard tariffs scheme as baseline

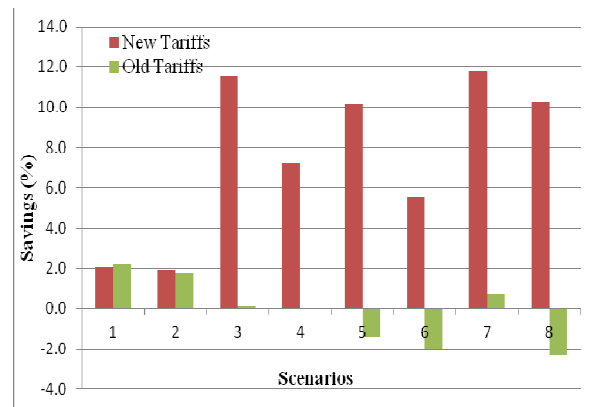


Fig. 4 Percentage savings in consumer's bill using new proposed tariffs and old tariffs for the eight scenarios

Figure 4 shows the percentage savings in electricity bill for the selected scenarios using scheme (v) of load shifting. It can be seen that, at the old tariffs level and with 23% of tier 2 load shifted to night period only a few consumers get a slight benefit. However, the adoption of a new proposed rate using scheme (v) of load shifting would benefit all consumers.

9. Conclusions

- A model for optimization of residential electricity tariffs in the presence of load shifting including problem formulation and solution were presented.
- The use of a local community simulator to investigate the Optimization of electricity tariffs for Peak Load

Balancing at the Domestic Level and the use of economy 7 tariffs as an incentive to generate demand response and showing its impact on consumers' behaviours in the UK domestic buildings has been presented.

- The analysis helps to determine the suitability of adopting demand response in the domestic sector.
- Current tariffs are not sufficient to change consumer behaviour at peak times as there is little benefit to them in terms of the bill.
- The results enable consumers to increase their awareness for electricity consumption and helps to save electricity as well as to decrease the amount of money spent for electricity
- The results enable electricity suppliers to focus on which particular types of households to market load shifting techniques.
- This area requires further study.

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