

TUBULAR LINEAR OSCILLATORY MACHINE FOR DRILLING APPLICATIONS

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Abstract: Tubular linear permanent magnet oscillatory machine is presented in this paper. It can transfer electric power to rock directly without any mechanical media. Based on equivalent model of the machine, mathematical model considering frictions is built by analyzing the forces for both piston and casing. The frequency of input voltage is given according to linearization of mathematical model. The different operating modes are defined on the basis of displacement of casing. Dynamics analysis is done by supplying sinusoidal voltage. Simulation results show that the machine can output all kinetic energy to rock during collision instant and it will be a good option for drilling application.

Key words: Linear machine, oscillatory machine, drilling application.

I. INTRODUCTION

Linear oscillatory machine is widely used in linear drive systems in the last decade and is playing an important role in applications of compressors, reciprocating pumps and renewable marine energy conversions [1]-[8]. The resonance frequency is analyzed for linear oscillatory motor based on both large power and good efficiency in [1]. The details of frequency, efficiency, system constants, forces, and control philosophy of linear oscillatory machine are analyzed in [2]-[6]. Transient forces calculation method for linear oscillatory motor is carried out in [7]. The linear oscillatory machine can be used as a generator in renewable marine application [8]. The tubular linear oscillatory machine presented in this work has the feature on driving loads directly without any mechanical transfer media, such as gear boxes, eccentrics, and crank-shafts. Its oscillatory motion is especially suitable for hard rock drilling application due to high energy density during collision instant.

In this work, the tubular linear oscillatory machine with internal permanent magnets and its dynamics are studied; and resonance frequency, its energy

transfer process and losses are analyzed. Finally, control philosophy based on collision is proposed.

II. TOPOLOGY AND EQUIVALENT MODEL

The topology of proposed tubular linear permanent magnet oscillatory machine is shown in Fig. 1. The functions of different components are listed in Table I. The equivalent model of tubular linear electric machine is shown in Fig. 2. For the purpose of simplification, we use dampers and equivalent spring to denote the frictions and gas springs respectively. Therefore, two mass-spring-damper subsystems are obtained: one is piston-gas springs-damper subject to electromagnetic force; another is casing-external spring-damper subject to the net force from piston-gas springs-damper subsystem. The whole system has a constant speed to the rock in order to get continuous drilling. That is to say, each collision can make more space between drill bit and rock. The whole system moves downward a little to cover this space such that next collision will come to the new surface of rock at the same situation as last collision.

We define middle point of the piston as the origin for casing and piston as shown in Fig. 2. All the other variables are based on these two variables. Fig. 3 shows an equivalent circuit of stator winding. The resistance and inductance of coil can be measured. The back EMF is induced when the relative motion between piston and casing occurs. The classical mass-spring-damper system is shown in Fig. 4. The mass is subject to an external force f . We define x as the displacement from a reference position and write Newton's law of motion:

$$m \cdot \ddot{x} = f - f_f - f_{sp} = f - b\dot{x} - kx \quad (1)$$

where b is friction coefficient and viscous friction force is only considered here, k_{sp} is stiffness coefficient of spring.

Table I

Functions of different component of prototype

Component	Function
Drill string	Holding the whole drilling system including external spring, casing, and the components inside casing.
External spring	Two functions: one is to connect drill string and casing, another is to work as a key part of mass-spring-damper subsystem. Here, mass, spring and damper are mass of casing, external spring and friction between casing and surroundings respectively.
Casing	Providing both a frame for stator winding and a smooth surface for piston. It has two ends which are used to seal the whole unit.
Permanent magnet	Providing the magnetic field for stator winding.
Iron	A part of magnetic circuit.
Stator winding	Engine coil for the whole system. The movement relative to piston will be obtained when the stator winding is carrying current.
Gas springs	Important parts for inner mass-spring-damper subsystem. Here, mass, spring, and damper are mass of piston, gas springs, and friction between piston and casing respectively.
Drill bit	Key part for hitting the rock.

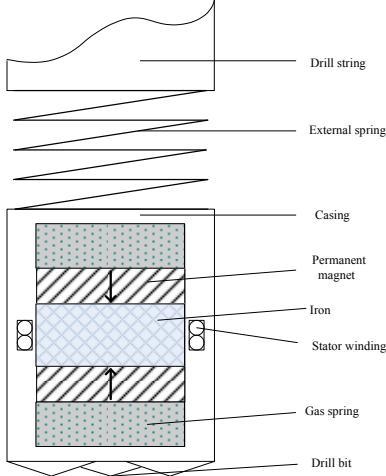


Fig. 1. Tubular linear electric machine system

If the external force is a periodic force, i.e. $f = F_m \sin(\omega t)$, the solution of (1) under the condition of $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$ is shown as follows:

$$x(t) = A \cdot e^{-\frac{b}{2m}t} \cdot \sin\left(\frac{\sqrt{4 \cdot m \cdot k_{sp} - b^2}}{2 \cdot m} \cdot t + \alpha\right) + B \cdot \sin(\omega \cdot t + \delta) \quad (2)$$

where:

$$A = \frac{x_0}{\sin \alpha} = \sqrt{\left(\frac{4 \cdot m \cdot k_{sp} - b^2}{4 \cdot m^2} \cdot x_0^2\right) + \left(\dot{x}_0 + \frac{b}{2 \cdot m} \cdot x_0\right)^2},$$

$$\alpha = \tan^{-1} \frac{\sqrt{4 \cdot m \cdot k_{sp} - b^2} \cdot x_0}{\dot{x}_0 + \frac{b}{2 \cdot m} \cdot x_0},$$

$$B = \frac{F_m}{m \cdot \sqrt{\left(\frac{k_{sp}}{m} - \omega^2\right)^2 + \frac{b^2}{m^2} \cdot \omega^2}}, \delta = \tan^{-1} \frac{-b \cdot \omega}{k_{sp} - m \omega^2}.$$

Here, B is a function of ω . By solving $\frac{dB}{d\omega} = 0$, the resonance frequency is obtained

$$\omega = \sqrt{\frac{k_{sp}}{m} - \frac{b^2}{2 \cdot m^2}} \approx \sqrt{\frac{k_{sp}}{m}}. \quad (3)$$

The coefficient of friction in (3) is neglected when it is small enough.

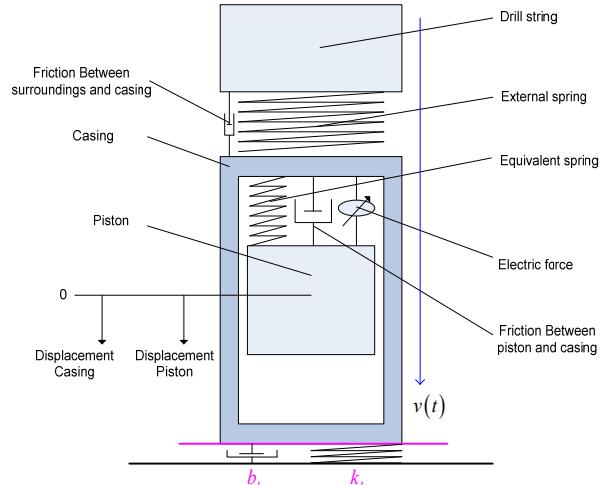


Fig. 2. Equivalent model of tubular electric machine system

Equivalent circuit of stator winding

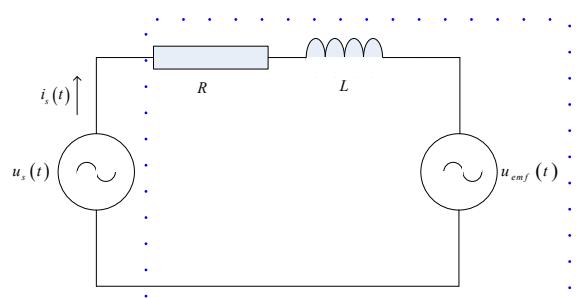


Fig. 3. Equivalent electric circuit

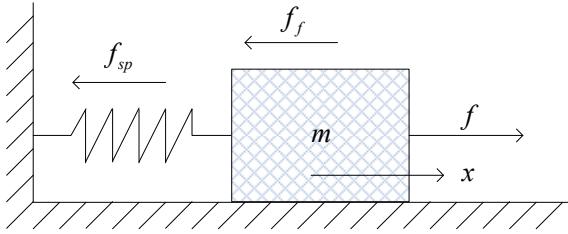


Fig. 4. Mass-spring-damper system

Equations (2) and (3) show that the frequency of external force must be same as the natural frequency in order to achieve the biggest stroke length. If not, small stroke length is obtained. The input voltage must be sine wave. This mass-spring-damper will have sinusoidal motion if we supply any ac input voltage. In principle, we can input ac voltage with square wave form. If so, we can get the Fourier series of square wave. Only the first-order component of input voltage can push the mass, the other components with high frequencies (harmonics) are no contribution to the motion. But these harmonics introduce many losses to heat the stator winding. Therefore, the whole system has poor efficiency because of harmonics.

III. MODELING

For simplifications of analysis, we assume:

- 1) Gas leakage between two gas chambers is zero.
- 2) Flux density is constant and electromagnetic force is only the function of current.
- 3) Neglect gravity.

A. Gas springs

Gas spring has the characteristic of $P \cdot V^\alpha = \text{Constant}$. Actually, gas springs are nonlinear components shown in (4).

$$f_{\text{gas}}(y) = A \cdot p_0 \cdot \left[\left(\frac{y_g}{y_g - y} \right)^\alpha - \left(\frac{y_g}{y_g + y} \right)^\alpha \right]. \quad (4)$$

where $A = \pi D^2 / 4$ is cross area of gas chamber, D is diameter of gas chambers, p_0 is initial pressure of gas springs, y_g is length of each gas chamber, y is displacement of piston from the origin. The characteristic curve of gas springs is shown in Fig. 5.

$$f_f(v) = F_N \cdot \left(\mu_s \cdot \tanh(s \cdot v) - \frac{1}{2} (\mu_s - \mu_c) \cdot \left(\tanh\left(s \cdot \left(v - \frac{6.5}{s}\right)\right) + \tanh\left(s \cdot \left(v + \frac{6.5}{s}\right)\right) \right) + \mu_v \cdot v \right) \quad (6)$$

B. Piston

Forces analysis for piston is shown in Fig. 6. The piston is subject to three forces: 1) electromagnetic force f_e ; and 2) friction force f_{fp} .

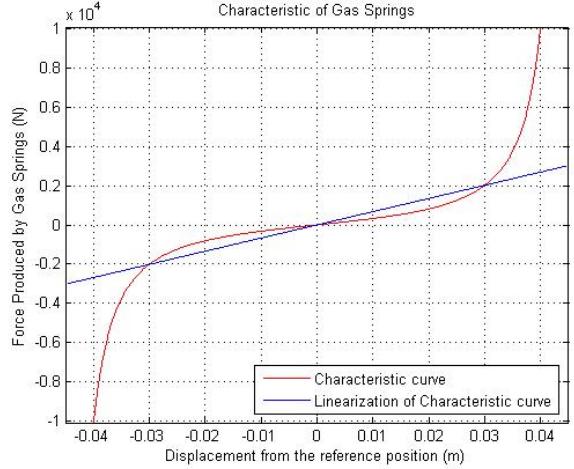


Fig. 5. Characteristic curve of gas springs

friction force f_{fp} between piston and casing; and 3) gas spring force f_{gas} . The motion equation of piston in the vertical direction is obtained as follows by using Newton's second law of motion:

$$\begin{aligned} m_p \cdot \ddot{y}_p(t) &= f_{\text{netp}}(t) \\ &= f_e(t) - f_{fp}(\dot{y}(t)) - f_{\text{gas}}(y(t)) \end{aligned} \quad (5)$$

where m_p is mass of piston, $f_e(t)$ is electromagnetic force between stator winding and permanent magnet, $f_{fp}(\dot{y}(t))$ is friction force between casing and piston, $\dot{y}(t) = \dot{y}_p(t) - \dot{y}_c(t)$ is piston velocity relative to casing, $y(t) = y_p(t) - y_c(t)$ is piston displacement relative to casing, $y_p(t)$ is displacement of piston, $y_c(t)$ is displacement of casing. Here, friction force includes three parts: static friction force, coulomb friction force, and viscous friction force. The friction force equation is given as function of velocity as follows [9]:

where μ_s , μ_c and μ_v are static, coulomb, and viscous friction coefficients respectively, s is the slope, and F_N is normal force.

C. Casing

Force analysis for casing is shown in Fig. 7. Casing is subjected to four forces shown in Fig. 7: 1) counter force from piston-gas springs-damper subsystem; and 2) friction force between casing and surroundings; and 3) external spring force; and 4) counter force from rock due to collision. The motion equation of casing in vertical direction is obtained as follows by using Newton's second law of motion:

$$m_c \cdot \ddot{y}_c(t) = f_{netc}(t) = -f_{netp}(t) - k_{ext} \cdot y_c(t) - f_{fc}(y_c(t)) - d \cdot f_c(t) \quad (7)$$

where m_c is mass of casing, k_{ext} is stiffness coefficient of external spring, $k_{ext} \cdot y_c(t)$ is external spring force, $f_{fc}(y_c(t))$ is friction force shown in (6) between casing and surroundings, $d \cdot f_c(t)$ is a counter force from rock due to collision and

$$d = \begin{cases} 1 & \text{during collision} \\ 0 & \text{no collision} \end{cases} .$$

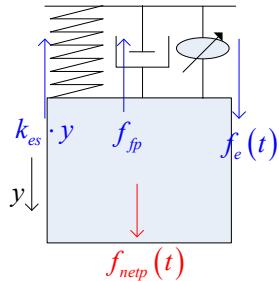


Fig. 6. Forces analysis for piston

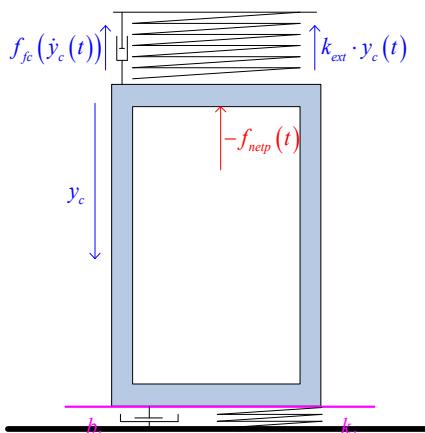


Fig. 7. Forces analysis for casing

D. Rock

Spring connected with damper in parallel is used as the equivalent model of the rock during the collision period. Different loads will be obtained by changing the coefficients of spring and damper. The force equation of equivalent model is as follows:

$$f_c(t) = k_L \cdot y_c(t) + b_L \cdot \dot{y}_c(t) \quad (8)$$

In the work, two different collisions are taken into account:

- 1) Completely inelastic collision ($k_L = 0$ and $b_L = \text{constant}$), only momentum of the two-body system is conserved;
- 2) Elastic collision ($k_L = \text{constant}$ and $b_L = \text{constant}$), both momentum and kinetic energy of the two-body system is conserved.

E. Electromagnetic force

The expression for Lorentz force is

$$F = IL \times B . \quad (9)$$

where F is force measured in Newton, I is current measured in Ampere, B is flux density measured in Tesla, \times is vector cross product, L is the length of wire measured in meter, and whose direction is along the wire, aligned with the direction of current flow. The electromagnetic force is obtained from Lorentz force equation as follows:

$$f_e(t) = B \cdot l \cdot i_s(t) = B \cdot \pi \cdot D \cdot i_s(t) = k_e \cdot i_s(t) \quad (10)$$

where $k_e = B \cdot \pi \cdot D \cdot N$, B is flux density, D is diameter of winding, N is number of turns of stator winding.

F. Electric circuit

By using Kirchhoff's voltage law (KVL) for the circuit in Fig. 3, the voltage equation is obtained as follows:

$$u_s(t) = R \cdot i_s(t) + L \cdot \frac{di_s(t)}{dt} + u_{emf}(t) \quad (11)$$

The current equation is obtained as follows:

$$\dot{i}_s(t) = \frac{1}{L} \cdot [u_s(t) - R \cdot i_s(t) - u_{emf}(t)] \quad (12)$$

Back EMF induced in stator winding is

$$u_{emf}(t) = B \cdot l \cdot \dot{y}(t) = B \cdot \pi \cdot D \cdot N \cdot \dot{y}(t) = k_e \cdot \dot{y}(t) \quad (13)$$

G. System model

By collecting (4)~(13) and defining state variables as $x_1(t) = y_p(t)$, $x_2(t) = \dot{y}_p(t)$, $x_3(t) = y_c(t)$, $x_4(t) = \dot{y}_c(t)$, $x_5(t) = i_s(t)$, we can get the mathematic model as follows:

$$\begin{cases} \dot{x}_1(t) = \frac{1}{m_p} \cdot [k_e \cdot x_5(t) - f_{gas}(x_2(t) - x_4(t)) - f_{fp}(x_1(t) - x_3(t))] \\ \dot{x}_2(t) = x_1(t) \\ \dot{x}_3(t) = \frac{1}{m_c} \cdot \left\{ -k_e \cdot x_5(t) + f_{gas}(x_2(t) - x_4(t)) + f_{fp}(x_1(t) - x_3(t)) \right. \\ \quad \left. - k_{ext} \cdot x_4(t) - f_{fc}(x_3(t)) - d \cdot [k_L \cdot x_4(t) + b_L \cdot x_3(t)] \right\} \\ \dot{x}_4(t) = x_3(t) \\ \dot{x}_5(t) = \frac{1}{L} \cdot \{u_s(t) - R \cdot x_5(t) - k_e \cdot [x_1(t) - x_3(t)]\} \end{cases} \quad (14)$$

where

$$f_{fp}(x_1(t) - x_3(t)) = F_{Np} \cdot \left(\mu_{sp} \cdot \tanh(s_p \cdot (x_1(t) - x_3(t))) - \frac{1}{2}(\mu_{sp} - \mu_{cp}) \begin{pmatrix} \tanh\left(s_p \cdot \left(x_1(t) - x_3(t) - \frac{6.5}{s_p}\right)\right) \\ + \tanh\left(s_p \cdot \left(x_1(t) - x_3(t) + \frac{6.5}{s_p}\right)\right) \end{pmatrix} + \mu_{vp} \cdot (x_1(t) - x_3(t)) \right)$$

$$f_{fc}(x_3(t)) = F_{Nc} \cdot \left(\mu_{sc} \cdot \tanh(s_c \cdot x_3(t)) - \frac{1}{2}(\mu_{sc} - \mu_{cc}) \begin{pmatrix} \tanh\left(s_c \cdot \left(x_3(t) - \frac{6.5}{s_c}\right)\right) \\ + \tanh\left(s_c \cdot \left(x_3(t) + \frac{6.5}{s_c}\right)\right) \end{pmatrix} + \mu_{vc} \cdot x_3(t) \right).$$

H. Frequency of input voltage

According to the oscillation analysis in chapter two, the frequency of input voltage must be same as natural frequency of mass-spring-damper system. We can get linearized model by neglecting static and coulomb friction forces and linearizing the other nonlinear components in (14). By analyzing linearized model, the estimation of natural frequency of real system is

$$\omega \approx \sqrt{\frac{k_{gas}}{m_p}} = \sqrt{\frac{k_{ext}}{m_c}}. \quad (15)$$

where k_{gas} is stiffness coefficient of equivalent spring of gas springs shown in Fig. 5. Here, the damper influence is neglected because the damper coefficient is decreased as small as possible from designing.

IV. DYNAMIC PROCESS ANALYSIS

A. Definitions of operating modes

We define that this machine has four operating

modes shown in Fig. 8, which is for no load mode. We assume that only completely inelastic collision occurs in each collision for different operating modes. And we define the starting point of one period is t_a . The collision can occur when the casing is moving downward at the period of $[t_a, t_c]$ and can not occur at the period of

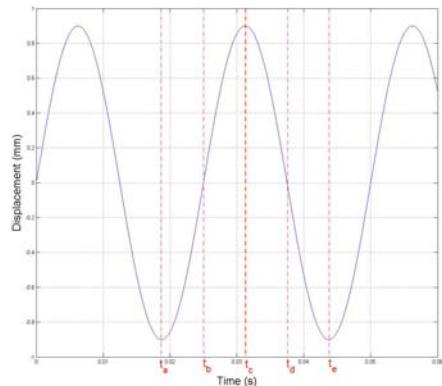


Fig. 8. Operating mode analysis

$[t_c, t_e]$ because the casing is moving up. According to different instants collision occurs, The four operating modes are: no load mode, light load mode, full load mode, and over load mode.

No load mode

There is no collision in this mode when the casing oscillates. The input power only covers losses. All losses finally transfer to heat in this case. We should decrease the power supply in this operating mode in order to avoid overheating the stator coil and the other components.

Light load mode

The collision point occurs at the period of (t_a, t_b) and (t_b, t_c) . The energy equation for the period of (t_a, t_b) or (t_b, t_c) is

$$\frac{1}{2}m_c\dot{y}_{cb}^2 + \frac{1}{2}k_{ext}y_{cb}^2 = \frac{1}{2}m_c\dot{y}_{ca}^2 + \frac{1}{2}k_{ext}y_{ca}^2 + w_{col}. \quad (16)$$

where $\frac{1}{2}m_c \cdot \dot{y}_{cb}^2$ is kinetic energy stored in casing before collision, $\frac{1}{2}k_{ext} \cdot y_{cb}^2$ is potential energy stored in external spring before collision, $\frac{1}{2}m_c \cdot \dot{y}_{ca}^2$ is kinetic energy stored in casing after collision and is zero due to completely inelastic collision, $\frac{1}{2}k_{ext} \cdot y_{ca}^2$ is potential energy stored in external spring after collision, w_{col} is energy transferred to the rock during collision.

The left-hand side of (16) is the energy stored in the system before collision. The right-hand side of (16) is the energy after collision. Kinetic energy is transferred to the rock. Potential energy is kept in the system and has no contribution to the collision.

Full load mode

The full load mode only occurs at the point of t_b . The energy transfer follows

$$\frac{1}{2}m_c\dot{y}_{cb}^2 = w_{col}. \quad (17)$$

The left-hand side of (17) is the energy stored in the system before collision. Kinetic energy only exists because the displacement of casing is zero at that point. And the velocity of casing reaches the biggest value at that point. All kinetic energy is transferred to the rock due to completely inelastic

collision. Therefore, the energy transfer is done with the highest efficiency.

Over load mode

It was mentioned that the whole system is moving downward at constant speed. The collision will not continue in next period if the external spring is fully compressed during the period of (t_c, t_e) . All input power will be transferred to losses in this situation. This is the worst situation which should be avoided.

B. Dynamic process analysis for energy conversion

The parameters of prototype are shown in Table II. We divide one period into three phases: collision phase, touching phase, and free oscillation phase. We assume that the tubular linear oscillatory motor operates at steady state with full load mode. One full period of collision from t_0 to t_3 shown in Fig. 9 is chosen to analyze the dynamic process of collision.

Collision phase from t_0 to t_1 :

The collision occurs at the point of t_0 and lasts to the instant of t_1 . The drill bit hits the rock during this period. The casing is subject to another force which is counter force from rock in the upward direction. The net force is the summation of forces from piston-gas springs-damper, friction force from surroundings and the counter force from rock. Therefore, the deceleration is not constant because the net force on the casing is not constant. The velocity of casing is decreasing from the peak point at the instant of t_0 to zero at the instant of t_1 . The displacement of casing is still increasing. The energy stored in system transfers to the rock so that the rock is collapsed into small pieces.

Touching phase from t_1 to t_2 :

The drill bit keeps on touching rock after collision because the net force from piston-gas springs-damper subsystem is larger than zero in the downward direction. Not until the net force is less than zero does casing continue to oscillate freely.

The net force on the casing is zero because the driving force from piston is equal to the counter force from the rock. The velocity of casing is zero from t_1 to t_2 . The displacement of casing is constant. There is no output energy during this phase.

Free oscillation phase from t_2 to t_3 :

The casing comes into free oscillation mode after keeping on touching rock because the net force driving casing is larger than zero in the upward direction. The values of velocity and displacement of casing depend on (14) where $d = 0$. The system is drawing power and storing energy which is prepared for next collision.

C. Dynamics of piston and casing

The net force of the piston is changed due to collisions. Therefore, the velocity and displacement of piston are changed shown in Fig. 10. The motion of casing has similar changes shown in Fig. 11. The peak of net force of casing is due to counter force from rock. Its value follows the law of conservation of momentum.

D. Losses analysis for system

The copper losses of stator winding, friction losses of piston and friction losses of casing are shown in Fig. 12. Friction losses are not drawn from electric power supply directly but from motion of mass.

E. Control philosophy

In order to reach maximum power transfer in this application, the constant velocity of whole system is needed. Therefore, drill string is controlled by another system. When drill bit hits hard rock input voltage can be increased to store more kinetic energy in casing.

V. CONCLUSION

Mathematics model of linear oscillatory machine is built in this work and it is important to both understanding dynamics and achieving good performance. In this work, it is especially important to improve the efficiency of machine and design control scheme as well.

The definition of operating modes for linear oscillatory machine in drilling application is given. It helps to understand the dynamics of whole system.

Dynamics of linear oscillatory machine is analyzed for completely inelastic collision mode. The simulation results show that this machine can work with hard rock and it is suitable for drilling application.

Table II
Parameters for Simulations

Input voltage	u_s	$32\sin(2\pi ft)$ V
Resistance of coil	R	2.35Ω
Inductance of coil	L	0.003 H
Mass of piston	m_p	0.950 kg
Mass of casing	m_c	1.100 kg
Coefficient of static friction of piston	μ_{sp}	0.3
Coefficient of coulomb friction of piston	μ_{cp}	0.2
Coefficient of viscous friction of piston	μ_{vp}	0.2
Coefficient of static friction of casing	μ_{sc}	0.03
Coefficient of coulomb friction of casing	μ_{cc}	0.02
Coefficient of viscous friction of casing	μ_{vc}	0.02
Slop constant	s	1000
Diameter of chamber	D	0.04 m
Length of chamber	l	0.045 m
Flux density	B	1.2 T
Number of turns	N	100
Coefficient of external spring	k_{ext}	70769 N/m
Coefficient of spring for equivalent load	k_L	0 N/m
Coefficient of damper for equivalent load	b_L	$50,000,000$

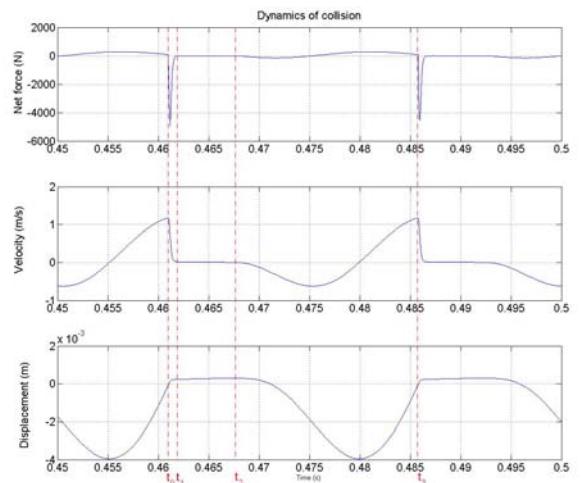


Fig. 9. Dynamic process of full load mode

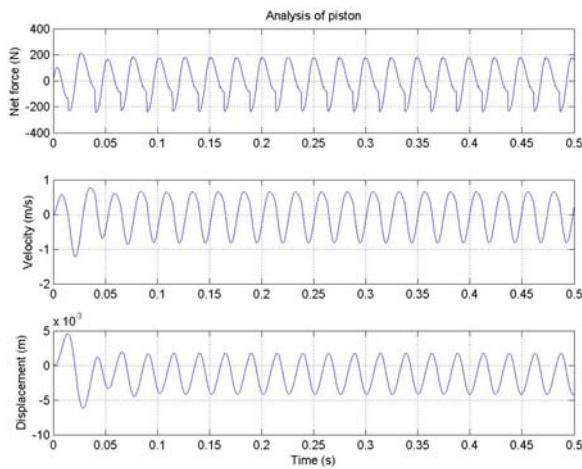


Fig. 10. Motion analysis of piston

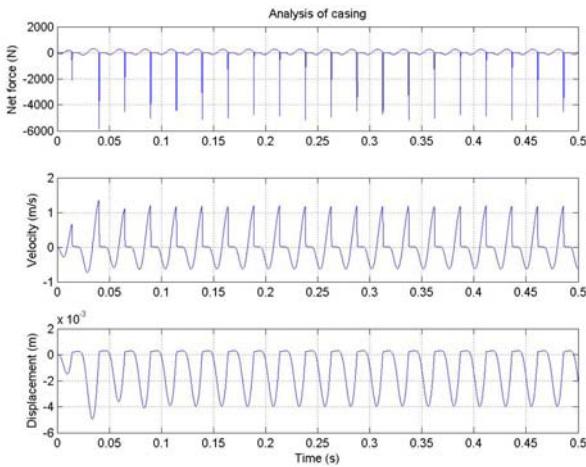


Fig. 11. Motion analysis of casing

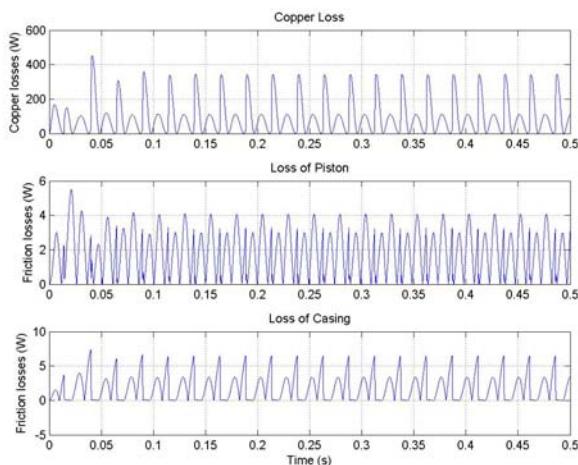


Fig. 12. Losses analysis

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