

Experimental Modelling of Air Breakdown Voltage Using Designs of Experiment

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ABSTRACT

Many experimental and numerical studies were devoted to the electric discharge of air, and some mathematical models were proposed for the critical breakdown voltage. As this latter depends on several parameters, it is difficult to find a formula, theoretical or experimental, which considers many factors. The aim of this paper is to model the critical breakdown voltage in a "Sphere-circular plane" electrodes system by using the methodology of experimental designs. Several factors were considered, such as geometrical factors (inter-electrodes interval, diameter of the electrodes) and climatic factors (temperature, humidity). Two factorial centred faces experimental designs (CCF) were carried out, a first one for the geometrical factors and a second one for the climatic factors. The obtained results made it possible to propose mathematical models and to study the interactions between the various factors.

Keywords: High voltage, electrical breakdown, modelling, design of experiments.

1 INTRODUCTION

The breakdown of air was the subject of many experimental, theoretical and simulation studies. The physical phenomenon is nowadays well known and researchers such as Townsend, Meek and Raether contributed to the comprehension and the explanation of the breakdown mechanism [1-4].

Many factors affect the breakdown, such as electrical, geometrical and climatic parameters. Nowadays, the influence of each one of them is well-known, but we do not appreciate the interactions existing between these factors. When the relative humidity and the temperature vary simultaneously for example, which one influences more than the other? Thus, we make use of the experimental designs methodology, a powerful tool which proved to be very helpful for modelling and analysing the interactions between factors [5-6]. We examine in this paper several factors: relative humidity, temperature, radius of electrodes and inter-electrodes interval.

2 DESCRIPTION OF THE EXPERIMENTAL DEVICE

All the experiments were carried out in a "sphere- circular plane" configuration of electrodes (Fig.1). A multimeter (FLUKE 867B, $U_{\max}=1000\text{V}$) and a high voltage probe (Metrix-HT212-typeB) are used to measure the applied high voltage. The high voltage is delivered by a reversible Direct Current supply ($U_{\max}= 50 \text{ kV}$, $I_{\max}=10 \text{ mA}$). Experiments carried out according to climatic conditions were done in a climatic room where temperature and humidity can be varied and controlled.

3 EXPERIMENTAL DESIGN METHODOLOGY

Methodology of the experimental designs makes it possible to determine the number of experiments to be achieved according to a well defined objective, to study several factors simultaneously, to reduce dispersion related to measurements, to appreciate the effects of

coupling between factors, to evaluate the respective influence of the factors and their interactions [7-8]. Many papers were written about the application of this methodology in electrical and electrostatic processes [9-18].

3.1 DEVELOPMENT OF THE METHOD

Regardless of the domain of application, design of experiments is useful for three objectives: 1) screening; 2) optimization; and 3) robustness testing. Initial screening experimental designs pave the way for optimization work, whereas robustness assessment is usually carried out only in the final stage of development of a new product or technology, as a last test to ensure quality. Within this framework, many different experimental strategies can be imagined, as an answer to the specific requirements of each application. In this paper, the experiments simulate a typical physical process, i.e. electric discharge of air, which disposes of many control variables.

Employed at the beginning of the investigation, screening experiments are designed either to explore many factors, in order to reveal whether they have an influence on the responses, or to identify their appropriate ranges.

Experimental methodology is particularly suitable for screening a large number of factors. To solve the problem formulated in the first paragraph of this section, the screening experiments should be designed with a different purpose in mind: defining the domain of variation of the three factors that can be adjusted. In the case of a relatively well-known process such as the electric breakdown of gases, classical “one-factor-at-a-time” experiments are expected to be more effective than other designs.

The optimization stage of an experimental procedure should enable the identification of the “set point,” i.e., the values of the control factors for which the response of the process is a maximum, is a minimum, or approaches a target.

Finding mathematical models of good quality with minimum efforts depends on the way in which intervals of input factors are selected. This method can be used as follows [19-20]:

- Selection of the most interesting and influent factors;
- Determination of variation interval of each factor, i.e. maximal, minimal and central values;
- Carry out a matrix of experiments with all the possible states and corresponding responses.

Before starting the experiments, it is necessary to opt for the best and suitable design which can model the process with the most possible precision. In this paper, we opted for the centred faces composite design (CCF) which allows using Response Surfaces Modelling (RSM). It is possible to determine a quadratic dependence between the output function to optimize (response) and the input variables u_i ($i = 1, \dots, k$):

$$y = f(u_i) = c_0 + \sum c_i u_i + \sum c_{ij} u_i u_j + \sum c_{ii} u_i^2 \quad (1)$$

Knowing that Δu_i and u_{i0} are respectively the step of variation and the central value of factor i , reduced centred values of input factors are defined by the following relation:

$$x_i = (u_i - u_{i0}) / \Delta u_i \quad (2)$$

With these new variables, the output function (response) becomes:

$$y = f(x_i) = a_0 + \sum a_i x_i + \sum a_{ij} x_i x_j + \sum a_{ii} x_i^2 \quad (3)$$

The coefficients can be calculated or estimated by a data-processing program, in such way to have a minimal variance between the predictive mathematical model and the experimental results.

3.2 PLANNING OF THE EXPERIMENTS

The main advantage of CCF composite designs is to carry out the experiments sequentially, i.e. to try initially modelling the process with a polynomial of first order [21]. The first step consists thus in a full factorial design. If the 1st order model is validated we stop modelling, if not we continue modelling with a polynomial of second order using a CCF composite design.

As figure 2 shows it, in the case of a design with 3 factors, the full factorial design (first order) corresponds to the experiments located at the tops of the cube (square points A,B...H) and 3 identical experiments done in the central point M (star point). The CCF composite design (second order) corresponds to the 11 experiments mentioned before, i.e. the preceding factorial design, and 6 experiments located in the centres of the cube faces (round points a,b...f). Thus, a CCF composite design with 3 factors includes 17 experiments.

3.3 SOFTWARE MODDE.05

We used software MODDE 5.0 (Umetrics AB, Umea, Sweden) which is a Windows program for the creation and the evaluation of experimental designs [22], the program assists the user for interpretation of the results and prediction of the responses. It calculates the coefficients of the mathematical model, draws surfaces of response (RSM) and identifies best adjustments of the parameters for optimizing the process.

Moreover, the program calculates two significant statistical criteria which make it possible to validate or not the mathematical model:

- The predictive power is given by Q^2 . This is a measure of how well the model will predict the responses for new experimental condition.
- The goodness of fit parameter given by R^2 .

A good mathematical model must have criteria Q^2 and R^2 which the numerical values closes to the unit.

4 RESULTS

4.1 EXPERIMENTAL DESIGN OF GEOMETRICAL FACTORS

As the insulation in high voltage remains the major problem of dielectrics, optimization of breakdown means the maximization of equation (3), expressing the response which is in our case the critical breakdown voltage U_c . The analysis concerning geometrical parameters was made using a CCF experimental design with 3 factors. According to the development of the method described below, we determine limits of variation of each factor:

- Diameter of the high voltage spherical electrode: $D_{1min} = 2 \text{ cm}$ & $D_{1max} = 4 \text{ cm}$;
- Diameter of the grounded circular-plane electrode: $D_{2min} = 4 \text{ cm}$ & $D_{2max} = 8 \text{ cm}$;
- Inter-electrodes interval: $d_{min} = 0.5 \text{ cm}$ & $d_{max} = 1.5 \text{ cm}$.

Measurements of voltage U_c obtained in positive polarity (U_{cp}) and negative polarity (U_{cn}) are deferred in table I. We represented in the same table the results of both factorial and composite designs.

Once experimental values of voltage U_c are measured, software MODDE.05 checks first if obtained experimental results are "reasonable" and detects any "doubtful" measurement result. Graph represented on figure 3 shows that all the experiments are located inside the validation limits of results and makes it possible to validate experiment's results.

MODDE.05 gives values of criteria R^2 and Q^2 lower than 0.6 for the factorial design. Consequently, mathematical models of first order for voltages U_{cp} and U_{cn} can not be validated. Thus we must carry out six additional experiments to accomplish a CCF composite design (grey lines of table I). The statistical tests led this time to a valid mathematical model since criteria $R^2 = 0.999$ and $Q^2 = 0.996$.

The models suggested by MODDE.05 are:

$$U_{cp} = 28.1 + 1.64 d^* - 0.38 D_1^* + 9.52 D_2^* - 0.4 d^{*2} + 0.28 (D_1^*)^2 - 2.8 (D_2^*)^2 - 0.06 (D_1^* d^*) + 1.43 (D_2^* d^*) - 0.29 (D_1^* D_2^*) \quad (4)$$

for positive polarity, and

$$U_{cn} = 27.4 + 1.55 d^* - 0.07 D_1^* + 9.4 D_2^* - 0.95 d^{*2} + 0.35 (D_1^*)^2 - 1.7 (D_2^*)^2 - 0.1 (D_1^* d^*) + 1.3 (D_2^* d^*) - 0.3 (D_1^* D_2^*) \quad (5)$$

for negative polarity.

4.2 EXPERIMENTAL DESIGN OF CLIMATIC FACTORS

The influence of climatic parameters, i.e. temperature and humidity, was also studied using a CCF composite experimental design. The experiments were carried out in a climatic room where temperature and humidity can be varied and controlled. According to the development of the method described below, we determine limits of variation of each factor:

- Inter-électrodes interval d : $d_{min}=0.5$ cm & $d_{max}=1.5$ cm ;
- Temperature T : $T_{min}=30^\circ$ & $T_{max}=50^\circ$;
- Humidity : $H_{min}=40$ % & $H_{max}=60$ %.

Measurements of voltage U_c obtained in positive polarity (U_{cp}) and negative polarity (U_{cn}) are deferred in table II. We represented in the same table the results of both factorial and composite designs. For this second design, software MODDE.05 gives also values of criteria R^2 and Q^2 lower than 0.6 for the factorial design. Consequently, mathematical models of first order have not been validated, and we carry out six additional experiments (grey lines of table II) to accomplish a CCF design. The statistical tests made by the software led this time to a valid mathematical model since criteria $R^2 = 1.0$ and $Q^2 = 0.997$.

The models suggested by MODDE.05 are:

$$U_{cp} = 26.3 + 8.05 d^* - 0.26 T^* + 0.07 H^* - 3.5 d^{*2} + 0.43 T^{*2} - 0.3 H^{*2} - 0.1 d^* T^* + 1.1 \cdot 10^{-6} d^* H^* - 0.02 T^* H^* \quad (6)$$

$$U_{cn} = 24.7 + 6.85 d^* - 0.06 T^* + 1.07 \cdot 10^{-7} H^* - 1.9 d^{*2} - 0.06 T^{*2} + 0.04 H^{*2} - 0.2 d^* T^* - 0.02 d^* H^* + 3.4 \cdot 10^{-7} T^* H^* \quad (7)$$

5 DISCUSSIONS

Values of the coefficients associated with the factors show the degree of influence of each factor. It arises from mathematical models given by equations 4 and 5 for geometrical factors, that within the variation limits of the selected intervals, the diameter of the grounded plane electrode is the most significant factor for the critical breakdown voltage, in the two polarities. The increase in surface of the electrode makes the electric field non uniform and produce corona discharge instead of breakdown. Its influence is most important than the influence of the diameter of the high voltage spherical electrode. Furthermore, notice from Response Surfaces (Figures 4 and 5) that the diameter of the grounded plane electrode is more significant than the inter-electrodes interval, and that the diameter of the spherical electrode has an insignificant effect when it varies within the interval limits. Among the interactions between factors, only, “interval-grounded plane diameter” d - D_2 interaction is significant because it affects the distribution of electric field between the electrodes.

Concerning the influence of humidity and ambient temperature, the obtained mathematical models given in equations 6 and 7, show that these factors are not so significant compared to the inter-electrode interval, this latter represents the most influential parameter. The humidity has a positive effect on the breakdown voltage, ie the voltage increases with humidity. This may seem paradoxical because the air becomes more conducting when humidity is higher. Indeed, part of the energy will be used to ionize the molecules of water vapour, which requires then a higher voltage. Furthermore, the temperature has a negative effect, ie the critical breakdown voltage decreases with temperature. This is because the conductivity of gaseous insulation increases when temperature is higher [23]. Note that the proposed mathematical models does not show strong interactions between temperature and humidity,

and that these two factors are independent. In addition, when comparing the rate of influence of climatic factors from iso-response diagrams (Fig.6 and 7), it is clear that the temperature has more significant effect than humidity.

6 CONCLUSION

Breakdown voltage U_c of gaseous dielectrics remains the subject of several research tasks in the world, because it depends on numerous factors. As it is difficult to find a formula, theoretical or experimental, which considers the various parameters, the aim of this paper consisted in modelling voltage U_c using the methodology of experimental designs. Several factors were considered in this study: geometrical factors (such as inter-electrodes interval and diameters of electrodes) and climatic factors (such as temperature and humidity). Obtained results in the two polarities made it possible to propose mathematical models and to analyze the various interactions between these factors.

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TABLE I.
Results of the 1st experimental design
(Geometrical factors)

<i>Exp.</i>	<i>d</i>	<i>D</i> ₁	<i>D</i> ₂	<i>U</i> _{cp}	<i>U</i> _{cn}
N°	[cm]	[cm]	[cm]	[kV]	[kV]
1	0,5	2	4	15,5	15,2
2	0,5	4	4	16	16
3	0,5	2	8	15,2	16,2
4	0,5	4	8	15,8	15,9
5	1,5	2	4	32,1	32
6	1,5	4	4	38,7	37,4
7	1,5	2	8	31	31,1
8	1,5	4	8	37	36,8
9	1,0	3	6	28,3	27,4
10	1,0	3	6	28,2	27,3
11	1,0	3	6	28,3	27,4
12	1	2	6	26,2	24,5
13	1	4	6	28,9	28,4
14	1	3	4	28,5	27,8
15	1	3	8	28	27,7
16	0,5	3	6	15,7	15,7
17	1,5	3	6	34,6	35,7

Factorial design

Composite design CCF

Table II
Results of the 2nd experimental design
(Climatic factors)

<i>Exp.</i>	<i>d</i>	<i>T</i>	<i>H</i>	<i>U</i> _{cp}	<i>U</i> _{cn}
N°	[cm]	[°C]	[%]	[kV]	[kV]
1	0,5	30	40	15,1	15,8
2	1,5	30	40	31,4	29,9
3	0,5	50	40	14,7	15,9
4	1,5	50	40	30,4	29,4
5	0,5	30	60	15,3	15,8
6	1,5	30	60	31,4	30
7	0,5	50	60	14,6	16,1
8	1,5	50	60	30,5	29,3
9	1	40	50	26,5	24,7
10	1	40	50	26,4	24,6
11	1	40	50	26,3	24,7
12	0,5	40	50	14,5	16,1
13	1,5	40	50	31	29,6
14	1	30	50	26,5	24,6
15	1	50	50	26,9	24,8
16	1	40	40	25,7	24,9
17	1	40	60	26,2	24,7

Factorial design

Composite design CCF

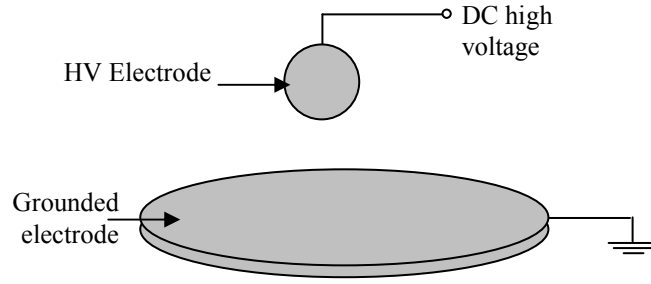


Figure 1: Schematic representation of the Experimental device

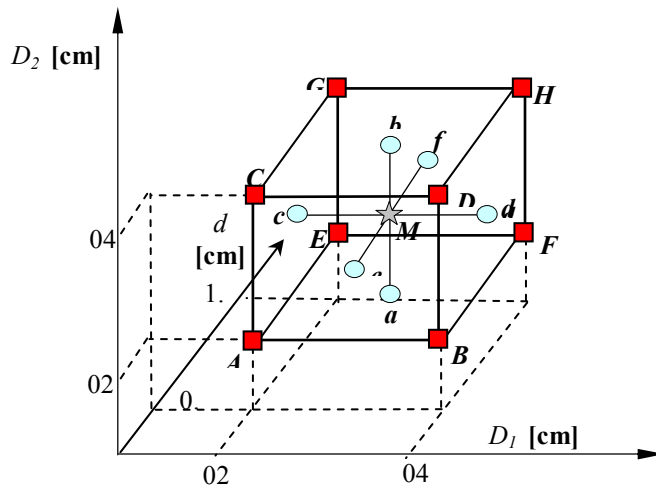


Figure 2: Diagram of experiments of a CCF design with 3 factors:
 D_1 : HV sphere diameter ($D_{1min} = 2$ cm & $D_{1max} = 4$ cm)
 D_2 : grounded plane diameter ($D_{2min} = 2$ cm & $D_{2max} = 4$ cm)
 d : inter-electrodes interval ($d_{min} = 0.5$ cm & $d_{max} = 1.5$ cm)

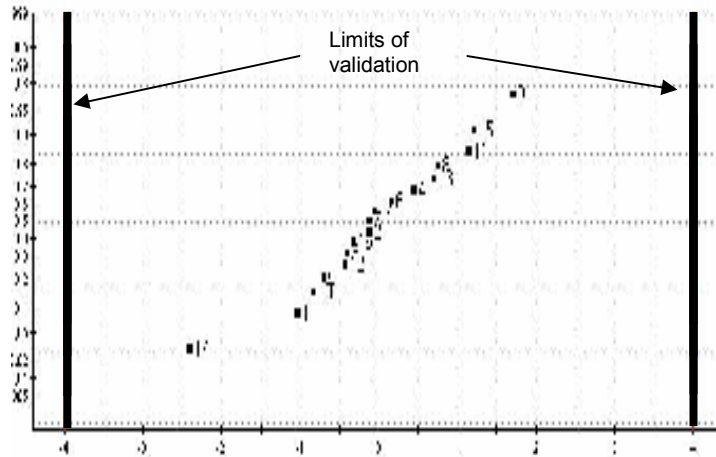


Figure 3: Graph for validation of measurements

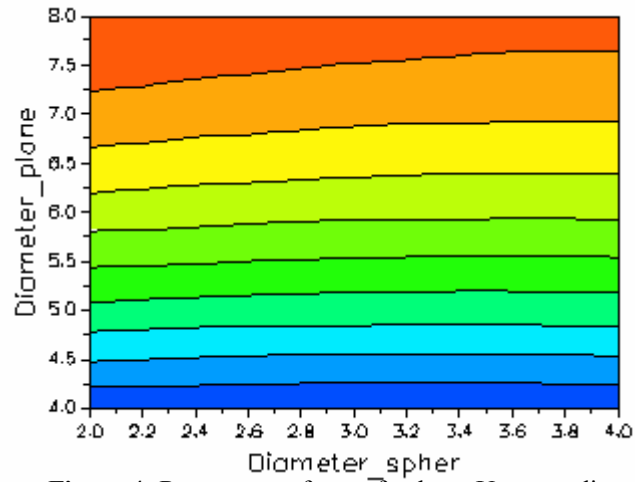


Figure 4: Response surfaces of voltage U_{cp} according to diameters of the electrodes (inter-electrodes interval $d=1\text{cm}$).

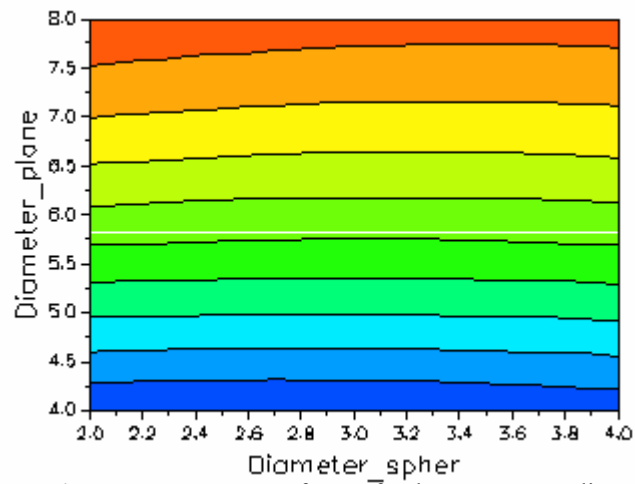


Figure 5: Response surfaces of voltage U_{cn} according to diameters of the electrodes (inter-electrodes interval $d=1\text{cm}$).

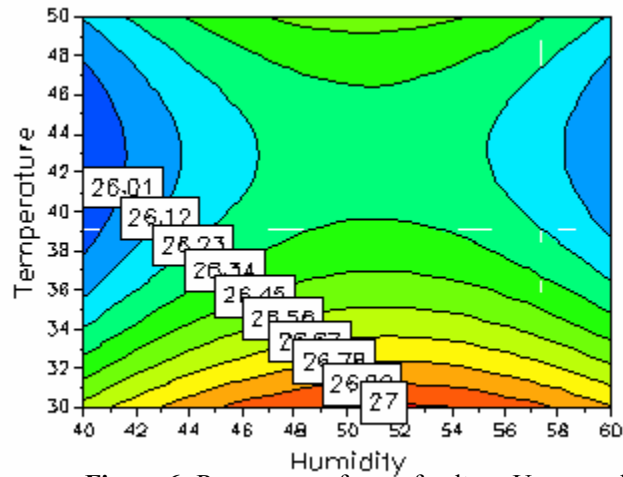


Figure 6: Response surfaces of voltage U_{cp} according to climatic factors (inter-electrodes interval $d=1\text{cm}$).

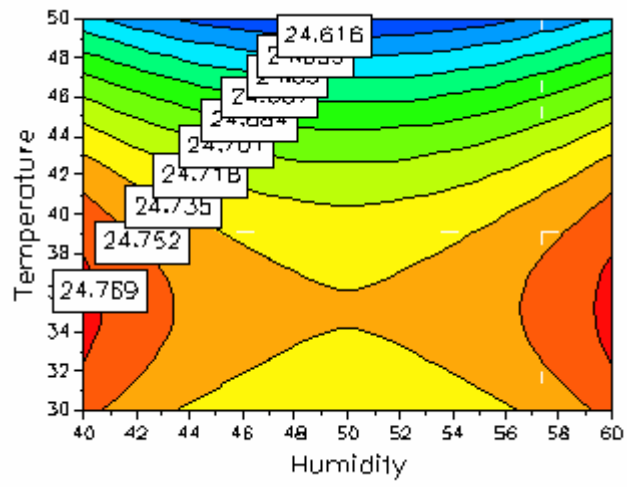


Figure 7: Response surfaces of voltage U_{cn} according to climatic factors (inter-electrodes interval $d=1$ cm).