AN ADAPTIVE STEP-SIZE USING MODIFIED FXLMS ALGORITHM FOR FEEDFORWARD ACTIVE NOISE CONTROL SYSTEMS BASED ON A NEW ONLINE SECONDARY PATH MODELING TECHNIQUE

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ABSTRACT
Several approaches have been introduced in literature for active noise control (ANC) systems. Since FxLMS algorithm appears to be the best choice as a controller filter, researchers tend to improve performance of ANC systems by enhancing and modifying this algorithm. This paper proposes a new version of FxLMS algorithm. In many ANC applications an online secondary path modeling method using a white noise as a training signal is required to ensure convergence of the system. This project also proposes a new approach for online secondary path modeling in feedforward ANC systems. The proposed algorithm based on varying the step size of the FxLMS algorithm by measuring the power of both signal and noise. Benefiting new version of FxLMS algorithm makes the system more desirable and improves the noise attenuation performance.

Index Terms— active noise control, FxLMS algorithm, noise reduction, convergence analysis, dynamic step size.

1. INTRODUCTION
Active noise are real time noise and they cannot be predictable (i.e. random). The traditional way to cancel the noise which is called passive noise control, which techniques based on the use of sound-absorbing materials, are effective in higher frequency noise. However, significant power of the industrial noise often occurs in the frequency range between 50–250Hz. Here the wavelength of sound is too long, so that passive techniques are no longer cost effective because they require material that is too heavy.

Active Noise Control System is working based on the principle of superposition. The system consists of a controller for which reference about the noise is given. The controller properly scales the reference noise and the phase reverses it. The phase reversed signal is then added to the input signal that has some noise along the original message signal so that the noise gets cancelled out. There are many methods used for ANC system include both feedback and feed-forward control.

ANC is based on either feed-forward control, where a coherent reference noise input is sensed before it propagates past the secondary source, or feedback control where the active noise controller attempts to cancel the noise without the benefit of an “upstream” reference input. The performance of the active control system is determined largely by the signal processing algorithm and the actual acoustical implementation. Effective algorithm design requires reasonable knowledge of algorithm behavior for the desired operating conditions. Since the active noise is random, the proper prediction of the noise cannot be possible, the controller should contain a adaptive filter part whose filter coefficients will be changing based on the error signal which the difference between the output of the controller and the output from an unknown plant. To achieve reduction of noise in complicated multiple noise source, we must use active noise control by multiple reference channel. That is input signal to each channel is correlated and the output also correlated

Active noise control (ANC) systems equipped with the Filtered-x Least Mean Square (FxLMS) adaptation algorithm cannot prescient from the online estimation of the secondary path (Akhtar M.T., Abe M., and Kawamata M. 2004). Two different approaches can be adopted for the secondary path modeling. A first approach involves the injection of an auxiliary white random noise in the ANC system and it uses a system identification method to model the secondary path (Akhtar M.T., Abe M., and Kawamata M. 2004). The second approach estimates the secondary path directly from the output of the control filter, without the injection of additional noise. It has been shown in (Kuo S.M. and Morgan D.R 1999) that the first approach
is superior for convergence speed of both the control filter and the secondary path modeling filter, for speed of response to modifications in the primary noise and the secondary path, and for independence between the primary noise attenuation and the online secondary path identification. The injection of an auxiliary noise for estimating the secondary path was proposed in (PooyaDavari and Hamid Hassanpour 2009), where two adaptive filters were used for adapting the control filter and for identifying the secondary path, respectively. The system suffers the slow convergence of the control filter and of the secondary path modeling filter and the low estimation accuracy of the optimal values of these filters. Indeed, with the injection of an auxiliary noise, the signal at the error microphone has two components: (1) the auxiliary noise filtered by the secondary path, (2) the residual noise of the ANC system. In the ANC system of the first component disturbs the adaptation of the control filter, while the second component disturbs the identification of the secondary path. In order to solve this problem the use of a third adaptive filter was proposed in (Koike S. 2002). In (Babu P. and Krishnan A. 2009) the third adaptive filter is used to improve the convergence performance and the estimation accuracy of the secondary path. In fact, this adaptive filter acts as a noise suppressor that removes the residual noise from the error signal of the secondary path modeling filter. The third adaptive filter is used for the same purpose also in (Hu A.Q., Hu X. and Cheng S. 2003), but a cross-update strategy is employed for removing also the auxiliary noise from the error signals of the control filter and of the noise suppressor. More recently, improved convergence performances were obtained with the ANC structure proposed in (PooyaDavari and Hamid Hassanpour 2009). The ANC system uses again only two adaptive filters, one for adapting the control filter and one for modeling the secondary path, but an improved convergence speed of the control filter is obtained by introducing the delay compensation scheme of, and by removing the auxiliary noise from the error signal of the control filter. When ANC is deployed in real applications, many practical problems arise and need to be addressed. An approach to adaptive ANC performance analysis that involves a hierarchy of techniques, starting with an ideal simplified problem and progressively adding practical constraints and other complexities, is essential (PooyaDavari and Hamid Hassanpour 2009).

In general, the filtered-X least mean square (FxLMS) algorithm can be used to adjust the generation of the secondary noise. However, the performance of the FxLMS algorithm is significantly affected by the difference between the estimated and the actual transfer functions representing the secondary path from the secondary source to the error sensor. Since this transfer function is unknown and continuously changing in a real environment, it is desirable to perform an on-line secondary path modeling. Moreover, in many practical cases, the primary noise exists even during off-line secondary path modeling, and this adversely affects the convergent performance of the modeling.

A general block diagram of the ANC system with on-line secondary path modeling [or \( y(n) = 0 \) for off-line modeling] using an additive random noise (Akhtar M.T., Abe M., and Kawamata M. 2005) is shown in Fig. 1. (we shall refer to this as Method A). A random noise generator is used to generate a zero-mean white noise \( v(n) \) that is uncorrelated with the primary noise \( d(n) \). The noise \( v(n) \) is mixed with the canceling signal \( y(n) \) to drive the secondary source. An adaptive filter \( \hat{S}(z) \), excited only by the noise \( v(n) \), is used to model the secondary path \( S(z) \) in parallel. The residual error for the ANC system is expressed as

\[
e(n) = d(n) - y(n) * s(n) = d(n) - y'(n) \tag{1}
\]

where \( y'(n) = y(n) * s(n) \), \( v'(n) = v(n) * s(n) \), and \( s(n) \) is the real transfer function of the secondary path. In Fig. \( e(n) \) is used as the signal for updating the adaptive filter \( \hat{S}(z) \), though only the component \( v'(n) \) in \( e(n) \) contains information on \( S(z) \). As \( d(n) \) and \( y'(n) \) do not provide information on \( \hat{S}(z) \), the use of \( e(n) \) can introduce distortion to the modeling, hence affecting its convergent performance. It is therefore important that this source of distortion be eliminated.

2. PROBLEM IDENTIFICATION

When ANC is deployed in real applications, many practical problems arise and need to be overcome. An approach to adaptive ANC performance analysis that involves a hierarchy of techniques, starting with an ideal simplified problem and progressively adding practical constraints and other complexities such as increases the convergence and noise attenuation by minimizing the unwanted noises by means of adapting new algorithms. The noises presented in ANC systems are classified into two categories, they are noises with small magnitude and noises with large magnitude, which is also called as impulsive noises. The first type of noises can be minimized by adapting the step-size of the ANC system and second one can be minimized by calculating probability density function (PDF) or replacing the magnitude of the signal by means of threshold function. In this project, new version of FxLMS algorithm implemented to improve the
Two types of noise exist in the environment, broadband noise, where its energy is more or less evenly distributed across the frequency spectrum, or narrowband noise, where the energy is mostly concentrated around specific frequencies. In Feedback ANC, a controller is used to modify the response of a system, for example by adding artificial damping. In Feedforward ANC, when the disturbance is deterministic, or in particular harmonic, a controller can be used to adaptively calculate a signal that cancels the disturbance. The basic idea of feedforward ANC is to account for primary acoustic disturbance. In this way vibration levels can be reduced even for a broadband random disturbance.

The adaptive filter actually consists of two parts. The digital filter \( W(z) \) calculates its output by using a reference \( x(n) \) and adjustable filter coefficients, or weights. The filter coefficients are updated by an adaptive algorithm, using \( x(n) \) and an error signal \( e(n) \) in such a way that the squared error \( e^2(n) \) is minimized. The error \( e(n) \) can be defined as

\[
e(n) = d(n) - y(n)
\]

where \( d(n) \) is an unwanted disturbance. The adaptive filter will try to calculate an output \( y(n) \) that is equal to the unwanted disturbance \( d(n) \), so this disturbance will be cancelled.

### 3. FxLMS ALGORITHM

The FxLMS algorithm can be applied to both feedback and feed forward structures. Block diagram of a feed forward FxLMS ANC system is Figure 2. Here \( P(z) \) accounts for primary acoustic path between references noise source and error microphone. \( \hat{S}(z) \) is obtained offline and kept fixed during the online operation of ANC.

The expression for the residual error \( e(n) \) is given as

\[
e(n) = d(n) - y'(n)
\]

where \( y'(n) \) is the controller output \( y(n) \) filtered through the secondary path \( S(z) \), then \( y'(n) \) and \( y(n) \) computed as

\[
y'(n) = s^T(n)y(n)
\]

And

\[
y(n) = w^T(n)x(n)
\]

where \( w(n) = [w_0(n)w_1(n)......w_{L-1}(n)]^T \) is tap weight vector, \( x(n) = [x(n)x(n-1)......x(n-L+1)]^T \) is the reference signal picked by the reference microphone and \( s(n) \) is impulse response of secondary path \( S(z) \). It is assumed that there is no acoustic feedback from secondary loudspeaker to reference microphone. The FxLMS update equation for the coefficients of \( W(z) \) is given as:

\[
w(n + 1) = w(n) + \mu e(n)x'(n)
\]

where \( x'(n) \) is reference signal \( x(n) \) filtered through secondary path model \( \hat{S}(z) \)

\[
x'(n) = \hat{s}^T(n)x(n)
\]
acoustic path from loudspeaker to error microphone, error microphone, preamplifier, anti-aliasing filter, and analog-to digital (A/D) converter. The schematic diagram for a simplified ANC system is shown in figure 3. From Fig. 3., the z-transform of the error signal is

$$E(z) = [P(z) - S(z)]W(z)X(z)$$ (8)

![Figure 3 Block diagram of simplified ANC system](image)

We shall make the simplifying assumption here that after convergence of the adaptive filter, the residual error is ideally zero [i.e., E (z) =0]. This requires W(z) realizing the optimal transfer function.

$$W^o(z) = \frac{P(z)}{S(z)}$$ (8)

In other words, the adaptive filter has to simultaneously model P(z) and inversely model S(z). A key advantage of this approach is that with a proper model of the plant, the system can respond instantaneously to changes in the input signal caused by changes in the noise sources.

However, the performance of an ANC system depends largely upon the transfer function of the secondary path. By introducing an equalizer, a more uniform secondary path frequency response is achieved. In this way, the amount of noise reduction can often be increased significantly [8]. In addition, a sufficiently high-order adaptive FIR filter is required to approximate a rational function 1/S(z) shown in (2). It is impossible to compensate for the inherent delay due to S(z) if the primary path P(z) does not contain a delay of at least equal length.

5. PROPOSED METHOD

The proposed method is based on varying the step size of FxLMS algorithm based on the secondary path modeling technique. This method utilizes VSS-FxLMS algorithm for modeling filter and uses f(n) as error signal for both \( \hat{S}(z) \) and \( W(z) \). The VSS-LMS algorithm is used to update modeling filter \( \hat{S}(z) \) coefficients. The FIR filter of tap-weight length L is used as the adaptive filter in this proposed algorithm. FIR adaptive filters very often perform well enough to satisfy the design criteria. An FIR filter for estimating a desired signal \( d(n) \) from a related signal \( x(n) \) is illustrated in fig is

$$\hat{d}(n) = \sum_{k=0}^{p} w_n^*(k)x(n-k) = w_n^T x(n)$$ (9)

Here it is assumed that \( x(n) \) and \( d(n) \) is non stationary random process and the goal is to find the coefficient vector \( w_n \) at time n that minimizes the mean-square error,

$$\hat{\xi}(n) = E[|e(n)|^2]$$ (10)

where

$$e(n) = d(n) - \hat{d}(n) = d(n) - w_n^T x(n)$$

The solution to minimization problem may be found by setting the derivative of \( \hat{\xi}(n) \) with respect to \( w_n^* (k) \) equal to zero for \( k = 0,1,\ldots,p \). The result is

$$E[e(n)x^*(n-k)] = 0; \quad k = 0,1,\ldots,p$$ (11)

Substituting equation (4.3) into equation (4.2) we have

$$E[(d(n) - \sum_{k=0}^{p} w_n^*(k)x(n-k))x^*(n-k)] = 0; \quad k = 0,\ldots,p$$ (12)

This, after rearranging the terms, becomes

$$\sum_{k=0}^{p} w_n^*(k)E[x(n-k)x^*(n-k)] = E[d(n)x^*(n-k)]; \quad k = 0,\ldots,p$$ (13)

Equation (4.5) is a set of \( p+1 \) linear equations in the \( p+1 \) unknowns \( w_n^* \). However, unlike the case of an FIR wiener filter where it was assumed that \( x(n) \) and \( d(n) \) is jointly WSS, the solution to these equations depends on \( n \). We may express these equations in vector form as follows

$$R_n w_n = r_{dn}$$ (14)

Where
ow and, as with respect to that minimizes the gradient is the \( \mu \), and

\[
\mathbf{R}_n(n) = \begin{bmatrix}
E[x(n)x'(n)] & E[x(n-1)x'(n)] & \cdots & E[x(n-p)x'(n)] \\
E[x(n)x'(n-1)] & E[x(n-1)x'(n-1)] & \cdots & E[x(n-p)x'(n-1)] \\
\vdots & \vdots & \ddots & \vdots \\
E[x(n)x'(n-p)] & E[x(n-1)x'(n-p)] & \cdots & E[x(n-p)x'(n-p)]
\end{bmatrix}
\]

is a \((p+1) \times (p+1)\) Hermitian matrix of autocorrelation and

\[
t_j(n) = [E[d(n)x'(n)], E[d(n)x'(n-1)], \ldots, E[d(n)x'(n-p)]]
\]

is a vector of cross correlations between \( d(n) \) and \( x(n) \). Note that in the case of jointly WSS processes, equation (6) reduces to the Wiener–Hopf equations, and the solution \( w_n \) becomes independent of time. Instead of solving equation (6) for each value of \( n \), which could be impractical in most real-time implementations? Hence we consider an iterative approach that is based on the method of steepest descent. In designing an FIR adaptive filter, the goal is to find the vector \( w_n \) at time \( n \) that minimizes the quadratic function

\[
\xi(n) = E[|e(n)|^2]
\]

Although the vector that minimizes \( \xi(n) \) may be found by setting the derivatives of \( \xi(n) \) with respect to \( w^*(k) \) equal to zero, another approach is to search for the solution using the method of steepest descent. The method steepest descent is an iterative procedure that has been used to find extreme of nonlinear functions. The basic idea of this method is as follows. Let \( w_n \) be an estimate of the vector that minimizes the mean-square error \( \xi(n) \) at time \( n \). At time \( n+1 \) a new structure is formed by adding a correction to \( w_n \) that is designed to bring \( w_n \) closer to the desired solution. The correction involves taking a step size \( \mu \) in the direction of maximum descent down the quadratic surface. Thus the update equation for \( w_n \) is

\[
w_{n+1} = w_n - \mu \nabla \xi(n)
\]

where \( \nabla \xi(n) \) is the gradient vector and it is given by

\[
\nabla \xi(n) = \begin{bmatrix}
\frac{\partial \xi(n)}{\partial w(0)} \\
\frac{\partial \xi(n)}{\partial w(1)} \\
\vdots \\
\frac{\partial \xi(n)}{\partial w(n)}
\end{bmatrix}
\]

The step size \( \mu \) affects the rate at which the weight vector moves down the quadratic surface and must be a positive number. For very small values of \( \mu \), the correction to \( w_n \) is small and the movement down the quadratic surface is slow and, as \( \mu \) is increased, the rate of descent increases. However, there is an upper limit on how large the step size may be. For values of \( \mu \) that exceed this limit, the trajectory of \( w_n \) becomes unstable and unbounded. The steepest descent algorithm may be summarized as follows:

1. Initialize the steepest descent algorithm with an initial estimate, \( w_0 \), of the optimum weight vector \( w \).
2. Evaluate the gradient of \( \xi(n) \) at the current estimate, \( w_n \), of the optimum weight vector.
3. Update the estimate at time \( n \) by adding a correction that is formed by taking a step size \( \mu \) in the negative gradient direction

\[
w_{n+1} = w_n - \mu \nabla \xi(n)
\]

4. Go back to (2) and repeat the process.

Let us now evaluate the gradient vector \( \nabla \xi(n) \). Assuming that \( w \) is complex, the gradient is the derivative of \( E[|e(n)|^2] \) with respect to \( w^* \). Hence

\[
\nabla \xi(n) = \nabla E[|e(n)|^2] = E[\nabla |e(n)|^2] = E[|e(n)|^2 e'(n)]
\]

and

\[
\nabla e^*(n) = -x^*(n)
\]

it follows that

\[
\nabla \xi(n) = -E[e(n)x^*(n)]
\]

Thus, with a step size of \( \mu \), the FxLMS algorithm becomes

\[
w_{n+1} = w_n + \mu E[e(n)x^*(n)]
\]
where \( w(n) = [w_0(n)w_1(n)\ldots w_{L-1}(n)]^T \) is tap weight vector, 
\( x(n) = [x(n)x(n-1)\ldots x(n-L+1)]^T \) is the reference signal picked by the reference microphone and \( s(n) \) is impulse response of secondary path \( S(z) \). 
An internally generated zero mean white Gaussian noise signal \( v(n) \), uncorrelated with the reference signal \( x(n) \), is injected at the output \( y(n) \) of the control filter. 
Thus, the residual error signal is given as 
\[
e(n) = [d(n) - y'(n) + v'(n)] \tag{21}
\]
where \( d(n) = p(n) \ast x(n) \) is the primary disturbance signal, \( y'(n) = s(n) \ast y(n) \) is the canceling signal, \( v'(n) = v(n) \ast s(n) \). 
It has been noted that if \( d(n) \neq y'(n) \), then it indicates the error is presence in the ANC system and if \( d(n) \approx y'(n) \) then the error \( e(n) \) is minimized. Initially, the error of the ANC system is large and hence the system deviate from the convergence. To overcome this problem initially, the method uses large step size to make the system to converge quickly. 
But here there is stability problem arises due to choose the large step size initially. To overcome this the system uses VSS-LMS algorithm in the secondary path to over the instability arises due to large step size. 
As the number of iterations continues, the error \( e(n) \) starts to decreases and at last the error \( e(n) \) becomes approximately zero, that is \( e(n) \approx 0 \). 
At this time the system retain its initial step size. In general the step size \( \mu \) can be selected as \( 0 < \mu < 0.1 \).

Assuming that the modeling filter \( \hat{S}(z) \) is an FIR filter of tap-weight length \( M \), and its output \( \hat{v}'(n) \) is given as 
\[
\hat{v}'(n) = S^T(n)v(n) \tag{22}
\]
The output of \( \hat{S}(z) \), \( \hat{v}'(n) \) is subtracted from \( e(n) \) to the error signal for the modeling filter \( \hat{S}(z) \) which is given as 
\[
f(n) = [d(n) - y'(n)] + [v'(n) - \hat{v}'(n)] \tag{23}
\]
The tap-weights of the modeling filter \( \hat{S}(z) \) are updated using LMS algorithm 
\[
\hat{S}(n+1) = \hat{S}(n) + \mu'(n)f(n)v(n) \tag{24}
\]
It has been noted that initially the error \( e(n) \) of the system is large then it allows large step size then as the number of iteration continuous then the error of system is going to decreases, then it retains original step size.

In modified FxLMS, the step size is varied dynamically with respect to the error signal. Since error at the beginning is large, the step size of the algorithm is also large. This in turn increases convergence rate. 
As the iteration progresses, the error will simultaneously decreases. Finally, the original step size will be retained. Figure 3 shows the block diagram for proposed method.

Finally the error is reduced greatly by the implementation of the dynamic step size algorithm. 
\[
w(n+1) = w(n) + \mu'(n)f(n)x'(n) \tag{25}
\]
The above equation represents the updating of the tap weights the modified FxLMS algorithm. This idea of dynamic step size calculation is represented in 
\[
\mu'(n) = \frac{\mu e(n)}{1 + |e(n-1)|} \tag{26}
\]
Thus the equation (26) is the dynamic step obtained based on secondary path modeling technique and it is used both in FxLMS and VSS LMS.
below

Step 1. Initially, step size $\mu = 0.001$

Previous error $e(n-1) = 0.99$

(calculated)

Current error $e1(n) = 0.95$

$\mu(n) = 0.001 \times 0.95 / (1 - 0.99) = 0.095$

Step 2.

Current error $e2(n) = 0.9$

Normalized current error $e2(n) = (0.9/0.95) \times 0.99 = 0.937$

$\mu(n) = 0.001 \times 0.937 / (1 - 0.95) = 0.01874$

Step 3

Current error $e3(n) = 0.85$

Normalized current error $e3(n) = (0.85/0.9) \times 0.99 = 0.884$

$\mu(n) = 0.001 \times 0.884 / (1 - 0.937) = 0.0140$

Using the dynamic step size both VSS LMS and FxLMS step size update dynamically.

5. RESULTS AND DISCUSSIONS

In this section the performance of the proposed modified FxLMS algorithm is demonstrated using computer simulation. The performance of the proposed algorithm is compared with that of FxLMS algorithm on the basis of noise reduction $R$ (dB) and convergence rate is given in “(27)” and “(28)”.

$$ R(dB) = -10 \log \left( \frac{\sum e^2(n)}{\sum d^2(n)} \right) $$

(27)

and

Convergence Rate $= 20 \times \log_{10} \left\{ \frac{\text{abs}(g)}{\text{abs}(g_0)} \right\}$

(28)

The large positive value of $R$ indicates that more noise reduction is achieved at the error . In this section the performance of the proposed modified FxLMS algorithm is demonstrated using computer simulation. The computer simulation for modified FxLMS algorithm performance is illustrated in figure (5) to (10). Figure 5 shows the characteristics of Noise reduction versus number of iteration times. It has been seen that the modified FxLMS with dynamic step-size produce better noise reduction compared with FxLMS with fixed step size. From the figure 5, note that the fixed step size FxLMS algorithm noise reduction is somewhat non-zero positive value (approximately +75dB), but in the dynamic step size FxLMS algorithm noise reduction is non-zero negative value (approximately -100dB), hence dynamic step size algorithm provides the better noise reduction than that of fixed step algorithm microphone.
Figure 6 shows the characteristics of convergence rate in dB with respect to number of iterations. It has been seen that the convergence rate of modified FxLMS with dynamic step-size increases by reducing the number of iterations compared with modified FxLMS with fixed step size.

Figure 7 shows the characteristic of error versus number of iterations. It has been seen that error at the beginning is large, the step size of the algorithm is also large. This in turn increases convergence rate. As the iteration progresses, the error will simultaneously decreases and approaches to zero. Finally, the original step size will be retained. Thus the convergence rate as well as error can be minimized.

Figure 8 shows the input noise source and figure 9 represents the generated white noise signal, which act as a reference signal for the noise minimization in the ANC system. Figure 10 shows the error measured at the controller filter, which is the...
difference between the $d(n)$ and $y'(n)$. Further the noise presence in the ANC system can be reduced by using modeling filter in the secondary path which uses the VSS LMS to update the filter coefficients and the error in modeling filter is the difference between $e(n)$ and $\hat{y}'(n)$ shown in figure 11.

CONCLUSION
Here we propose a modified FxLMS structure for ANC system. This structure combines the concept of modified FxLMS algorithm with the dynamic variable step size. It shows better tracking performance and convergence rate than the modified FxLMS algorithm with fixed step and conventional FxLMS algorithm. The main feature of this method is that it can achieve improved performance in noise reduction than the existing methods.

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