EVALUATING FLOW-GATE MARGINAL PRICING AS AN EFFICIENT TOOL FOR SELLING AND MANAGEMENT OF ELECTRICAL ENERGY IN Deregulated ELECTRICITY MARKET

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Abstract: Flow-gate marginal pricing (FMP) is a suitable and useful index in deregulated power systems that has not been sufficiently investigated and reviewed. There is a very tight association between power system performance and this parameter. FMP can be evaluated as an effective index in power system studied. In this regard, this paper aims at investigating FMP features in power system operation and planning. In this paper, FMP formulation is expressed and the relationship between FMP with LMP, voltages, powers, congestion, and other variables is studied. In addition, FMP in power systems is investigated from different points such as effects of renewable energy uncertainties on the FMP, impacts of the reactive power on FMP, FMP sensitivity analysis, FMP and N-1 contingency, and power system expansion considering FMP. This paper aims at signifying the importance of FMP in power systems as well as illustrating the meaningful relationship between FMP with other parameters. The results and mathematical formulations demonstrate that FMP is a useful index and can be used in power system planning, operation, and control as an alternative index for conventional indexes.

Keywords: Electricity Market; Flow-Gate Marginal Pricing; Optimal Power Flow; Power System Performance Constraints; Renewable Energy Resources; Transmission Line Congestion;

1. Introduction

Deregulation of electric power systems toward liberalized electricity market has created many new concepts and issues such as locational marginal pricing (LMP) [1-3], financial transmission rights and flow gate marginal pricing (FMP) [3, 4], and congestion surplus [5]. These issues mainly impact on the system performance, operation, planning, and participants’ profit. In this respect, many studies have been performed to evaluate the electricity market concepts and applications of market concepts (including LMP and FMP) in power system planning, operation, and control.

Power system expansion planning is mainly addressed by the three terms of generation expansion planning (GEP) [6], transmission expansion planning (TEP) [7], and distribution expansion planning [8]. GEP and TEP have been widely presented under deregulated market. The objective of planning is to maximize the profit for each participant (generation company (GENCO) or transmission company (TRANSCO)) in deregulated environment, while in non-deregulated power systems, the objective is to minimize the costs [6, 7, 9-11]. TEP has been investigated from different aspects such as considering LMPs [12], FMPs [13, 14], load curtailment cost [15], transmission congestion cost [16-19], congestion surplus [20, 21], and minimizing market risk [22] in deregulated environments. TEP in the electricity market can also be studied from view of the planners’ rights [23]. A transmission line affects the system’s transfer capability through two characteristics of the capacity and the admittance of the line. Therefore, the transmission expansion project under deregulated electricity market provides the rights such as the right to collect the value of the capacity provided by the expansion project as well as the value arising from the admittance of the expansion project for the developer [23].

GEP has been widely investigated in deregulated market where each GENCO tries to maximize its revenue while satisfying the independent system operator (ISO) criteria such as reliability and reserve margin [24]. In electricity market, GEP is presented in pool market [10, 25-27] or the LMPs are used to signify the GENCOs profit [13, 14]. It has been shown that generation and transmission topologies have a great effect on the electricity market parameters such as LMPs, load payment, generation revenues, congestion rents, and FMPs [28]. It has been shown by [28] that the market principles such as LMP, FMP can be varied by changing the system topology. Therefore, it is possible to modify the market principles by choosing certain transmission elements to be open in order to achieve a better dispatch.

Power system operation has also been reconfigured in electricity market. The problems such as unit commitment [29, 30], security constrained unit commitment [31, 32], and price-based unit commitment [33] have been presented in deregulated environment. In such problems, the objective is to maximize the profit obtained from selling energy and spinning reserve in market. Other power system concepts have also been investigated on the basis of market environments such as optimal power flow [34], harmonic losses [2], renewable energies [35], frequency control [36], FACTS devices [37], congestion management, and distributed generation (DG) planning [38].

Regarding this large number of investigations, following issues are pointed out: (i) conventional problems should be considered regarding deregulated market to obtain appropriate results; (ii) the deregulated market problems are reconfigured and presented regarding market concepts such as LMP, FMP, congestion surplus; (iii) the main purpose of the deregulated environment is to focus on the economical parameters and not on the technical parameters. Therefore, it is worth studying market concepts and providing a relationship between the market parameters and power system variables.

In this regard, flow gate marginal pricing (FMP), which has not been adequately reviewed and studied, is a useful parameter in market studies. FMP is a parameter similar to LMP; while, LMP provides suitable information about the power system nodal or buses; and FMP presents a suitable view about the
transmission lines and congestion [39]. Although FMP has not been investigated as much as LMP, the importance of FMP is not less than that of LMP. FMP is a useful index, which can be used in power system planning, operation, control, and managing as a decision-making factor. In addition, FMP provide a view from flows in the transmission lines and system congestion based on the currency [39].

With respect to the importance of FMP, this paper aims at providing a comprehensive study on FMP and its effects on power systems. In this regard, the mathematical formulation of FMP and its relationship with LMP; the correlation of FMP with voltages, powers, and congestion are studied. Simulation results demonstrate that FMP is a useful index for power system planning, operation, and control; it can be considered as an alternative index to replace the conventional indexes.

2. Problem Formulation

LMP is a market-pricing approach used to manage the efficient use of the transmission network when congestion occurs. Congestion is mainly arisen when transmission constraints prevent the transmission of power to the demand in a certain location. LMP comprises the cost of supplying more expensive electricity to those locations. LMP presents a pricing signal for GENCOs and TRANSCOs in order to invest in new generation units and transmission lines. In addition, FMP is the shadow price associated with a flow gate generally defined as any transmission element or set of elements. The FMP is equivalent to the change in the social benefit of transactions settled through the spot market when the transmission constraint is relaxed by an increment [39]. FMP reveals a valuable pricing signal for TRANSCOs in order to invest in new transmission lines.

LMP and FMP can be calculated by solving optimal power flow (OPF) problem, where LMP and FMP are Lagrangian multipliers of OPF problem. The OPF is basically a non-linear constrained optimization problem and consists of a scalar objective function with a set of equality and inequality constraints. A typical OPF problem for minimizing the congestion surplus can be mathematically formulated as follows:

Min \( P_{iO,\theta O} - P_{iO,\theta G} \)  

Subject to

\[
\begin{align*}
\left( P_{iO,\theta O}, P_i, Q_i, Q_{bo,\theta O} \right) & = 0, \quad i, j \in N \\
P_{max} & \leq P_i \leq P_{min}, \quad i, j \in N \\
Q_{max} & \leq Q_i \leq Q_{min}, \quad i, j \in N \\
V_{min} & \leq V \leq V_{max} \\
S_{from} & \leq S \leq S_{max} \\
\end{align*}
\]

where the objective function (1) minimizes the difference between consumers’ payments and generation revenue. This difference is also known as the congestion surplus. In (1), \( P_{iO} \) and \( \pi_{iO} \) show the vectors of demand and its price, \( P_{iO} \) and \( \pi_{iO} \) indicate the vectors of generation and its cost respectively. Constraints (2)-(5) introduce the active and reactive power balance of each bus used in the AC power flow; where, \( P_{iO} \) and \( Q_{iO} \) show the vectors of real and reactive loads, \( P_{iG} \) and \( Q_{iG} \) indicate the vectors of real and reactive power generation. Equations (4)-(5) limits the real and reactive power of generators; where, \( P_{max} \) and \( Q_{max} \) show the maximum limits of real and reactive powers and \( P_{min} \) and \( Q_{min} \) represent the minimum limits. Equation (6) limits the voltage magnitude; where, \( V_{max} \) and \( V_{min} \) show the maximum and minimum limits. The thermal limit of the transmission lines is represented by (7)-(8); where, \( S_{from} \) and \( S_{max} \) show the apparent power flow through the branches in both terminals and \( S_{max} \) indicates the flow limit. Also, \( P(V, \theta, n) \) and \( Q(V, \theta, n) \) given in (2) and (3) are calculated as follows:

\[
\begin{align*}
P_{iO,\theta O} & = \sum_{j=1}^{N_{b}} \left[ G_{ij}(\theta) \cos \theta_{ij} + B_{ij}(\theta) \sin \theta_{ij} \right] \\
Q_{iO,\theta O} & = \sum_{j=1}^{N_{b}} \left[ G_{ij}(\theta) \sin \theta_{ij} + B_{ij}(\theta) \cos \theta_{ij} \right]
\end{align*}
\]

where, \( i, j \) show the bus indices, \( N_b \) indicates the set of all buses, \( V \) and \( \theta \) show the magnitude and angle of voltages. The elements \( G \) and \( B \) are calculated as follows:

\[
\begin{align*}
G_{ij}(\theta) & = \left( n_{ij} g_{ij} + n_{ij}^G g_{ij}^G \right) \\
B_{ij}(\theta) & = \left( b_{ij} + n_{ij}^B b_{ij}^B \right) + (\cos \theta_{ij} + \sin \theta_{ij}) \\
\end{align*}
\]

Where, \( b_{ij}^B \) shows the shunt susceptance of line \( ij \), \( b_{ij}^B \) shows the shunt susceptance at bus \( i \), \( g_{ij} \) and \( b_{ij} \) indicate the conductance and susceptance of the transmission line \( ij \) and \( j \) shows the set of all load buses.

\[
\begin{align*}
S_{from} & = \sqrt{\left( P_{from} \right)^2 + \left( Q_{from} \right)^2} \\
S_{max} & = \sqrt{\left( P_{max} \right)^2 + \left( Q_{max} \right)^2}
\end{align*}
\]

where,

\[
\begin{align*}
P_{from} & = V_{i}^2 g_{ij} - V_{j}^2 g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij} \\
Q_{from} & = V_{i}^2 b_{ij} + V_{j}^2 b_{ij} \cos \theta_{ij} - b_{ij} \sin \theta_{ij} \\
P_{max} & = V_{i}^2 g_{ij} - V_{j}^2 g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij} \\
Q_{max} & = V_{i}^2 b_{ij} + V_{j}^2 b_{ij} \cos \theta_{ij} - b_{ij} \sin \theta_{ij}
\end{align*}
\]

2.1. LMP and FMP definition as Lagrange multipliers

The proposed OPF problem can be solved by Lagrange multiplier method. Lagrange multiplier method is a methodology to find the optimal solution of a constrained optimization problem as follows:

Min \( f(x, y) \)  

Subject to

\[
\begin{align*}
g(x, y) & = 0, \quad (x, y) \in \mathbb{R}^n \\
h(x, y) & < 0, \quad (x, y) \in \mathbb{R}^n
\end{align*}
\]

The Lagrangian multiplier method can be used to solve the above optimization problem as follows:

\[
\lambda(x, y) = f(x, y) + \sum \lambda(x, y) + \sum \mu(x, y) \]

where, \( x \) and \( y \) are the Lagrange multipliers. Therefore, the proposed optimization problem presented in (1) to (8) can be solved by Lagrange multiplier method. In the proposed OPF problem, the Lagrangian multiplier of constraint (2) shows the LMPs and that of constraints (7) and (8) indicates the FMPs. It worth noting that only one constraint of (7) and (8) is satisfied for each condition.

2.2. Mathematical relationship between FMP and LMP

In order to find the relationship between LMP and FMP, at first the power transfer distribution factor (PTDF) should be defined. PTDF is the sensitivity of a flow to an injection relative to a reference location. This index is useful to find the impact of transactions on flow-gates, where the flow-gate is a congested line or corridor. The regular method to find the PTDFs is to find the base power flows and then, to change an injection in the system to find the new flows and eventually, the PTDFs are calculated by dividing the change in the flow by a change in the
injection. In addition, a mathematical model to find PTDFs can be presented as follows:

\[
PTDF = J^T f
\]

(23)

Where \( J \) is the ordinary Jacobian and \( f \) is the Jacobian with respect to flows. In addition, the PTDFs can be approximately calculated as follows:

\[
PTDF = B^{-1} B' f
\]

(24)

Where \( B \) is the reduced nodal susceptance matrix and \( B' \) is a reduced matrix with the branch susceptances. The reduced matrix means that rows (and columns) corresponding to a reference location are eliminated (results are insensitive to this choice). Now, the relationship between LMP and FMP can be defined on the basis of the PTDF concept; this relationship is as follows:

\[
LMP_{k} - LMP_{i} = \sum_{j=1}^{NF} FMP_{ij} \left( PTDF_{ij} - PTDF_{kj} \right)
\]

(25)

where, \( LMP_{k} \) and \( LMP_{i} \) show LMP at bus \( k \) and \( i \), respectively, \( NF \) represents the number of flow-gates with FMP. \( FMP_{ij} \) indicates the FMP at flow-gate \( j \). \( PTDF_{ij} \) shows the PTDF of bus \( i \) to line \( j \) and \( PTDF_{kj} \) indicates the PTDF of bus \( k \) to line \( j \). The proposed relationship shows that FMP can provide suitable information about the system, and it can be directly used to improve the system performance from the view of electricity market.

2.3. Transmission pricing schemes

Defining a pricing schemes for the generation sectors (or power system buses) and transmission lines can be very useful and effective in electric power systems. The pricing schemes for the power system buses are known as locational marginal pricing (LMPs) and all the power system operators and consumers are now aware of the advantages of LMPs. From the view of transmission network, congestion management and tradable transmission rights are the fundamental elements in the design of restructuring power markets. The regional transmission organizations (RTOs) in US and European Union are envisioned to define a suitable transmission pricing schemes [40].

LMP shows the spot price at each bus of the system reflecting the marginal cost of energy at that time and location, while the transmission cost equals the difference between LMPs at the points of withdrawal and injection. While there is growing appreciation of LMP, there is still significant disagreement over the appropriate method for specifying the transmission rights so that scarce transmission capacity can be allocated and the market participants can hedge congestion costs. In general, the two basic types of financial transmission rights currently being discussed are the point-to-point right and the flow-based right [40].

The definition of transmission rights mainly depends on how transmission capacity is measured. Two common methods are used to specify the transfer capability of the power system. One way is to determine the point-to-point transfer capabilities, and the other method is to compute the power-flow-carrying capacity for each link of the power system. The disadvantage of the point-to-point financial right can be pointed out as: (i) the transfer capability between any two points in a network changes continuously and (ii) there are a large number of potential point-to-point combinations. In contrast, the capacity of each link or flow-gate is mainly denoted by physical factors associated with the link (e.g. thermal limit, voltage stability, and dynamic stability) and is generally insensitive to the power flow pattern. Each power transfer requires approximately a constant fraction (PTDF) of the capacity of each link in the network [40].

3. FMP analysis and calculation

In order to calculate and evaluate FMPs, a typical six-bus test system is considered as Fig. 1. Buses 1, 2, and 3 are the generation buses and the other buses are load buses. The system data are derived from [41]. The market data are listed in Tables 1-2. The maximum permitted power of each line can also be found in Table 2.

![Image](Fig. 1. Single line diagram of six-bus test system)

| Table 1: Generation and demand data for the test system |
|----------|-----------|------------|-------------|
| Bus     | Generation MW | Offer Price [$/MWh] | Demand MW | MVar |
| 1       | 90         | 90         | 9.7        | -     |
| 2       | 100        | 100        | 8.8        | -     |
| 3       | 60         | 60         | 7.0        | -     |
| 4       | -          | -          | 65         | 45    |
| 5       | -          | -          | 85         | 70    |
| 6       | -          | -          | 90         | 60    |

| Table 2: Line data for the test system |
|----------|-------------|----------------|------------|
| Line No | From Bus | To Bus | R (p.u.) | X (p.u.) | Imax (p.u.) |
| 1       | 2         | 3       | 0.05     | 0.25     | 0.06       | 0.30       |
| 2       | 3         | 6       | 0.02     | 0.10     | 0.02       | 0.60       |
| 3       | 4         | 5       | 0.20     | 0.40     | 0.08       | 0.17       |
| 4       | 5         | 3       | 0.12     | 0.26     | 0.05       | 0.20       |
| 5       | 5         | 6       | 0.10     | 0.30     | 0.06       | 0.20       |
| 6       | 2         | 4       | 0.05     | 0.10     | 0.02       | 1.37       |
| 7       | 1         | 2       | 0.10     | 0.20     | 0.04       | 0.25       |
| 8       | 1         | 4       | 0.05     | 0.20     | 0.04       | 0.70       |
| 9       | 1         | 5       | 0.08     | 0.30     | 0.06       | 0.84       |
| 10      | 2         | 6       | 0.07     | 0.20     | 0.05       | 0.90       |
| 11      | 2         | 5       | 0.10     | 0.30     | 0.04       | 0.71       |

By using the proposed methodology provided in section 2, the system FMPs and LMPs are calculated. The OPF problem is solved by PSAT software [41] and the results are listed in Tables 3-4. It is clear that only one line is congested and FMP is obtained for the proposed line between bus 3 and bus 5. It is also clear that LMP at generation buses is less than the other buses.

<table>
<thead>
<tr>
<th>Table 3: The results of OPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generated power (MW)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>LMP ($/MWh)</td>
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<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>FMP ($/MWh) From Bus 3 to Bus 5 (line 4)</td>
</tr>
<tr>
<td>Congestion surplus ($/h)</td>
</tr>
</tbody>
</table>

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With respect to the results, it is clear that FMP is only defined for the congested lines and FMP of line 4 is 2.3492 ($/MWh). In addition, the value of LMP at bus 3 is the lowest while it is the highest at bus 5. Therefore, there is a meaningful relation between LMPs and FMPs. In order to conduct more analyses, it is useful to observe the effects of congestion on the system. In this respect, the constraint of the maximum power at congested line 4 is relaxed (constraints (7) and (8)); the results for such condition are listed in Table 5. It is clear that without the congested lines, there are no FMPs and LMPs are significantly changed and the congestion surplus is reduced by almost 50%. The total generation of the system is also reduced due to the removal of the system constraints. In this case, the LMP at bus 3 is increased, while the LMP at bus 5 is reduced in comparison with the previous case. These results originate from an increase in the search space of the optimization problem through the removal of the constraints.

<table>
<thead>
<tr>
<th>Line No</th>
<th>I (p.u.)</th>
<th>Imax (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2250</td>
<td>0.3082</td>
</tr>
<tr>
<td>2</td>
<td>0.5589</td>
<td>0.6000</td>
</tr>
<tr>
<td>3</td>
<td>0.1365</td>
<td>0.1796</td>
</tr>
<tr>
<td>4</td>
<td>0.2000</td>
<td>0.2000</td>
</tr>
<tr>
<td>5</td>
<td>0.0453</td>
<td>0.2000</td>
</tr>
<tr>
<td>6</td>
<td>0.4654</td>
<td>1.3740</td>
</tr>
<tr>
<td>7</td>
<td>0.1622</td>
<td>0.2591</td>
</tr>
<tr>
<td>8</td>
<td>0.3766</td>
<td>0.7000</td>
</tr>
<tr>
<td>9</td>
<td>0.4135</td>
<td>0.8478</td>
</tr>
<tr>
<td>10</td>
<td>0.4946</td>
<td>0.9000</td>
</tr>
<tr>
<td>11</td>
<td>0.3327</td>
<td>0.7114</td>
</tr>
</tbody>
</table>

With respect to the results, it is clear that FMP is only defined for the congested lines and FMP of line 4 is 2.3492 ($/MWh). In addition, the value of LMP at bus 3 is the lowest while it is the highest at bus 5. Therefore, there is a meaningful relation between LMPs and FMPs. In order to conduct more analyses, it is useful to observe the effects of congestion on the system. In this respect, the constraint of the maximum power at congested line 4 is relaxed (constraints (7) and (8)); the results for such condition are listed in Table 5. It is clear that without the congested lines, there are no FMPs and LMPs are significantly changed and the congestion surplus is reduced by almost 50%. The total generation of the system is also reduced due to the removal of the system constraints. In this case, the LMP at bus 3 is increased, while the LMP at bus 5 is reduced in comparison with the previous case. These results originate from an increase in the search space of the optimization problem through the removal of the constraints.

In order to evaluate the wind turbine uncertainties on the FMPs, a 200 (MW) wind generation unit is considered at bus 3. Fig. 3 shows the FMPs in line 4 following changing the wind turbine output from zero to the nominal value. It is clear that by a change output power of wind turbine, the FMP is increased from 2.34 to 12.18. The FMP is constant for powers above 0.1 p.u. Fig. 3 shows that FMPs are associated with generation system. Therefore, it is necessary to evaluate the FMP in the presence of such uncertainty. Such FMP studies should be carried out based on the probabilistic methodologies [13].

### Table 5: The results following relaxation of the maximum power constraint at line 4

<table>
<thead>
<tr>
<th>Generated power (MW)</th>
<th>LMP ($/MWh)</th>
<th>FMP ($/MWh)</th>
<th>Congestion surplus ($/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 1</td>
<td>86.57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bus 2</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bus 3</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bus 1</td>
<td>9.7000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bus 2</td>
<td>9.8509</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bus 3</td>
<td>10.0223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bus 4</td>
<td>10.1163</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bus 5</td>
<td>10.3435</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bus 6</td>
<td>10.2770</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. **Applications of FMP in electric power systems**

There is a meaningful relationship between FMP and power system parameters such as voltage limits, transmission lines permitted powers, generation limits, etc. Based on this relationship, FMP can be suitably used in electric power systems as an effective index in power systems. Many power system problems such as transmission expansion planning (TEP), coordinated GEP-TEP, congestion management, bilateral transactions, financial transmission rights (FTR), and transmission service settlement can be carried out while considering FMP. In order to show the FMP effects on power system, some FMP effects are studied and simulated as follows;

#### 4.1. Effects of the renewable energy uncertainties on the FMP

Renewable energies have been widely developed in electric power system due to their benefits such as reducing generation cost, reducing losses and increasing reliability. Renewable generation mainly has a significant effect on the net transfer capacity and congestion of network which is more important in electricity market [42]. One of the most important renewable sources is wind power. Wind turbines are associated with uncertainty and it is necessary to consider this uncertainty in system. Wind turbine generator (WTG) output is nonlinearly related to wind speed. The nonlinear model of a WTG has three parameters: cut in speed ($V_{ci}$), rated speed ($V_{co}$), and cut out speed ($V_{co}$).

\[
\begin{align*}
\frac{V_{ci}}{V_{co}} & \leq V \leq \frac{V_{co}}{V_{ci}} \\
P &= \begin{cases} \\
0 & \text{if } V < V_{ci} \\
\frac{V_{co}}{V_{ci}} (V - V_{ci}) & \text{if } V_{ci} \leq V \leq V_{co} \\
0 & \text{if } V > V_{co} \\
\end{cases}
\end{align*}
\]

Fig. 2. Relationship between wind speed and WTG output power

In order to evaluate the wind turbine uncertainties on the FMPs, a 200 (MW) wind generation unit is considered at bus 3. Fig. 3 shows the FMPs in line 4 following changing the wind turbine output from zero to the nominal value. It is clear that by a change output power of wind turbine, the FMP is increased from 2.34 to 12.18. The FMP is constant for powers above 0.1 p.u. Fig. 3 shows that FMPs are associated with generation system. Therefore, it is necessary to evaluate the FMP in the presence of such uncertainty. Such FMP studies should be carried out based on the probabilistic methodologies [13].

#### 4.2. Effects of the reactive power on the FMP

FMP is an index to show the system congestion and therefore, the reactive power of lines may have an effect on the FMPs. In order to show the effects of reactive power on the FMPs, the reactive loads are changed from zero to nominal values; the FMPs are shown in Table 6 for such conditions. Table 6 shows that along with an increase in reactive demands, the capacity of lines are more occupied by reactive power and the FMP is increased. In addition, for large values of the reactive demands, the system cannot serve the demands OPF diverges. Based on the results, FMP can be used as a suitable index for...
reactive power management, location, and expansion under electricity market environment.

<table>
<thead>
<tr>
<th>Total reactive power (p.u.)</th>
<th>FMP of line 4 ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0.00</td>
</tr>
<tr>
<td>30% of the nominal values</td>
<td>$0.22</td>
</tr>
<tr>
<td>80% of the nominal values</td>
<td>$0.81</td>
</tr>
<tr>
<td>100% of the nominal values</td>
<td>$2.34</td>
</tr>
<tr>
<td>120% of the nominal values</td>
<td>$3.43</td>
</tr>
<tr>
<td>150% of the nominal values</td>
<td>OPF divergence</td>
</tr>
</tbody>
</table>

### 4.3. FMP sensitivity analysis

In this section, the sensitivity of FMP to the system constraints is investigated. It is worth mentioning that the system constraints are defined in section 2. Table 7 presents the sensitivity of FMP to the voltage constraint. It is clear that along with a decrease in the constraint range, the FMP is increased. This is due to a reduction in the search space of the optimization problem. Table 8 shows the sensitivity of FMP to the reactive power of generators. It is clear that along with a decrease in the constraint range, the FMP is increased as in the previous case.

<table>
<thead>
<tr>
<th>Permitted limits of voltage</th>
<th>FMP of line 4 ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8&lt;V&lt;1.2</td>
<td>$1.098</td>
</tr>
<tr>
<td>0.9&lt;V&lt;1.1</td>
<td>$2.34</td>
</tr>
<tr>
<td>0.95&lt;V&lt;1.05</td>
<td>$3.47</td>
</tr>
</tbody>
</table>

Table 7: The sensitivity of FMP to the voltage constraint

<table>
<thead>
<tr>
<th>Permitted limits of reactive power</th>
<th>FMP of line 4 ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal ranges</td>
<td>$2.34</td>
</tr>
<tr>
<td>20% reducing nominal ranges</td>
<td>$2.34</td>
</tr>
<tr>
<td>40% reducing nominal ranges</td>
<td>$2.89</td>
</tr>
<tr>
<td>60% reducing nominal ranges</td>
<td>OPF divergence</td>
</tr>
<tr>
<td>80% reducing nominal ranges</td>
<td>OPF divergence</td>
</tr>
</tbody>
</table>

Table 8: The sensitivity of FMP to the reactive power of the generators

### 4.4. FMP and N-1 contingency

FMP is an index related to the transmission line congestion. Therefore, it is valuable to assess the FMP following outage of transmission lines as N-1 contingency. Table 9 shows the FMP following N-1 contingency for some lines. It is clear that line 4 is the most critical line from the view of FMP and contingency. In addition, when line 10 is subject to outage, the highest FMPs occur in lines 1 and 4; therefore, outage of line 10 can be denoted as the critical condition for this network. If the power system planner tends to reinforce the network from the view of security, installing a parallel line with line 10 is the best choice. It is worth noting that some lines are not presented in Table 9, since the OPF diverges for such lines.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Outage line</th>
<th>Congested lines</th>
<th>$S^m$ (p.u.)</th>
<th>$S^m$ (p.u.)</th>
<th>$S^n$ (p.u.)</th>
<th>FMP ($/MWh$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No outage</td>
<td>Line 4</td>
<td>0.1815</td>
<td>0.2</td>
<td>0.2</td>
<td>$2.3492$</td>
</tr>
<tr>
<td>2</td>
<td>Line 1</td>
<td>Line 4</td>
<td>0.1697</td>
<td>0.2</td>
<td>0.2</td>
<td>$0.9332$</td>
</tr>
<tr>
<td>3</td>
<td>Line 5</td>
<td>Line 4</td>
<td>0.1796</td>
<td>0.2</td>
<td>0.2</td>
<td>$1.9937$</td>
</tr>
<tr>
<td>4</td>
<td>Line 7</td>
<td>Line 4</td>
<td>0.1747</td>
<td>0.2</td>
<td>0.2</td>
<td>$1.6523$</td>
</tr>
<tr>
<td>5</td>
<td>Line 10</td>
<td>Line 4</td>
<td>0.2081</td>
<td>0.3</td>
<td>0.3</td>
<td>$12.0185$</td>
</tr>
<tr>
<td></td>
<td>Line 4</td>
<td>Line 4</td>
<td>0.1644</td>
<td>0.2</td>
<td>0.2</td>
<td>$11.2992$</td>
</tr>
</tbody>
</table>

Table 9: The FMP following outage of transmission lines as N-1 contingency

As stated above, line 4 is the most congested line in the network and comprises FMP. In order to show the relationship between the maximum permitted power of line 4 and FMP, this relationship is shown in Fig. 4. It is clear that along with an increase in the maximum permitted power of line 4, FMP is increased and eventually becomes zero. This figure provides a suitable view of congestion at line 4 for power system operator and each investment at this corridor (corridor between bus 3 and bus 5) can be easily assessed. For instance, the capacity of this corridor can be increased by series capacitor and therefore, cost of an increase in capacity can be easily calculated. The revenue obtained through reducing FMP can also be computed. Then, the net profit of this expansion is obtained as revenue minus cost.

![Fig. 4. The FMP of line 4 following changing the maximum permitted power of line 4](image)

### 4.5. Power system expansion considering FMP

In deregulated systems, the main objective of ISO is to provide a nondiscriminatory and competitive environment for all stakeholders such as TRANSCOs, GENCOs and consumers; while the main objective of private participants is to maximize their profit. In this regard, when a section of the network needs to invest, ISO should provide suitable incentives to convince the private participants to invest on the proposed section. Respecting this issue, FMP is an index that can be suitably used by ISO for persuading TRANSCOs to invest on the transmission system. In the proposed test system, line 4 is congested and FMP of this line is $2.34$ ($/MWh$). Therefore, installing a new line in this corridor is suitable for ISO as well as TRANSCO. Since TRANSCO obtains profit and ISO improves the performance of its network by reducing congestion at this line. The system analyses following installing a new line in the proposed corridor are listed in Table 10. It is clear that the LMPs, FMPs, and congestion surplus are reduced following installing a new line. The ISO and TRANSCO costs and profits can be easily calculated with respect to the FMP. The TRANSCO’s profit is calculated as follows:

$$TP = (FMP\cdot P_L) - C_{r, O}$$  (27)
Equation (27) comprises three terms, where, the first term
denotes the revenue from installing new line and two other terms
represent the investment and operation-maintenance costs. In
(27), \( TP \) shows the TRANSCO’s profit, \( FMP_L \) indicates the
FMP at line \( L \), \( C_L \) represents the investment cost of new line \( L \),
and \( O_L \) shows the operation and maintenance cost. The lifetime
and investment cost of line are equal to 15 years and 30 Million
dollars [45]. The operation-maintenance costs are equal to 10% of
the investment cost. By using this data, the profit of
TRANSCO is obtained as 28.73 million dollar for 15 years or
1.915 million dollar as annually profit. The ISO is also impart at
this planning and ISO profit is calculated as follows;
\[
ISOP=(CS_0-CS_S)^{LT_L} \tag{28}
\]
Where, \( ISOP \) shows the ISO profit, \( CS_0 \) and \( CS_S \) show the
congestion surplus of the network before and after expansion and
\( LT_L \) shows the lifetime of new line per hours. The profit of
ISO is calculated as 4.129 million dollar for 15 years or 0.275
million dollars as annually profit.

Regarding the presented analysis, installing a new line based
on the FMP concept can satisfy the TRANSCOs and ISO. This
methodology not only provides suitable profit for TRANSCO
but also has advantages for ISO such as reducing the system
congestion surplus.

Table 10: The results following installing a new line at corridor 4

<table>
<thead>
<tr>
<th>Generated power (MW)</th>
<th>LMP ($/MWh)</th>
<th>FMP ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 1</td>
<td>86.12</td>
<td></td>
</tr>
<tr>
<td>Bus 2</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Bus 3</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Bus 1</td>
<td>9.7000</td>
<td></td>
</tr>
<tr>
<td>Bus 2</td>
<td>9.8547</td>
<td></td>
</tr>
<tr>
<td>Bus 3</td>
<td>10.0509</td>
<td></td>
</tr>
<tr>
<td>Bus 4</td>
<td>10.1164</td>
<td></td>
</tr>
<tr>
<td>Bus 5</td>
<td>10.3254</td>
<td></td>
</tr>
<tr>
<td>Bus 6</td>
<td>10.2885</td>
<td></td>
</tr>
<tr>
<td>Congestion surplus ($/h)</td>
<td></td>
<td>0.154</td>
</tr>
</tbody>
</table>

Installing a new line in the network was successfully
introduced in the previous example. However, in the large-scale
power systems with many lines and FMPs, the proposed planning
can be defined as an optimization problem, which aims at
maximizing the TRANSCOs, profit as follows;
\[
\text{Max} (FMP_1 + P_L - C_L - O_L) \tag{29}
\]
Subject to
\[
\begin{align*}
(N+N_C)S_{\text{max}} & s(N+N_C)S_{\text{max}} \tag{30} \\
(N+N_C)S_{\text{max}} & s(N+N_C)S_{\text{max}} \tag{31}
\end{align*}
\]
Equations (2) to (6)

Where, objective function (29) aims at maximizing the profit
of TRANSCOs subject to system constraints. In this optimization
planning, \( N \) and \( N_0 \) show the existing and new lines
in each corridor and the other parameters have already been
defined.

5. Discussions
FMP is a suitable index in deregulated market, which has not
been adequately investigated. This index provides a suitable
view about the system congestion. The importance of each
congested line is signified with an economic weight per money.
FMP provides alternative methods for congestion management
and financial transmission rights. In conventional congestion
management, all congested lines have similar weight and value.
However, by the FMP concept, each congested line is signified
with a weighted value and the priority of lines is signified for
the planner. Based on the FMP concept, the following issues
are suggested as further and novel studies:
- **Congestion management**: In conventional congestion
management, all congested lines have similar weight and value
for planner. However, by using FMP, each congested line is
associated with a FMP and the priority of the congested lines
is signified. In addition, the net profit of TRANSCOs can be
calculated by FMPs of lines.
- **Transmission expansion planning**: In deregulated
environment, one of the challenging issues is to signify the
TRANSCOs profit. However, FMP can be suitably used to
signify the TRANSCOs profit.
- **DG planning**: In deregulated market, DG planning can be
performed to maximize the profit of individual participants by
maximizing the financial transmission rights.
- **Reactive power planning**: This planning can be easily
carried out to maximize the profit of individual participants by
maximizing the financial transmission rights as the previous
case.
- **Optimal power flow**: Optimal power flow (OPF) can be
reconfigured by considering FMP at objective function.
- **Electricity market**: FMP can be used to denote the power
transfer rights in deregulated power system, especially for
bilateral transactions in electricity market. In such transactions,
the right of transmission can be assessed based on the FMP of
lines. In addition, the effects of reserved power for bilateral
transaction can be evaluated on the system performance.

6. Conclusions
This paper addressed a comprehensive evaluation of FMP in
electricity market. Mathematical formulation for calculating
FMP was presented as an OPF problem. Simulations were
carried out on a six-bus system. FMPs were affected by the
power system constraints such as voltage and power. The
sensitivity of FMP was investigated and the results demonstrated
that FMP is increased following reduction in the constraint
ranges. This paper showed that FMP could be considered as
a suitable index in the power system planning, operation,
management, and control; since this index was thoroughly
associated with all system parameters and provided a weight for
each congested lines based on the currency. The paper highlights
can be concluded as follows:
- Providing a mathematical formulation for LMP and FMP
calculation;
- Addressing the mathematical relationship between LMP and
FMP;
- Defining the transmission pricing schemes;
- Showing the effects of the renewable energy uncertainties on
FMP; and providing a new idea for further research activities.
- The effect of the reactive power on FMP was investigated. The
reactive power is a suitable parameter to control FMP
without changing load. This issue provides a new idea for
further research activities.
- FMP sensitivity analysis showed the effects of power system
elements on FMP.
- The power system expansion considering FMP has also been
addressed, this research opens the doors to more works in
this regard.

7. References
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