Sensorless DC Voltage Control with Backstepping Design Scheme for Shunt Active Power Filter

Ahmed A. Elkousy  Sherif A. Zaid
Faculty of Engineering, Cairo University, Cairo 12613
{aelkousy&sherifzaid3}@yahoo.com

Shokry M. Saad  Ashraf A. Hagras
Atomic Energy Authority, AboZaabal, Cairo 13759
ashrafa1973@yahoo.com & shokryms@hotmail.com

Abstract: In this paper a novel sensorless DC voltage regulation controller using backstepping design is proposed for three phase shunt active power filters (SAPF). The control scheme consists of nonlinear backstepping controllers and DC voltage observer to achieve the purpose of DC voltage regulation. The DC voltage observer and the backstepping controllers are developed and designed recursively based on adaptive backstepping design algorithm. The proposed control scheme not only stabilizes the operation of SAPF but also drives the DC voltage error to converge to zero asymptotically. Furthermore, some simulation results are given to illustrate the excellent performance of the nonlinear backstepping control applied to SAPF by comparison between the adaptive backstepping and sensorless backstepping control.

Key words: Nonlinear Control, Adaptive Backstepping Design, Shunt Active Power Filter.

1. Introduction

Nonlinear loads such as controlled and uncontrolled diode/thyristor rectifiers, cycloconverters and arc furnaces are utilized widely in industrial, commercial and consumer environment. These power electronics devices constitute the current source power supplies of the Egypt MGC-20 cyclotron and draw nonsinusoidal currents from the local power network. These currents contain harmonics and reactive components which degrade power quality of the utility system and interfere with the radio frequency generator system of the cyclotron. Also, harmonics increase power system losses, damage sensitive loads, create significant interferences with communication systems and generate noise on regulating devices and control systems. Therefore, compensation of harmonics and reactive currents has become a major concern for power systems specialists [1-4].

Usually, high performance active filter requires fast accurate response, quick recovery from any disturbance and insensitive to parameter variation. The dynamic behavior can be improved significantly using vector control where the active filter variables are transformed into an orthogonal set of d-q coordinates such that the active and reactive current can be controlled separately. This gives the active filter, the high dynamic performance capabilities of reactive power and harmonics compensation etc. [5-6].

However, precise DC voltage control becomes a complex issue due to the uncertain parameters and the nonlinearity between the active, reactive current and the DC voltage. The system nonlinearity becomes severe as the system offers uncertainties and parameter variations. Application of some nonlinear control methods may lead to cancellation of useful nonlinearities and the results may not be achieved easily.

There have been significant developments in nonlinear control theory applicable to active filter. Interesting the d-q transformation applicable to active filter can be considered as feedback linearization [7-10]. However with the recent development in nonlinear control theory, a modern control engineer has not only found a system approach like combining feedback linearization with linear adaptive control technique in dealing with nonlinearities [11, 12] but has managed to develop approaches which had not been considered previously [13-18]. But these methods had limitations in compensating uncertainties or didn’t realize it at all retaining the achievement of the conventional control objectives.

Backstepping control is a newly developed technique and recursive design methodology for the control of uncertain nonlinear systems, particularly those systems that do not satisfy matching conditions [19-28]. The main idea of this technique is to design a controller recursively by considering some of state
variables as “virtual controls” and this method is usually suitable for parameter strict feedback systems.

As SAPF possesses well defined nonlinear model characteristics, they have become good candidate for the application of newly developed nonlinear control techniques. If the knowledge of such nonlinearities can be incorporated into the design of nonlinear controller, an enhanced dynamic behavior of the active filter can be accomplished.

The design of an APF requires a DC voltage sensor to achieve the control targets but very few works that don’t have DC voltage measurement devices. Eliminating the DC voltage sensor simplifies the structure of control of SAPF and widens the application fields. Most sensorless active filters operate without current sensors or AC voltage sensors and proved the stable operation of AF [29-31].

This paper presents a new sensorless control technique to SAPFs. This technique is based on backstepping design applied to the uncertain nonlinear model of SAPF which provides an efficient control design processes for both regulation and tracking problems of uncertain parameters of R and L.

This paper is organized as follows: firstly the SAPF model in the d-q reference frame is presented in section II. In section III, adaptive backstepping control methodology is developed. Also the sensorless backstepping control was developed in section IV. Finally section V gives the simulation results and the conclusions are drawn in section VI.

2. Mathematical Modelling of SAPF

The SAPF shown in Fig. 1 can be modeled in the d-q reference frame with the aim of reducing control complexity if compared with modeling in the stationary a-b-c reference frame.

The dynamic model as described in [32-33] is obtained as follows:

\[
\frac{di_d}{dt} = \frac{-R}{L_c} i_d + wi_q - \frac{v_{dc}}{L_c} d_m + \frac{v_q}{L_c} \quad (1)
\]

\[
\frac{di_q}{dt} = \frac{-R}{L_c} i_q - wi_d - \frac{v_{dc}}{L_c} d_n + \frac{v_d}{L_c} \quad (2)
\]

\[
\frac{dv_{dc}}{dt} = d_d \frac{i_d}{C} + d_q \frac{i_q}{C} \quad (3)
\]

Where

\[i_d, i_q\]: The three phase inverter currents in the d-q reference frame.

\[v_{dc}\]: The DC voltage.

\[R_c, L_c\]: The resistance and inductance of the filter.

\[d_d, d_q\]: The three phase switching state functions in the d-q reference frame.

Fig. 1 SAPF with a voltage source inverter

The model represented by (1), (2) and (3) is nonlinear because of the existence of multiplication terms between the state variables \{i_d, i_q, v_{dc}\} and the inputs \{d_d, d_q\}.

Furthermore the principle of operation of active filter requires that the three state variables have to be controlled independently. Therefore each of the currents \{i_d, i_q\} has to follow a varying reference frame extracted from nonlinear load current. In addition, the DC voltage level has to be regulated at a set point in order to maintain the compensation performance of the active filter during dynamic variations.

3. Adaptive Backstepping Control Design

The theory of adaptive backstepping nonlinear control design (ABNC) is a systematic approach to construct a Lyapunov equation and unperturbed controller so that the system is uniformly asymptotic stable at the equilibrium point. In the design, constructing suitable function enables system design to achieve expected purpose. The design objective of the controller is to obtain the switching state functions so as to achieve high-quality DC voltage and current tracking performance. This will be achieved when the active filter injects the load current in the opposite phase.

\[i_{dref} = -i_{dih} + i_{do} \quad (4)\]

\[i_{qref} = -i_{iq} \quad (5)\]

As the nonlinear model of SAPF has extended matching since the uncertainties enter the system one integrator before the control does. This leads to the adaptive backstepping being applicable to it. This control technique can be effectively employed to linearize any nonlinear system in the presence of uncertainties. Unlike the other feedback linearization methods, the advantage of adaptive backstepping is its flexibility as useful nonlinearities can be kept intact.
during stabilization.

The key idea of backstepping is systematically to decompose a complex control problem into smaller and simpler ones to select recursively appropriate functions of state variables as so-called “virtual controls” to be dealt with a decomposed subsystem problem and this virtual control becomes a reference to the next design step for another subsystem. But in this case we don’t need to decompose the control problem into smaller ones because the virtual controls are known.

In this work, the virtual controls are the reference currents which stabilize the operation of SAPF as indicated by equations (4) and (5). The pertinent control objective is the regulation of the DC voltage. The DC voltage error and its derivative are given as:

\[ e = v_{dcref} - v_d \]

\[ \dot{e} = \dot{v}_{dcref} - \dot{v}_d = \dot{v}_{dcref} - \frac{dv_k}{dt} = \frac{1}{C} \left[ \frac{d}{dt} d_d + i_d d_d \right] \]

The d-q current error variables can be defined as:

\[ e_d = i_{dref} - i_d \]

\[ \dot{e}_d = i_{dref} - \frac{di_d}{dt} = i_{dref} - \left[ -\frac{R}{L_c} i_d + \frac{v_{dref}}{L_c} d_d + \frac{v_d}{L_c} \right] \]

\[ e_q = i_{qref} - i_q \]

\[ \dot{e}_q = i_{qref} - \frac{di_q}{dt} = i_{qref} - \left[ \frac{R}{L_c} i_q - \frac{v_{qref}}{L_c} d_q + \frac{v_q}{L_c} \right] \]

As the active filter inductance and resistances have to be measured adaptively. Therefore, let these estimates be \( \hat{L}_c \) and \( \hat{R}_c \). One can define the following lyapunov function including the current and dc voltage error variables:

\[ V = \frac{1}{2} \left( e^2 + e_d^2 + e_q^2 + \frac{1}{\gamma_1} \hat{L}_c^2 + \frac{1}{\gamma_2} \hat{L}_c^2 \right) \]

Where \( \hat{L}_c = \hat{L}_c - L_c, \hat{R}_c = \hat{R}_c - R_c, \gamma_1 \) and \( \gamma_2 \) are adaptive gains.

By differentiating the lyapunov function and using the error dynamics one can get:

\[ \dot{V} = \dot{e} \dot{e} + e_d \dot{e}_d + e_q \dot{e}_q + \frac{1}{\gamma_1} \hat{R}_c \dot{\hat{R}}_c + \frac{1}{\gamma_2} \hat{L}_c \dot{\hat{L}}_c \]

Where \( k, k_1 \) and \( k_2 \) are closed loop feedback constants. The input switching functions can be derived from (9):

\[ d_d = \frac{L_c}{v_{dref}} \left[ -i_{dref} + \frac{R_c}{L_c} i_d + \frac{v_{dref}}{L_c} - \frac{k e_d - \frac{k e^2}{e_d}}{L_c} \right] \]

\[ d_q = \frac{L_c}{v_{dref}} \left[ -i_{qref} - \frac{R_c}{L_c} i_q + \frac{v_{qref}}{L_c} d_d + \frac{k e_q - \frac{k e^2}{e_d}}{L_c} \right] \]

Substituting for the estimated values for L and R:

\[ d_d = \frac{\hat{L}_c}{v_{dref}} i_{dref} + \frac{\hat{L}_c}{v_{dref}} \frac{\hat{R}_c}{v_d} \frac{v_{dref}}{v_d} d_d - \frac{\hat{L}_c}{v_{dref}} \frac{\hat{R}_c}{v_d} \frac{v_d}{v_d} k e_d \]

\[ d_q = \frac{\hat{L}_c}{v_{dref}} i_{qref} + \frac{\hat{L}_c}{v_{dref}} \frac{\hat{R}_c}{v_d} \frac{v_{qref}}{v_d} \frac{v_{dref}}{v_d} \]

Substituting in the general lyapunov equation (9) with further simplification:
\[ \dot{V} = -k e^2 - k_1 e_d^2 - k_2 e_q^2 \]
\[ + e_d \left( \frac{i_{\text{dref}}(1 - \frac{1}{L_c}) + \frac{i_d}{L_c} - \frac{k e_d}{e_d} \hat{L}_c}{i_{\text{dref}}(1 - \frac{1}{L_c}) + \frac{i_d}{L_c} - \frac{k e_d}{e_d} \hat{L}_c} \right) \]
\[ + e_q \left( \frac{i_{\text{qref}}(1 - \frac{1}{L_c}) + \frac{i_q}{L_c} - \frac{k e_q}{e_q} \hat{L}_c}{i_{\text{qref}}(1 - \frac{1}{L_c}) + \frac{i_q}{L_c} - \frac{k e_q}{e_q} \hat{L}_c} \right) \]
\[ + e_q \left( \frac{i_{\text{qref}}(1 - \frac{1}{L_c}) + \frac{i_q}{L_c} - \frac{k e_q}{e_q} \hat{L}_c}{i_{\text{qref}}(1 - \frac{1}{L_c}) + \frac{i_q}{L_c} - \frac{k e_q}{e_q} \hat{L}_c} \right) \]
\[ + \frac{i_d d e}{C} \left( \frac{L_c}{L_c} - 1 \right) + \frac{i_d d e}{C} \left( \frac{L_c}{L_c} - 1 \right) + \frac{1}{\gamma_1} \hat{R} \hat{L}_c + \frac{1}{\gamma_2} \hat{L}_c \]

Equation (10) can be simplified and arranged in the following form:
\[ \dot{V} = -k e^2 - k_1 e_d^2 - k_2 e_q^2 \]
\[ + e_d \left( \frac{e_d(1 - \frac{1}{L_c}) + \frac{i_d}{L_c} - \frac{k e_d}{e_d} \hat{L}_c}{e_d(1 - \frac{1}{L_c}) + \frac{i_d}{L_c} - \frac{k e_d}{e_d} \hat{L}_c} \right) \]
\[ + e_q \left( \frac{e_q(1 - \frac{1}{L_c}) + \frac{i_q}{L_c} - \frac{k e_q}{e_q} \hat{L}_c}{e_q(1 - \frac{1}{L_c}) + \frac{i_q}{L_c} - \frac{k e_q}{e_q} \hat{L}_c} \right) \]
\[ + \frac{\hat{R}}{L_c} \left( -e_d i_d - e_q i_q + \frac{L_c}{\gamma_1} \hat{L}_c \right) \]

From (11) the following update laws for the estimated adaptive values can be derived as:
\[ \hat{L} = \frac{\gamma_2}{L_c} \left[ e_d \left( i_{\text{dref}} - \frac{e_d}{L_c} \right) + k e_d - k_1 e_d^2 \right] \]
\[ + e_q \left( i_{\text{qref}} - \frac{e_q}{L_c} \right) + k e_q - k_2 e_q^2 \]
\[ + \frac{\hat{R}}{L_c} \left( -e_d i_d - e_q i_q + \frac{L_c}{\gamma_1} \hat{L}_c \right) \]
\[ \hat{\gamma} = \frac{\gamma_1}{L_c} \left[ e_d i_d + e_q i_q \right] \]

Based on the algorithm mentioned earlier the block diagram of the proposed ABNC is shown in Fig. 2.

4. Sensorless Backstepping DC voltage Control Design

Since the objective of this work is to realize the sensorless DC voltage control, a DC voltage observer must be employed to estimate the actual DC voltage. Hence, the DC voltage estimation error is defined by
\[ \tilde{v}_{\text{dc}} = v_{\text{dc}} - \hat{v}_{\text{dc}} \]

One can use the following Lyapunov Eq.:
\[ V = \frac{1}{2} \left( e^2 + e_d^2 + e_q^2 + \frac{1}{\gamma_1} \tilde{v}_{\text{dc}}^2 \right) \]

Differentiating this equation including the error dynamics and letting \( x = v_{\text{dc}} \), we can get:
\[ \dot{V} = -k e^2 - k_1 e_d^2 - k_2 e_q^2 + \dot{e} e + e_d \dot{e}_d + e_q \dot{e}_q + \frac{1}{\gamma_1} \tilde{v}_{\text{dc}} \]

Clearly \( \dot{V} \) is negative definite, so it implies that the resulting closed loop system is asymptotically stable and the errors \( \hat{L} \) and \( \hat{R} \) will converge to zero asymptotically.

In (12) as long as \( k, k_1 \) and \( k_2 > 0 \) the system has global asymptotic stability. Based on the algorithm mentioned earlier, the block diagram of the proposed ABNC is shown in Fig. 2.
Substituting Eq. (15) and replacing the errors variables equations (6), (7) and (8) into Eq. (17) yields:
\[
\dot{V} = -k_e^2 - k_i e_q^2 - k_e e_q^2
\]
\[
+ \left( v_{dref} - v_{dc} \right) \left[ -\frac{1}{C} \left( e_d i_{dref} + e_q i_{qref} \right) + k e \right] + e_d \left[ i_{dref} + \frac{R}{L_c} i_d - w_i + \frac{v_{dref}}{L_c} d_d - \frac{v_d}{L_c} + k_i e_d \right]
+ e_q \left[ i_{qref} + \frac{R}{L_c} i_q + w_i + \frac{v_{dref}}{L_c} d_d - \frac{v_d}{L_c} + k_i e_q \right]
+ \frac{1}{\gamma} \left( \dot{v}_{dc} - \dot{v}_{dc} \right) \left[ + \frac{1}{C} \left( e_d i_{dref} + e_q i_{qref} \right) + k e \right] \tag{18} 
\]
Replacing \( v_{dc} = \ddot{v}_{dc} + \dot{v}_{dc} \) into eq. (18) results:
\[
\dot{V} = -k_e^2 - k_i e_q^2 - k_e e_q^2
\]
\[
+ \left( v_{dref} - v_{dc} - \ddot{v}_{dc} \right) \left[ -\frac{1}{C} \left( e_d i_{dref} + e_q i_{qref} \right) + k e \right] + e_d \left[ i_{dref} + \frac{R}{L_c} i_d - w_i + \dot{v}_{dc} d_d - \frac{v_d}{L_c} + k_i e_d \right]
+ e_q \left[ i_{qref} + \frac{R}{L_c} i_q + w_i + \dot{v}_{dc} d_d - \frac{v_d}{L_c} + k_i e_q \right]
+ \frac{1}{\gamma} \left( \ddot{v}_{dc} - \dot{v}_{dc} + \frac{\gamma}{L_c} d_d e_d + \frac{\gamma}{L_c} d_d e_d \right) \tag{19} 
\]
Simplifying Eq. (19) results:
\[
\dot{V} = -k_e^2 - k_i e_q^2 - k_e e_q^2
\]
\[
+ \left( v_{dref} - \ddot{v}_{dc} \right) \left[ -\frac{1}{C} \left( e_d i_{dref} + e_q i_{qref} \right) + k e \right] + e_d \left[ i_{dref} + \frac{R}{L_c} i_d - w_i + \dot{v}_{dc} d_d - \frac{v_d}{L_c} + k_i e_d \right]
+ e_q \left[ i_{qref} + \frac{R}{L_c} i_q + w_i + \dot{v}_{dc} d_d - \frac{v_d}{L_c} + k_i e_q \right]
+ \frac{1}{\gamma} \left( \ddot{v}_{dc} - \ddot{v}_{dc} + \frac{\gamma}{L_c} \left( d_d e_d + d_d e_q \right) \right) \tag{20} 
\]
Letting, \( e_e = v_{dref} - \ddot{v}_{dc} \), then, we can conclude the nonlinear backstepping controllers and the DC voltage observer as follows:
\[
d_d = \frac{L_c}{\dot{v}_{dc}} \left[ -i_{dref} - \frac{R}{L_c} i_d + w_i + \frac{v_d}{L_c} - k e e_q \right] + \frac{1}{L_c} \frac{e_d}{\dot{v}_{dc}} i_d d_d \tag{21} 
\]
\[
d_q = \frac{L_c}{\dot{v}_{dc}} \left[ -i_{qref} - \frac{R}{L_c} i_q - w_i + \frac{v_d}{L_c} + k e e_q \right] + \frac{1}{L_c} \frac{e_q}{\dot{v}_{dc}} i_q d_q \tag{22} 
\]
\[
\ddot{v}_{dc} = \frac{\gamma}{L_c} \left( d_d e_d + d_d e_q \right) + \frac{\gamma}{L_c} \left( i_d e_d + i_q e_q \right) - k e \gamma \tag{23} 
\]
The equation (20) can be simplified and approximated by \( \ddot{v}_{dc} \equiv e \) to give:
\[
\ddot{v}_{dc} = \frac{\gamma}{L_c} \left( d_d e_d + d_d e_q \right) + \frac{\gamma}{L_c} \left( i_d e_d + i_q e_q \right) \tag{23} 
\]
Substituting (21), (22) and (23) into (20) results the following:
\[
\dot{V} = -k_e^2 - k_i e_q^2 - k_e e_q^2 - \frac{k}{\gamma} \ddot{v}_{dc}^2 
\]
According to the algorithm mentioned previously, the block diagram of the sensorless DC voltage control can obtained as in Fig. 3.
Clearly, \( \dot{V} \) is negative definite, so it implies that the
closed loop system is asymptotically stable and hence, all the error variables and the estimation error will converge to zero asymptotically. Therefore the DC voltage will converge to the reference speed. In addition, because the estimation voltage error \( \bar{v}_{dc} \) converges to zero, the estimation DC voltage \( \hat{v}_{dc} \) will converge to the actual DC voltage. As a result, the desired control objective of sensorless DC voltage control is indeed achieved by the proposed sensorless backstepping controller.

4. Simulation Results

The simulation model is developed using Matlab/Simulink software. The simulation model parameters are given in table I. A comparison between the two schemes, Adaptive backstepping nonlinear control (ABNC) and sensorless backstepping control is achieved by simulating the two schemes.

The goal of simulation is to examine three different aspects:

Table I

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage (rms)</td>
<td>120 V</td>
</tr>
<tr>
<td>Supply resistance ( R_s )</td>
<td>1 ( \Omega )</td>
</tr>
<tr>
<td>Supply inductance ( L_s )</td>
<td>0.01 mH</td>
</tr>
<tr>
<td>Active filter inductance ( L_c )</td>
<td>2 mH</td>
</tr>
<tr>
<td>Active filter resistance ( R_c )</td>
<td>1 ( \Omega )</td>
</tr>
<tr>
<td>DC link capacitance</td>
<td>0.1 mF</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>5000 Hz</td>
</tr>
</tbody>
</table>

A. Step change of load

In Fig. 4 the designed feedback controller based on adaptive backstepping method enables the system to get better regulation and tracking performance. The simulation results demonstrate that the capacitor DC voltage has fast transient response and no steady state error and offers robust performance against step load change at \( t = 0.2 \) s. The source current after compensation exhibits better THD content (5%) than without compensation (20.5%) and good current tracking. The THD of the line current was affected due to step change of load.

Fig. 5 shows the dynamic response of the sensorless DC voltage of SAPF when it was subjected to step load change at \( t = 0.3 \) s. The DC voltage offers longer rise time (40 ms) for the DC voltage response than ABNC (16 ms) with steady state error. Nevertheless, at the same time it exhibits rather better THD than the adaptive backstepping and lower DC voltage for achieving the same control objectives. The THD of the line currents were not affected by the load change.
Fig. 5 The simulation results of sensorless DC voltage control due to step load change; (a) The filter current; (b) The source current; (c) The nonlinear load current; (d) The capacitor DC voltage.

B. Compensation of $R_c$ and $L_c$ uncertainties

Fig. 6 shows the effect of $R_c$ and $L_c$ uncertainties for ABNC. The DC voltage response offers robust robustness against parameters variations with little increased rise time but the THD of the line currents was affected from 5.5% to 8%. The filter current was greatly decreased.

Fig. 7 shows the effect of parameters uncertainties of 100% and 50% for $L_c$ and $R_c$ respectively for sensorless backstepping control. The DC voltage offered sensitivity to uncertainties and steady state error w.r.t. the reference value 150 V. The THD of the line currents was increased to 7% compared to 5% without uncertainties.
C. Robustness against frequency variations

This aspect tests the capability of active filter to achieve its control objectives with robustness against step of frequency change from the operating switching frequency 5 kHz to 9 kHz at $t = 0.1$ ms.

Fig. 8 shows the DC voltage response of ABNC isn’t affected at all by step change of frequency but the THD of the line currents was little affected from 5.5% to 6.5% and the filter current was little decreased regardless of the large change. At small step changes little than 2 kHz, these effects will not appear.

Fig. 9 shows the response of DC voltage, line current, filter current and nonlinear load current of sensorless backstepping control when the frequency was step changed from the operating switching frequency 5 kHz to 7 kHz at $t = 0.2$ ms. It is shown that the THD of the line currents was improved from 5.6% to 5.2% and the DC voltage was very little affected. The values of the line and filter currents were not affected.
laws are developed subsequently using Lypunov stability theory. The simulation results verify the effectiveness of the proposed adaptive controller and its robustness to parameters variations, step change of load and switching frequency variation.

The sensorless backstepping controller achieves the same control objectives without utilization of the sensor to measure the DC voltage and lower DC voltages. It offers also robustness against frequency variations but it is sensitive to step load change and parameters uncertainties. Besides, the two control techniques operate at lower switching frequency 5 kHz and didn’t consider the known parameters of the SAPF in the design stage. Future development of this work through implementation using DSP will be tried to investigate its applicability to the Egypt MGC-20 cyclotron and eliminating the problems of power quality in the local power network.

References