Numerical and Analytical Analysis of Eddy Current Non Destructive Testing in JSAEM Benchmark


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Abstract: In this paper, sample comparison is presented by an analytical and numerical models to describe the response of an eddy current (EC) testing for a conducting plate. The 2D electromagnetic analysis is applied in the case of axial symmetry. Harmonic electromagnetic analysis is based on the magnetic vector potential (MVP) formulation. The impedance of the sensor coil is plotted by varying the thickness and the lift-off. This impedance for two models (analytic and numeric) is measured for one frequency equal to 1 MHz. The variation of thickness and lift-off parameters are studied in the model for harmonic excitation and the results are discussed.

Keywords – Non-destructive testing, Eddy current, Finite Element Methods, Electromagnetic.

I. INTRODUCTION

Non-destructive evaluation (NDE) is the science of inspecting materials without compromising their usefulness. These techniques are used in NDE applications by Eddy current for estimating the conductivity and the thickness of a nonmagnetic metallic plate.

This paper describes the harmonic electromagnetic analysis by calculating the electromagnetic field that interacts between sensor coil and target. Through harmonic electromagnetic analysis in ANSYS [6], some useful electromagnetic properties of metallic plate and coil are obtained, such as the eddy currents on the metallic plate, the electromagnetic field distribution, the impedance and the inductance. Hence, the results can identify the working principle of the eddy current sensor and evaluate its impedance. The 2D electromagnetic analysis is applied in the case of axial symmetry. For the eddy current measurement system, two situations with target and without target can be used. The relation of the normalized impedance of the sensor is determined for 2D simulation.

A sample comparison is presented by an analytical and numerical model to describe the response of an eddy current testing. The model computes the induced eddy current distribution in the specimen as well as the resulting impedance change in the coil [7]. Calculation and visualization of impedance plane loci are used for comparison with actual test signals.

II. ANALYTICAL MODEL

The geometry of the problem considered is illustrated schematically in Fig.1. An axisymmetric air-core coil of rectangular cross-section is placed above the evaluated plate. The principle coil parameters are number of turn’s n, inner radius of the coil r1, and outer radius r2, the coil height h, the lift-off l0, the conductivity σ, and the thickness t of the plate. The width and the length of the plate are supposed infinitely large.

![FIG.1. Geometry and dimensions of the 2D axisymmetric model (coil with plate).](image)

The forward problem consists in the determination of the coil impedance in function of the plate parameters and the probe. The calculation of the coil impedance is based on the analytical model developed by Dodd & Deeds [4] [5]. The coil is excited by a constant alternative current of angular frequency ω; its impedance is measured as a function of frequency. So the probe impedance Z is theoretically given by [1], [2]:

\[
Z = jK \int_0^\pi \frac{D^2}{\alpha^2} (2h + 1) (2e^{-\alpha h} - 2 + e^{-\alpha h} - e^{-\alpha h} \int \phi(\alpha) d\alpha) d\alpha
\]

Where

\[
K = \frac{\pi \mu_0 n^2}{h^2 (r_2 - r_1)}
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\[
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\]
\[ D = \int_{\alpha} x J_1 \, dx \]  \hspace{1cm} (3)

Where \( J_1(x) \) is a first-order Bessel function.

\[
\phi(\alpha) = \frac{\left(\alpha + \alpha_1\right)\alpha_1 + \alpha + \alpha_1 e^{2\alpha_1 t}}{\left(\alpha - \alpha_1\right)\alpha_1 - \alpha}\left(\alpha + \alpha_1\right)\left(\alpha + \alpha_1 + \alpha\right) e^{2\alpha_1 t} \]  \hspace{1cm} (4)

\[
\alpha_1 = \sqrt{\alpha^2 + j\omega \mu_0 \sigma} \]  \hspace{1cm} (5)

During the simulation, the coil and plate parameters are: \( r_1 = 20\, \text{mm}, \ r_2 = 30\, \text{mm}, \ h = 10\, \text{mm} \), whereas \( e \) was allowed to vary from 0.11mm to 5mm and the lift-off equal at 0.5mm, 1mm and 1.5mm respectively, the conductivity of coil and the plate are equals to 1.62 and 1.73 respectively.

The electrical impedance of the coil is measured at one excitation frequency \( (f_1=1\, \text{MHz}) \).

### III. NUMERICAL MODEL

The numerical method used to calculate the impedance of a plate, air-cored eddy current coil during the variation of thickness and lift-off over a plate structure is based on the finite element analysis (FEA) method [3] [4].

The eddy current problem can be described mathematically by the following partial differential equation in terms of the magnetic vector potential [3] [4].

\[
\text{rot}(-\text{rot}(A)) = -\sigma(\omega A + \text{grad}(V)) \]  \hspace{1cm} (6)

Where, \( A \) represents the magnetic vector potential, \( V \) electrical scalar potential, \( j \) is the imaginary unit, \( \omega \) is the angular frequency of the excitation current (rad/s), \( \mu \) is the magnetic permeability of the media involved (H/m), \( \sigma \) is the electrical conductivity (S/m).

The FEA model is composed of an exciting coil forced by voltage, plate, near field air and far-field infinite air (see Fig 2). Element type is plane53 with voltage-fed or current–fed, and element INFIN110 is used for define infinite far-field air shown in out-layer of the FEA model [6]. The results in the form of 2D flux (see Fig.3) present clearly the influence of the target on the electromagnetic field.

### IV. DETERMINATION OF THE PROBE IMPEDANCE

For the calculation of the sensor coil impedance, ANSYS can offer an inductance \( L_s \) and an unloaded resistance \( R_s \) calculated by their geometric structure [6]. Also, ANSYS can provide the real \( \text{Re}(I_{es}) \) and imaginary part \( \text{Im}(I_{es}) \) of the current through the coil. Hence, we can obtain the inductance and the resistance of the whole system through electric circuit equation because the voltage applied to the coil is fixed. The related calculation principles are as follows:

\[
I_{es} = \text{Re}(I_{es}) + j\text{Im}(I_{es}) \]  \hspace{1cm} (7)

The probe impedance is given by:

\[
Z_e = \frac{V_e}{I_{es}} = R_e + jX_e \]  \hspace{1cm} (8)
With
\[
R_e = \frac{V}{\sqrt{\text{Re}(I_e)^2 + \text{Im}(I_e)^2}} \cos(\theta) \tag{9}
\]
\[
L_e = \frac{V}{\sqrt{\text{Re}(I_e)^2 + \text{Im}(I_e)^2}} \sin(\theta) \tag{10}
\]

Where
\[
\theta = \tan^{-1}\left(\frac{\text{Im}(I_e)}{\text{Re}(I_e)}\right) \tag{11}
\]

In absence of target, the impedance of coil in air is given by:
\[
\overline{Z}_0 = \frac{\overline{V}}{\overline{I_e}} = R_0 + jX_0 \tag{12}
\]

The impedance is normalized, using the inductive reactance of the coil in air as the normalizing factor.

Where:
\[
\overline{Z}_n = \frac{\overline{Z}_0}{\overline{I_e}} = \frac{Z_e - R_0}{X_0} = \frac{R_e - R_0}{X_0} + j\frac{X_e}{X_0} \tag{13}
\]
\[
R_n = \frac{R_e - R_0}{X_0} \tag{14}
\]
\[
X_n = \frac{X_e}{X_0} \tag{15}
\]

V. RESULTS

The Figures are the normalized impedance plot for varying the thickness and the lift-off; these figures validate the numerical and analytical models. The dashed lines are the lift-off curves and represent the impedance variation with coil lift-off. The computation of coil impedance based on analytical and numerical formulation is validated by computing the model using the Dodd and Deeds solution.

Fig 4: Coil impedance display for various thickness and lift-off equal 0.5mm.

Fig 5: Coil impedance display for various thickness and lift-off equal 1mm.

Fig 6: Coil impedance display for various thickness and lift-off equal 1.5mm.

Fig 7: Coil impedance display for various thickness and lift-off.
VI. CONCLUSION

This paper presents a simple comparison for two models; this comparison is between the analytic model developed by Dodd & Deeds and the numerical model using the Finite Elements. The variation of the coil impedance is shown by varying the thickness of the plate and lift-off parameters. The results obtained indicate that the comparison have a good results accuracy for two models.

VII. REFERENCES


