PSO AND GA BASED INTUITIONISTIC FUZZY OPTIMIZATION PROBLEMS

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Abstract: Linear programming techniques are widely used to solve a number of business, industrial, military, economic, decision making, marketing and advertising problems and optimization plays an important role in Linear programming. In decision making, the decision maker can not decide and choose a best alternative without optimization. In this paper a linear programming problem is constructed in decision making and then Genetic Algorithm & Particle Swarm Optimization are applied to find the optimal solution through which we can have the better results.

Keywords: Intuitionistic Fuzzy set, decision making, Intuitionistic fuzzy optimization, Genetic Algorithm, Particle Swarm Optimization.

Mathematics Subject Classification: 03F55, 08A72.

1. INTRODUCTION

Decision maker is the one who has to choose the best choice among finite number of choices through some assessment based on the requirement. Having a finite set of alternatives and analyzing it based on some evaluative criteria, they are ranked in terms of how attractive they are. Finding the relative priority of each alternative based when all criteria are considered simultaneously is another way to find the best alternative. Solving this type of problems is the main process of multiple-criteria decision analysis (MCDA). When we apply different MCDA methods on the same data, we may get different results and hence this area of decision-making, even though very old, has fascinated many researchers and practitioners. In 1965, Lofti A. Zadeh [1] gave the idea of a fuzzy subset of a set as a method for representing the real world’s uncertainty. As an extension of fuzzy set K. T. Atanassov [2] proposed the definition of intuitionistic fuzzy set. By considering both the grade of certainty and the grade of uncertainty which are not complement to each other Atanassov in his research emphasized the two independent properties for handling vagueness. H-J. Zimmermann [3] suggested a conversion of crisp decision making model into fuzzy decision model. As an extension of fuzzy optimization, Plamen G Angelov [4] considered a transportation problem and obtained the solution through Intuitionistic Fuzzy set which is better than fuzzy environment. S. K. Bharati and S. R. Singh [5] used linear membership function and linear non membership function and compared it with non linear functions. Arindam Garai and Tapan Kumar Roy [6] included the hesitation degree of intuitionistic fuzzy sets for solving the same problem considered by Plamen P. Angelov [4]. Manish Agarwala, Kanad K. Biswas, Madasu Hamandulu, introduced a new score function and the generalized parameter to Intuitionistic fuzzy numbers to show the effectiveness of Generalised Intuitionistic Fuzzy softs sets in decision making [7]. By formulating several linear programming models for generating optimal weights for criteria, Deng-Feng Li [8] investigated multiattribute decision-making using intuitionistic fuzzy sets. Lin Lin, Xue-Hai Yuan, Zun-Quan Xia [9] represented the features of the choices (alternatives) in terms of Intuitionistic Fuzzy sets through which the degree of each alternative is measured. In this method they calculated the optimal weights for criteria using score function and accuracy function and then ranked the alternatives through which a best alternative is identified. After that many researchers have investigated the applications of IFS for decision making. Even though many methods are available for decision making we can have better solutions through these proposed methods. Moreover time consumption is very less in these methods on comparing with the existing methods.

2. GENETIC ALGORITHM:

Genetic Algorithms (GA) [10] are generalized search algorithms based on the mechanics of natural genetics. GA were formerly introduced and developed by J. Holland and his colleagues at the University of Michigan. GA is carried out by maintaining a population of individuals which represent the candidate solutions to the given problem and evaluated by giving some measure of fitness from the objective function. The evaluation by GA is done by using genetic
operators namely, selection, crossover and mutation to obtain the optimality.

2.1 TOURNAMENT SELECTION:

GAs uses a selection method to choose individuals from the population to insert into a mating pool. A new offspring is generated by these individuals from the mating pool which forms the basis of the next generation. Tournament selection is a process done by holding a tournament among S competitors, S being the tournament size. The competitor (individual) with the highest fitness is the winner of the tournament among the S tournament competitors. The winner is then inserted into the mating pool. Being comprised of tournament winners, now the mating pool has a better average fitness than the average population fitness. This difference provides the selection pressure, which drives the GA to improve the fitness of each succeeding generation. Improved selection pressure can be given by simply increasing the tournament size S, as the average from a larger tournament will have a higher fitness than the average of a smaller tournament.

2.2 CROSS OVER:

Cross over is a method of considering more than one parent solutions and producing a child solution from them. Arithmetic crossover is considered in the proposed work.

2.2.1 ARITHMETIC CROSSOVER:

The arithmetic crossover (AC) is the process of creating children which are the weighted arithmetic mean of two parents. Children are feasible with respect to linear constraints and limits. Alpha is random value between [0,1]. When two parents are considered (parent1 and parent2) and if parent1 has the better fitness value, then the function returns the child.

\[
\text{offspring} = \alpha \text{parent1} + (1-\alpha) \text{parent2}
\]

2.3 MUTATION:

Mutation is an operator that preserve genetic diversity between one generation of a population of genetic algorithm chromosomes and to the next genetically. It is similar to biological mutation. Mutation changes one or more gene values in a chromosome from its starting state. In mutation sometimes, the solution may change totally from the previous solution. Hence by mutation GA can come to better solution. The user-definable mutation probability should be set low during which mutation occurs. If it is set too high, the search will turn into the earliest random search. Uniform mutation is considered in this paper.

3. PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization (PSO) is a very simple algorithm that consists of group of variables (particles) and the position of each particle is updated by its velocity over a number of iterations. The idea came from the studies on the synchronous flocking of birds and schooling of fish. By considering a population of candidate solutions, it solves a problem by assuming initial particles with initial velocities and moving these particles by following the current optimum particles. Each particle's movement is affected by its local best known solution and also forwarded to the best known positions that are upgraded as better positions. Particle swarm optimization (PSO) is a technique introduced by Kennedy and Eberhart in 1995. Actually they started creating software simulations of bird flocking and then they applied the algorithm for optimization problems. At present, PSO algorithm is widely used in optimization of vehicle mechanical systems, transportation network design, combination optimization, smart antenna array systems, Graph coloring[11], power systems, parameter search in dynamical systems, fuzzy system control and so on [12]. A.Shumgalatha & S.Mary Raja Slochanal [13] proposed an hybrid PSO to minimize the power transmission losses.

Many researchers applied GA and PSO using Fuzzy mathematics such as control engineering, multi modal optimization, designing of filters, gain tuning of integrated flight, neural networks image segmentation[16], etc. and we have now used it under intuitionistic Fuzzy environment. In this paper, we started with the definition of intuitionistic fuzzy sets, formulation of linear programming in decision making using intuitionistic fuzzy sets and then the corresponding optimization are discussed.

4. INTUITIONISTIC FUZZY SETS

An Intuitionistic fuzzy set (IFS) A in an universe E is of the form \( A = \{< x, \mu_A(x), \gamma_A(x) > | x \in E \} \) where \( \mu_A(x) \in [0, 1] \) is the grade membership and \( \gamma_A(x) \in [0, 1] \) is the grade of non
membership of the element \( x \in A \) satisfying \( 0 \leq \mu A(x) + \nu A(x) \leq 1 \).

Also for each IFS \( A \) in \( E \) the degree of hesitancy (indeterminacy) is given by \( \pi_i(x) = 1 - \mu_i A(x) - \gamma_i A(x) \). Clearly that \( 0 \leq \pi_i(x) \leq 1 \) for each \( x \in E \). In particular, if \( \pi_i(x) = 0 \), for all \( x \in A \) then the IFS \( A \) is reduced to a fuzzy set.

5. MODELLING OF MULTI ATTRIBUTE DECISION MAKING USING INTUITIONISTIC FUZZY VALUES

The construction of decision making problem as intuitionistic Fuzzy problems [8] is as follows. For the sake of simplicity we have considered the same problem which was discussed in Deng-Feng Li [8] and Lin Lin, Xue-Hai Yuan, Zun-Quan Xia [9]. Assuming the collection of \( n \) alternatives \( X = \{ x_1, x_2, \ldots, x_n \} \) from which a best alternative is to be chosen. Let \( A = \{ a_1, a_2, \ldots, a_m \} \) be the set of all attributes on which every alternative is assessed.

Let \( \mu_i \) and \( \nu_i \) be the grade of belonging and the grade of non belonging of the alternative \( x_j \in X \) with respect to the attribute \( a_i \in A \) to the fuzzy definition “excellence”, respectively, where \( 0 \leq \mu_i \leq 1 \), \( 0 \leq \gamma_i \leq 1 \), and \( 0 \leq \mu_i + \gamma_i \leq 1 \) hence \( X_{ij} = \{ x_j, \mu_{ij}, \gamma_{ij} \} \) denotes the intuitionistic fuzzy set of alternatives with respect to the attribute \( a_i \in A \). If the intuitionistic fuzzy index \( \pi_i(x) = 1 - \mu_i A(x) - \gamma_i A(x) \) is high then the decision maker has a larger hesitation margin to the “excellence” of the alternative \( x_j \in X \) with respect to the attribute \( a_i \in A \). The decision maker can improve the evaluation by accumulating the intuitionistic index and hence the assessment value will be in the extended interval \( [\mu_{ij}, \mu_{ij}^0] = [\mu_{ij}, \mu_{ij} + \pi_{ij}] \), where \( \mu_{ij}^0 = \mu_{ij} + \pi_{ij} \) and \( \mu_{ij}^0 = \mu_{ij} + \nu_{ij} \) for all \( x_j \in X \) and \( a_i \in A \). In the same way we denote the degree of belonging and the degree of non belonging of the attribute \( a_i \in A \) by \( \rho_i \) and \( \tau_i \) be to the fuzzy definition of “significance” respectively where \( 0 \leq \rho_i \leq 1 \), \( 0 \leq \tau_i \leq 1 \), and \( 0 \leq \rho_i + \tau_i \leq 1 \).

When the intuitionistic indices \( \eta_i = 1 - \rho_i - \tau_i \) are high then the decision maker has the higher hesitation margin to the “significance” of the attribute \( a_i \in A \). Intuitionistic indices are used to calculate the weight whether it is bigger or smaller. As in case of alternative evaluation the decision maker can improve the evaluating weights by accumulating the intuitionistic index. Hence the assessed weight will be in the extended interval \( [\omega_i^0, \omega_i^0] = [\rho_i, \rho_i + \eta_i] \), where \( \omega_i^0 = \rho_i \) and \( \omega_i^0 = \rho_i + \eta_i \). Clearly \( 0 \leq \omega_i^0 \leq 1 \) for each attribute \( a_i \in A \). In addition to this we assume that \( \sum_{i=1}^n \omega_i^0 \leq 1 \) and \( \sum_{i=1}^n \omega_i^0 \geq 1 \) to calculate the weights \( \omega_i \in [0,1] \), \( (i = 1, 2, \ldots, m) \) satisfying \( \omega_i^0 \leq \omega_i \leq \omega_i^0 \) and \( \sum_{i=1}^m \omega_i = 1 \). Constructing the above problem into a linear programming problem by one of the conventional methods [8] we get.

\[
\text{max } z = \sum_{i=1}^n \sum_{j=1}^m (\mu_{ij}^0 - \mu_{ij}^0) \omega_i,
\]

\[
(i = 1, 2, \ldots, m), \sum_{i=1}^m \omega_i = 1
\]

Now GA & PSO are applied to find the optimal weights of the attributes.

EXAMPLE:

We may have the problem of air-condition selection [8]. Having the three alternate systems \( x_1, x_2 \) and \( x_3 \) denoted by \( X = \{ x_1, x_2, x_3 \} \), let \( A = \{ a_1, a_2, a_3 \} \) be the set of three attributes where \( a_1, a_2, a_3 \) denote the economical factor, working condition, and operative convenience respectively. Statistically we can have the degrees \( (\mu_{ij}, \gamma_{ij}) \) as follows

\[
\begin{bmatrix}
\mu_{ij} & \gamma_{ij} \\
(0.75, 0.10) & (0.80, 0.15) & (0.40, 0.45) \\
(0.60, 0.25) & (0.68, 0.20) & (0.75, 0.05) \\
(0.80, 0.20) & (0.45, 0.50) & (0.60, 0.30)
\end{bmatrix}
\]

Similarly \( (\mu_{ij}^0, \gamma_{ij}^0) \) is taken as

\[
\begin{bmatrix}
\mu_{ij}^0 & \gamma_{ij}^0 \\
(0.75, 0.90) & (0.80, 0.85) & (0.40, 0.55) \\
(0.60, 0.75) & (0.68, 0.80) & (0.75, 0.95) \\
(0.80, 0.80) & (0.45, 0.50) & (0.60, 0.70)
\end{bmatrix}
\]

In the same way
\[
\begin{bmatrix}
a_1 \\
(0.25, 0.25) \\
(0.35, 0.40) \\
(0.30, 0.65)
\end{bmatrix}
\]

i) By the method proposed by Deng-Feng Li[8] the following linear programming problem is obtained.

\[
\text{max } z = -0.35a_1 - 0.87a_2 + 0.45a_3
\]

subject to the constraints

\[
\begin{align*}
0.25 & \leq a_1 \leq 0.75 \\
0.35 & \leq a_2 \leq 0.60 \\
0.30 & \leq a_3 \leq 0.35 \\
a_1 + a_2 + a_3 &= 1
\end{align*}
\]

The optimal solution for this problem by conventional methods, is \( a_1 = 0.25, a_2 = 0.40, a_3 = 0.3 \). After that the index for each alternative is calculated as \( \xi_1 = 0.7335, \xi_2 = 0.6563, \xi_3 = 0.6616 \) which shows that the best alternative is \( x_1 \).
ii) Now by the second method [9] the following linear programming is constructed in the same problem.

\[
\text{max } z = 1.41 \omega_1 + 1.765 \omega_2 + 0.925 \omega_3,
\]

subject to the constraints

\[
\begin{align*}
0.25 &\leq \omega_1 \leq 0.75 \\
0.35 &\leq \omega_2 \leq 0.60 \\
0.30 &\leq \omega_3 \leq 0.35 \\
\omega_1 + \omega_2 + \omega_3 &= 1
\end{align*}
\]

Here the optimal solution by already existing methods is

\[
\begin{align*}
\omega_1 &= 0.25, \\
\omega_2 &= 0.40, \\
\omega_3 &= 0.35
\end{align*}
\]

Based on these values the ranks of the alternatives are calculated as

\[
\begin{align*}
R(a_1) &= 0.5525, \\
R(a_2) &= 0.40425, \\
R(a_3) &= 0.47125
\end{align*}
\]

and \(x_1\) is chosen as the best.

**GENETIC ALGORITHM:**

No. Of generations: 60, population size: 20

Cross over probability: 0.85,

mutation probability: 0.01

The algorithm was run 20 times and the optimal values of control variables are given in the table.

<table>
<thead>
<tr>
<th>Table: 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Variable</td>
</tr>
<tr>
<td>(W_1)</td>
</tr>
<tr>
<td>(W_2)</td>
</tr>
<tr>
<td>(W_3)</td>
</tr>
</tbody>
</table>

**PARTICLE SWARM OPTIMIZATION:**

<table>
<thead>
<tr>
<th>Table: 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>No. of variables</td>
</tr>
<tr>
<td>Population Size</td>
</tr>
<tr>
<td>No. of iterations</td>
</tr>
<tr>
<td>(C_1)</td>
</tr>
<tr>
<td>(C_2)</td>
</tr>
</tbody>
</table>

The highest 10 values are chosen out of 100 values.

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.N o.</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
</tbody>
</table>
RESULTS:

Using conventional method we got $\omega_1 = 0.25, \omega_2 = 0.40, \omega_3 = 0.35$ and $\max z = 0.10933$ By GA method the required solution is $\omega_1 = 0.2589, \omega_2 = 0.4114, \omega_3 = 0.3296$ and $\max z = 0.1111$ and the corresponding indices for each alternative is calculated as $\xi_1 = 0.73163, \xi_2 = 0.66816, \xi_3 = 0.661930$ which shows that the best alternative is $x_1$.

Similarly by PSO method the optimal solution is $\omega_1 = 0.2525, \omega_2 = 0.4446, \omega_3 = 0.3030$ and $\max z = 0.1143$ and the indices are $\xi_1 = 0.7268, \xi_2 = 0.6767, \xi_3 = 0.6683$. Hence we conclude $x_1$ is the best alternative.

In the same way by the second method [9], the solution by conventional methods is $\omega_1 = 0.25, \omega_2 = 0.40, \omega_3 = 0.35$ and $\max z = 1.38225$

By GA method the solution is $\omega_1 = 0.2589, \omega_2 = 0.4114, \omega_3 = 0.3296$ and $\max z = 1.39605$ And by using PSO method the solution is $\omega_1 = 0.2525, \omega_2 = 0.4446, \omega_3 = 0.3030$ and $\max z = 1.4210$

SUMMARY OF RESULTS:

The effectiveness of Genetic Algorithm and Particle Swarm Optimization are shown in Table 4 and Table 5 and both are compared with the results of conventional methods.

### Table 4

<table>
<thead>
<tr>
<th>Objective</th>
<th>Optimal solution[9]</th>
<th>GA method</th>
<th>PSO method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max $z$</td>
<td>$\omega_1 = 0.25$</td>
<td>$\omega_1 = 0.25$</td>
<td>$\omega_1 = 0.2589$</td>
</tr>
<tr>
<td></td>
<td>$\omega_2 = 0.40$</td>
<td>$\omega_2 = 0.4114$</td>
<td>$\omega_2 = 0.4446$</td>
</tr>
<tr>
<td></td>
<td>$\omega_3 = 0.35$</td>
<td>$\omega_3 = 0.3296$</td>
<td>$\omega_3 = 0.3030$</td>
</tr>
<tr>
<td>Subject to</td>
<td>$\max z = 0.10933$</td>
<td>$\omega_1 = 0.3296$</td>
<td>$\max z = 0.1111$</td>
</tr>
<tr>
<td></td>
<td>$0.25 \leq \omega_1 \leq 0.75$</td>
<td>$\omega_1 = 0.1111$</td>
<td>$\omega_1 = 0.1143$</td>
</tr>
<tr>
<td></td>
<td>$0.35 \leq \omega_2 \leq 0.60$</td>
<td>$\omega_2 = 0.4114$</td>
<td>$\omega_2 = 0.4446$</td>
</tr>
<tr>
<td></td>
<td>$0.30 \leq \omega_3 \leq 0.35$</td>
<td>$\omega_3 = 0.3296$</td>
<td>$\omega_3 = 0.3030$</td>
</tr>
<tr>
<td></td>
<td>$\omega_1 + \omega_2 + \omega_3 = 1$</td>
<td>$\omega_1 = 0.3296$</td>
<td>$\omega_1 = 0.3030$</td>
</tr>
</tbody>
</table>

### Table 5

<table>
<thead>
<tr>
<th>Objective</th>
<th>Optimal solution[9]</th>
<th>GA method</th>
<th>PSO method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max $z$</td>
<td>$\omega_1 = 0.25$</td>
<td>$\omega_1 = 0.2589$</td>
<td>$\omega_1 = 0.2525$</td>
</tr>
<tr>
<td></td>
<td>$\omega_2 = 0.40$</td>
<td>$\omega_2 = 0.4114$</td>
<td>$\omega_2 = 0.4446$</td>
</tr>
<tr>
<td></td>
<td>$\omega_3 = 0.35$</td>
<td>$\omega_3 = 0.3296$</td>
<td>$\omega_3 = 0.3030$</td>
</tr>
<tr>
<td>Subject to</td>
<td>$\max z = 1$</td>
<td>$\omega_1 = 1.38225$</td>
<td>$\omega_1 = 1.39605$</td>
</tr>
<tr>
<td></td>
<td>$0.25 \leq \omega_1 \leq 0.75$</td>
<td>$\omega_1 = 1.38225$</td>
<td>$\omega_1 = 1.39605$</td>
</tr>
<tr>
<td></td>
<td>$0.35 \leq \omega_2 \leq 0.60$</td>
<td>$\omega_2 = 0.4114$</td>
<td>$\omega_2 = 0.4446$</td>
</tr>
<tr>
<td></td>
<td>$0.30 \leq \omega_3 \leq 0.35$</td>
<td>$\omega_3 = 0.3296$</td>
<td>$\omega_3 = 0.3030$</td>
</tr>
<tr>
<td></td>
<td>$\omega_1 + \omega_2 + \omega_3 = 1$</td>
<td>$\omega_1 = 0.3296$</td>
<td>$\omega_1 = 0.3030$</td>
</tr>
</tbody>
</table>

### CONCLUSION

In this paper, GA & PSO Algorithms are applied for finding the optimal solution of a linear programming problem in decision making. Simulation results show the effectiveness of the proposed methods on comparison with the existing methods.

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### CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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