SPACE VECTOR BASED VARIABLE SWITCHING FREQUENCY PWM ALGORITHMS FOR INDUCTION MOTOR DRIVES WITH REDUCED COMPLEXITY

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Abstract: This paper presents a simple space vector based variable switching frequency PWM algorithm for induction motor drives. The classical space vector approach needs the calculation of angle, sector and reference voltage in each sampling period, which increases the complexity of the PWM algorithm. To reduce the complexity, this paper presents a simple space vector approach in which by using the phase voltages the switching times are calculated in a simple form. By using the proposed approach various switching sequences such as space vector PWM (SVPWM) basic bus-clamping PWM (BBCPWM) and advance bus-clamping PWM (ABCPWM) algorithms are derived. Then, the rms flux ripple which is a measure of current ripple characteristics were plotted for all the sequences. Based on the rms flux ripple characteristics, the zones of superior performance are identified. Then, in each sampling time period, the suitable sequence, which gives reduced flux ripple is applied to the. To validate the proposed algorithm simulation studies have been carried on v/f controlled induction motor drive at various modulation indices and the results are presented. The results show the effectiveness of the proposed work.

Key words: ABCPWM, BBCPWM, Induction Motor, SVPWM, VSFPWM.

1. Introduction

The adjustable speed drives (ASD) are becoming popular in many applications. In order to control the speed of the drive continuously, an ac voltage with controllable magnitude and frequency is required. This required ac voltage can be obtained by using a pulse width modulated voltage source inverter (PWM-VSI). Various PWM algorithms are developed during the fast five decades. A detailed study on various PWM algorithms is presented in [1] and concluded that among the various PWM techniques, the space vector PWM (SVPWM) gives superior waveform quality. In the SVPWM technique, based on the magnitude of the reference voltage and position of the reference voltage vector, the dwelling times of the switching states are calculated [2]. Though, the SVPWM technique gives superior waveform quality, it gives more harmonic distortion at higher modulation indices and also it gives more switching losses of the inverter due to the continuous modulating signal. In the recent years, the basic bus-clamping PWM (BBCPWM) techniques became popular. The BBCPWM techniques give superior waveform quality at higher modulation indices and gives reduced switching losses of the inverter. For the real time implementation of the PWM algorithms, there are two popular approaches namely; triangular comparison (TC) approach and space vector (SV) approach. The correlation between these two approaches is given in [3]. To reduce the complexity involved in the classical SVPWM and BBCPWM algorithms, simple TC approaches are presented in [4-9] by using the concept of voltage magnitude tests and imaginary switching times.

The SVPWM technique divides the zero state time equally among the two possible zero states. But, the BBCPWM algorithms use only one zero state at any time. Thus, by using the freedom in the division of zero state time various PWM algorithms are generated. These can be generated through either TC or SV approach. Recently, advanced bus-clamping PWM (ABCPWM) techniques are attracting many researchers. These ABCPWM algorithms utilize the degrees of freedom in the division of active state. These ABCPWM algorithms can be generated easily through SV approach when compared with the TC approach as explained in [10, 15]. To study the harmonic analysis of various PWM algorithms two approaches are presented in the literature [4-5, 11-14]. Based on the instantaneous current ripple, the rms ripple characteristics can be plotted for each sequence and then the harmonic analysis can be carried out as explained in [11]. Moreover, by using the concept of flux ripple, which is a measure of current ripple, the harmonic analysis can be carried out as explained in [4-5, 12-14].

This paper first reveals a simple space vector approach for the generation of various PWM
techniques such as SVPWM, BBCPWM and ABCPWM algorithms. Then, by utilizing the concept of stator flux ripple, the rms stator flux ripples are plotted, from which finally variable switching frequency PWM (VFSPWM) techniques are presented.

2. Proposed Space Vector Approach

In the classical SV approach, due to involvement of various calculations, the complication in the algorithm increases. To reduce the complexity, a simple SV approach is presented in this paper, in which the dwelling times can be calculated directly from the instantaneous phase voltages. The procedure for the calculation of switching times is as follows:

The instantaneous phase voltages can be given as

\[ V_{in} = V_{ref} \cos(\theta - 2(r-1)\pi/3) \]

for \( i=a,b,c \) and \( r=1,2,3 \)

The maximum, minimum and middle values of \( V_{in} \) are calculated in each sampling time period as follows:

\[ V_{\text{max}} = \max(V_{\text{in}}) \]
\[ V_{\text{mid}} = \text{mid}(V_{\text{in}}) \]
\[ V_{\text{min}} = \min(V_{\text{in}}) \]

Then, the switching dwell times can be calculated as

\[ T_1 = \frac{T_z}{V_{dc}}(V_{\text{max}} - V_{\text{mid}}) \]
\[ T_2 = \frac{T_z}{V_{dc}}(V_{\text{mid}} - V_{\text{min}}) \]
\[ T_z = T_s - T_1 - T_2 \]

By utilizing the degrees of freedom in the division of zero state and active state times various PWM algorithms such as SVPWM, BBCPWM and ABCPWM techniques can be generated and these are given in Table 1.

Table 1: PWM Sequences in all six sectors

<table>
<thead>
<tr>
<th>Sector Number</th>
<th>SVPWM</th>
<th>BBC PWM-1</th>
<th>BBC PWM-2</th>
<th>ABCPWM</th>
<th>ABCPWM-1</th>
<th>ABCPWM-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0127- 7210</td>
<td>012-210</td>
<td>721-127</td>
<td>0121-1210</td>
<td>7212-2127</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0327- 7230</td>
<td>032-230</td>
<td>723-327</td>
<td>0323-3230</td>
<td>7322-3227</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0347- 7430</td>
<td>034-430</td>
<td>743-347</td>
<td>0343-3430</td>
<td>7434-3434</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0547- 7450</td>
<td>054-450</td>
<td>745-547</td>
<td>0545-5450</td>
<td>7545-4547</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0567- 7650</td>
<td>056-650</td>
<td>765-565</td>
<td>0565-5650</td>
<td>7656-5656</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0167-7610</td>
<td>016-610</td>
<td>761-1610</td>
<td>0161-1610</td>
<td>7616-6167</td>
<td></td>
</tr>
</tbody>
</table>

From Table 1, it can be observed that the number of commutations in each sampling time period for SVPWM and ABCPWM algorithms is three while for BBCPWM algorithms is two. Hence the sampling frequency of the SVPWM and ABCPWM algorithms is the twice the average switching frequency \( f_s = 2f_{sw} \), whereas the sampling frequency of the BBCPWM algorithms is three times the average switching frequency \( f_s = 3f_{sw} \). The SVPWM and BBCPWM algorithms cannot switch more than once in each \( T_s \). Hence, these can be generated with triangular comparison approach also. But, in ABCPWM algorithms, one of the phases will be clamped to the dc bus while the one of the other phases will switch twice in each \( T_s \). These sequences cannot be generated with triangular comparison approach. These can be generated by using the space vector approach very easily. Thus, the space vector approach offers more degrees of freedom compared to the carrier comparison approach.

3. Proposed VFSPWM Algorithms

A. Analysis of Stator Flux Ripple

The total harmonic distortion (THD) of a line current \( I_{THD} \) is a widely used measure for the quality of waveform generated by a PWM inverter, which is defined as

\[ I_{THD} = \frac{1}{T} \sum_{n=1}^{\infty} I_n^2 \]

where, \( I_1 \) and \( I_n \) are the rms values of the fundamental and the \( n \)th harmonic components of the no-load current respectively.

The SV approach generates the reference voltage in an average manner and hence there is always a difference voltage will exists between the applied voltage and reference voltage. This difference voltage is known as ripple voltage vector, which depends on angle and modulation index \( M_f \). The ripple voltage vectors and trajectory of the stator flux ripple for SVPWM and BBCPWM sequences are shown in Fig. 1. The corresponding d-axis and q-axis components of the stator flux ripple vector are as shown in Fig. 2. Here \( T_1, T_2, T_0 \) and \( T_f \) are the switching times of the voltage vectors \( V_1, V_2, V_0 \) and \( V_f \). The \( T_0 \) and \( T_f \) are taken as \( T_0 = kT_s \) and \( T_f = (1-k)T_s \) for \( k=0.5, 0 \) and 1, the expressions for SVPWM, BBCPWM-1 and ABCPWM-2 techniques can be derived.

The error volt-seconds corresponding to the ripple voltage vectors are as follows:

\[ V_r T_1 = \left( \frac{2}{3} V_{dc} \sin \alpha \right) T_1 \]
\[ V_r T_2 = \left( \frac{2}{3} V_{dc} \sin(60^\circ - \alpha) \right) T_2 + \]
\[ \left( \frac{2}{3} V_{dc} \cos(60^\circ - \alpha) - V_{ref} \right) T_2 \]
\[ V_r T_0 = -jV_{ref} T_0 = -j \frac{2M_f V_{dc}}{\pi} T_0 = jQ_0 \]
\[ V_{r1}T_2 = -jV_{ref}T_2 = -j \frac{2M_I V_{dc}}{\pi} T_2 = jQ_2 \quad (12) \]

By using the switching times expressions of classical SVPWM technique [4-5], the (9) and (10) can be expressed in terms of switching times as given in (13) - (14).

![Fig. 1 Voltage ripple vectors and trajectory of the flux ripple](image1)

\[ V_{r1}T_1 = \frac{\pi V_{dc}}{3\sqrt{3} M_i} \frac{T_1 T_2}{T_s} + j \left( \frac{2V_{dc} \pi (T_1 + 0.5T_2)}{9M_i T_s} - \frac{2V_{dc} M_i}{\pi} \right) T_1 = D + jQ_1 \quad (13) \]

\[ V_{r2}T_2 = -\frac{\pi V_{dc}}{3\sqrt{3} M_i} \frac{T_1 T_2}{T_s} + j \left( \frac{2V_{dc} \pi (0.5T_1 + T_2)}{9M_i T_s} - \frac{2V_{dc} M_i}{\pi} \right) T_2 = -D + jQ_2 \quad (14) \]

By using the expressions (11) - (14) and Fig 2, the expressions of the rms stator flux ripple can be derived as in (15).

![Fig. 2 q-axis and d-axis components of the flux ripple vectors](image2)

\[
y^2_{rms} = \frac{1}{3T_s} \left( \frac{V_{dc}^2}{9T_s} (8T_1^4 + 4T_1 T_2 + 4T_2^2) + \frac{4V_{dc}^2 M_i^2}{\pi^2} \left( T_0^3 + 2T_1^3 + T_2^3 + 2T_1 T_2 + T_0 T_1 + T_0 T_2 - 2T_1 T_2 \right) \right) \quad (15)\]

Similarly, the harmonic analysis of ABCPWM algorithms can be carried out. The variation of stator flux ripple in case of ABCPWM-I and ABCPWM-II algorithms are shown in Fig. 3 and Fig 5. The corresponding d- and q-axes stator flux ripple trajectories of AZSPWM-I and AZSPWM-II are as shown in Fig 4 and Fig 6 respectively. Similar to the SVPWM and BBCPWM algorithms, the rms stator flux ripple expression can be calculated as given in (16) and (17).

![Fig. 3 voltage ripple vectors and trajectory of the flux ripple for ABCPWM-I](image3)

\[
y^2_{rms} = \frac{1}{3T_s} \left( \frac{V_{dc}^2}{9T_s} (8T_1^4 + 4T_1 T_2 + 4T_2^2) + \frac{4V_{dc}^2 M_i^2}{\pi^2} \left( T_0^3 + 2T_1^3 + T_2^3 + 2T_1 T_2 + T_0 T_1 + T_0 T_2 - 2T_1 T_2 \right) \right) \quad (15)\]

![Fig. 4 q-axis and d-axis components of the flux ripple vectors for ABCPWM-I](image4)
Thus, the rms stator flux ripple characteristics can be obtained for all the switching sequences. The current pulsations mainly depend on both d- and q-axes flux ripples and the torque pulsation depends on q-axis flux ripple only.

### 3. Proposed VFSPWM Algorithms

To achieve better waveform quality, the harmonic distortion must be reduced. To achieve lower THD, the rms flux ripple must be low. The rms stator flux ripple characteristics of SVPWM, BBCPWM and ABCPWM algorithms can be plotted as shown in Fig. 7 – Fig. 11 at different modulation indices. From Fig. 7 – Fig. 11, the following points can be concluded:

- Replacing $\alpha$ by $(60^\circ - \alpha)$ in the rms stator flux ripple expressions of SVPWM does not change its value, which can be given as in (18).

$$\psi_{\text{SVPWM}}^2(\alpha) = \psi_{\text{SVPWM}}^2(60^\circ - \alpha)$$  \hspace{1cm} (18)

- Swapping of $T_1$ and $T_2$ and that of $Q_1$ and $Q_2$, in the rms stator flux ripple expression of ABCPWM-I (ABCPWM-1) lead to the rms stator flux ripple expression of BBCPWM-II (ABCPWM-2) as given in (19) and (20).

$$\psi_{\text{BBCPWM}}^2(\alpha) = \psi_{\text{ABCPWM}}^2(60^\circ - \alpha)$$  \hspace{1cm} (19)

$$\psi_{\text{ABCPWM}}^2(\alpha) = \psi_{\text{ABCPWM}}^2(60^\circ - \alpha)$$  \hspace{1cm} (20)

- Moreover, from Fig. 7 - Fig. 11, it can be concluded that at lower modulation indices, the SVPWM algorithm gives superior performance whereas at medium modulation indices BBCPWM algorithms give superior performance and at higher modulation indices the ABCPWM algorithms give superior performance.

To minimize the harmonic distortion in the line current, the rms current ripple or rms stator flux ripple over every sampling time period should be reduced. The proposed variable switching frequency PWM (VFSPWM) algorithms employ the best sequence out of the considered possible sequences to minimize the rms current ripple in each sampling time period. The development of proposed VFSPWM techniques for reduced current ripple involves determination of superior performance for every sequence. The zone of superior performance for a given sequence is the spatial zone within a sector where the given sequence results in less mean square stator flux ripple than the other sequences considered. By taking the combination of different sequences two types of VFSPWM algorithms are generated as follows:

- 3-zone VFSPWM algorithm
- 5-zone VFSPWM algorithm
The 3-zone VSFPWM technique involves the use of switching sequences 0127, 012 and 721. From Fig. 7 – Fig. 11, the zones of superior performance of 0127, 012 and 721 sequences can be identified as shown in Fig. 12. Similarly, for the 5-zone VSFPWM algorithm, the zones of superior performance can be represented as shown in Fig. 13, which uses all five switching sequences.
4. Simulation Results and Discussion

To validate the proposed space vector based VSFPWM algorithms, the simulation studies have been carried out on v/f controlled induction motor drive. For the simulation studies, the average switching frequency is taken as 3 kHz and dc link voltage is taken as 540V. The simulation studies are carried out at different operating frequencies (modulation indices). The simulation results of SVPWM and proposed VSFPWM algorithms are as shown in Fig. 14 – Fig. 28 at various modulation indices.

From the simulation results, it can be concluded that the proposed VSFPWM algorithms give superior performance when compared with the SVPWM algorithm. As the proposed VSFPWM algorithms use the bus-clamping sequences, the switching losses of the inverter can be decreased. Moreover, the proposed VSFPWM algorithms give spread spectra and also give reduced harmonic amplitudes at the multiples of switching frequencies when compared with the SVPWM algorithm. This leads to the reduction of acoustic noise.
Fig. 17 Harmonic spectra of line current for SVPWM based induction motor drive at $M_i=0.906$

Fig. 18 Simulation results for 3-zone VSFPWM based induction motor drive at $M_i=0.69$

Fig. 19 Harmonic spectra of line current for 3-zone VSFPWM based induction motor drive at $M_i=0.69$

Fig. 20 Simulation results for 3-zone VSFPWM based induction motor drive at $M_i=0.906$

Fig. 21 Harmonic spectra of line current for 3-zone VSFPWM based induction motor drive at $M_i=0.906$

Fig. 22 Simulation results for 5-zone VSFPWM based induction motor drive at $M_i=0.69$
The spectral density results for SVPWM and proposed VSFPWM algorithms are shown in Fig. 26 – Fig. 27. From these, it can be observed that the proposed VSFPWM algorithms will give reduced acoustical noise of the drive system.

5. Conclusions
A simple SV approach is presented in this paper, which uses instantaneous phase voltages only for the calculation of switching times of the inverter devices. By using the proposed approach various PWM techniques such as SVPWM, BBCPWM and ABCPWM algorithms are generated. Then, by using the concept of stator flux ripple, the rms flux ripple expressions are derived for all sequences. Based on the magnitude of rms stator flux ripple, a suitable switching sequence, which results in reduced harmonic distortion, is applied in each sampling time interval. Thus, the proposed VSFPWM algorithms use a suitable sequence in each sampling period. From the simulation results, it can be observed that the proposed VSFPWM algorithms will give reduced harmonic distortion and acoustical noise when compared with the SVPWM algorithm.
References


