Non linear control Design of the 2-DOF Helicopter (TRMS system)

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Abstract- In this paper, a backstepping controller is designed to position the yaw and pitch angles of a Twin Rotor Multi-input Multi-output System (TRMS). With the coupling effects considered as the uncertainties, the highly coupled nonlinear TRMS is decomposed into a horizontal subsystem and a vertical subsystem. The reaching conditions and the stability of the TRMS with the proposed controller are guaranteed. Finally Simulation results are included to indicate that TRMS with the proposed controller can remain robust to the external disturbances.

Keywords: TRMS system, Backstepping control, Dynamic model.

I-INTRODUCTION

Helicopters are one of the most manoeuvrable and versatile platforms. They can take-off and landing without a runway and can hover in place. These capabilities have brought about the use of autonomous miniature helicopters. For these reasons, there is currently great interest in using these platforms in a wide range of civil and military applications that include traffic surveillance, search and rescue, air pollution monitoring, area mapping, agriculture applications, bridge and building construction inspection. For performing safely many types of these tasks, high manoeuvrability and robustness of the controllers with respect to disturbances and modelling errors are required. This has generated considerable interest in the robust flight control design. The twin rotor multi-input multi-output system (TRMS) is an aero-dynamical system similar to a helicopter it is characterized by the complicated nonlinearity and the high coupling effect between two propellers [1],[2], many efforts have been made to control the TRMS and some strategies have been developed to solve the path following problems for this type of system. First of this works is in [3] the authors present a comparison of classical control and intelligent control based on fuzzy logic control and genetic algorithm applied to the TRMS system. In [4] presents the evolutionary computation based the genetic algorithm for the parameters optimization of the proportional-integral differential (PID) control to the TRMS system. The goals of control are to stabilize the TRMS in significant cross-couplings. In [5] the design procedures of the fuzzy takagi-sugeno model of TRMS are detailed. Based on the derived fuzzy takagi-sugeno model, parallel distributed fuzzy LQR controller are designed to control the position of the pitch and yaw angles in TRMS.

In [6] a multivariable nonlinear $H_\infty$ controller is designed for the angle control of the TRMS. Since the rotor speeds are assumed to be constant, in [7] investigates the development of an adaptive dynamic nonlinear model inversion control law for a TRMS system utilizing artificial neural networks and genetic algorithms, In [8] a stable neural network based observer for TRMS system is proposed to approximate the nonlinearities of the system. A learning rule for neural network is given which guarantee robustness of the observer. In [9], fuzzy controllers are designed for the tracking of pitch and yaw angles of the TRMS system.

On the other hand, the sliding mode control has been applied extensively to control the non linear system. The advantage of this approach is its insensitivity to the model errors, parametric uncertainties, ability to globally stabilize the system and other disturbances [10], [11]. In [12] a fuzzy sliding and fuzzy integral sliding controller is designed to position the yaw and pitch angles of a TRMS system using the linear surface. To simplify the design of an effective controller for the position control of the pitch and yaw angles, the TRMS is pseudo-decomposed into the horizontal and vertical subsystems. Instead of ignoring the coupling effects between the horizontal and the vertical subsystems, the coupling effects are considered as the uncertainties in the horizontal and the vertical subsystems.

The contribution of our work is used the backstepping control in order to ensuring the locally asymptotic stability and desired tracking trajectories. Unlike to However, Finally all the control laws synthesized are highlighted by simulations which gave results considered to be satisfactory.

The remainder of this paper is organized as follows. The dynamics of the TRMS is described in Section II. In Section III, the decomposed model of the TRMS is introduced. Section IV present the backstepping designed and simulation results to demonstrate the effectiveness of our approach. Finally we arrive to the conclusion of the whole work in section V.

II- MODEL DESCRIPTION OF THE TRMS

Similar to most flight vehicles, the helicopter consists of several elastic parts such as rotor, engine and control surfaces. The nonlinear aerodynamic forces and gravity act on
the vehicle, and flexible structures increase complexity and make a realistic analysis difficult. For control purpose, it is necessary to find a representative model that shows the same dynamic characteristics as the real aircraft [8]. The behaviour of a nonlinear TRMS, (shown in Fig. 1), in certain aspects resembles that of a helicopter. It can be well perceived as a static test rig for an air vehicle with formidable control challenges.

This TRMS consists of a beam pivoted on its base in such a way that it can rotate freely in both its horizontal and vertical planes. There are two rotors (the main and tail rotors), driven by DC motors, at each end of the beam. If necessary, either or both axes of rotation can be locked by means of two locking screws provided for physically restricting the horizontal or vertical plane rotation. Thus, the system permits both 1 and 2 degree-of-freedom (DOF) experiments. The two rotors are controlled by variable speed electric motors enabling the helicopter to rotate in a vertical and horizontal plane (pitch and yaw). The tail rotor could be rotated in either direction, allowing the helicopter to yaw right or left. The motion of the helicopter was damped by a pendulum, which hung from a central pivot point. In a typical helicopter, the friction in the system is of the viscous type. The propeller – air subsystem could be described in accordance with the postulates of the flow theory. The propellers, and the rotation can be described in principle as the motion of a pendulum. We can write the equations describing this motion as follows.

a) The main rotor model

\[ M_{v1} = g \left( \frac{m_t}{2} l_t + m_{sh} l_{sc} \right) \sin \alpha_t \]

With:

\[ A = \left( \frac{m_t}{2} l_t + m_{sh} l_{sc} \right) l_t \]

\[ B = \left( \frac{m_{sh} + m_{sw} + m_{ms}}{2} \right) l_m \]

\[ C = \left( \frac{m_t}{2} l_t + m_{sh} l_{sc} \right) l_{sc} \]

\[ M_{v2} = I_m S_f F_i (\omega_i) \]

The mechanical system of TRMS is simplified using a four point-mass system shown in Fig. 2, includes main rotor, tail rotor, balance-weight and counter-weight. Based on Lagrange’s equations, we can classify the mechanical system into two parts, the forces around the horizontal axis and the forces around the vertical axis.

The parameters in the simplified four point-mass system are \( M_{v1} \) is the return torque corresponding to the force of gravity, \( M_{v2} \) is the moment of a centrifugal forces, \( M_{v3} \) is the moment of a centrifugal forces, \( M_{v4} \) is a Moment of friction, \( m_{ms} \) is the mass of the DC motor within the main rotor, \( m_{cb} \) is the mass of the counter-weight beam, \( l_{cb} \) is the distance between the counter-weight and joint, and \( g \) is the gravitational acceleration.

Consider the rotation of the beam in the vertical plane (around the horizontal axis). The driving torqueses are produced by the propellers, and the rotation can be described in principle as the motion of a pendulum. We can write the equations describing this motion as follows.

\[ M_{v1} = g \left( \frac{m_t}{2} l_t + m_{sh} l_{sc} \right) \sin \alpha_t \]

\[ M_{v2} = I_m S_f F_i (\omega_i) \]

\[ M_{v3} = g \left( A - B \right) \cos \alpha_t - C \sin \alpha_t \]

With:

\[ A = \left( \frac{m_t}{2} l_t + m_{sh} l_{sc} \right) l_t \]

\[ B = \left( \frac{m_{sh} + m_{sw} + m_{ms}}{2} \right) l_m \]

\[ C = \left( \frac{m_t}{2} l_t + m_{sh} l_{sc} \right) l_{sc} \]
The angular velocity $\omega_m$ of main propeller is a nonlinear function of a rotation angle of the DC motor describing by:

$$\omega_m(u_{m}) = 90.90 u_{m}^5 + 599.73 u_{m}^4 - 129.26 u_{m}^3 - 1238.64 u_{m}^2 + 63.45 u_{m} + 1238.41$$

(5)

Also, the propulsive force $F_i$ moving the joined beam in the vertical direction is describing by a nonlinear function of the angular velocity $\omega_m$

$$F_i(\omega_m) = -3.48 \times 10^{12} \omega_m^8 + 1.09 \times 10^9 \omega_m^6 + 4.123 \times 10^6 \omega_m^4 - 1.632 \times 10^5 \omega_m^2 + 9.544 \times 10^2 \omega_m$$

(6)

The model of the motor-propeller dynamics is obtained by substituting the nonlinear system by a serial connection of a linear dynamics system. This can be expressed as:

$$\frac{du_{m}}{dt} = \frac{1}{T_{mr}} (-u_{m} + u_{i})$$

(7)

$u_{i}$ is the input voltage of the DC motor, $T_{mr}$ is the time constant of the main rotor and $K_{mr}$ is the static gain DC motor.

Fig 3. The relationship between the input voltage and the propulsive force for the main rotor [13]

$$M_{3} = -\Omega_{b}^{2} (\frac{m_{1} + m_{2} + m_{3}}{4} + \frac{m_{4} + m_{5} + m_{6}}{4})^{2} \sin \alpha \cos \alpha$$

(8)

$$M_{3} = -\Omega_{b}^{2} \frac{H}{\sin \alpha \cdot \cos \alpha}$$

(9)

With:

$$H = \frac{m_{1} + m_{2} + m_{3}}{4} + \frac{m_{4} + m_{5} + m_{6}}{4} - \frac{m_{b}^{2} + m_{b}^{2} + m_{b}^{2}}{4}$$

(10)

$$\Omega_{b} = \frac{d\alpha_{b}}{dt}$$

(11)

$$M_{v4} = -\Omega_{v} K_{v}$$

(12)

$$\Omega_{v} = \frac{d\alpha_{v}}{dt}$$

(13)

$$\frac{dS_{v}}{dt} = \frac{1}{J_{v}} \sum_{i=1}^{4} M_{vi}$$

(14)

$$\frac{dS_{v}}{dt} = \frac{1}{J_{v}} l_{w} S_{v} F_i(\alpha) - K_{v} \Omega_{v} + \frac{1}{J_{v}} (A - B \cos \alpha - C \sin \alpha)$$

(15)

$$-\Omega_{v}^{2} \frac{H}{\sin \alpha \cdot \cos \alpha}$$

$$\frac{d\alpha_{v}}{dt} = \Omega_{v} = S_{v} + \frac{J_{v} \omega_{v}}{J_{v}}$$

(16)

Where $\omega_{v}$ is the angular velocity of tail propeller, $S_{v}$ the angular momentum in the vertical plane of the beam, $J_{v}$ the sum of inertia moments in the horizontal plane, $J_{mr}$ the moment of inertia in DC motor tail propeller subsystem, $K_{v}$ the Friction constant, and $S_{v}$ the balance scale.

b) The tail rotor model

Similarly, we can describe the motion of the beam in the horizontal plane (around the vertical axis) as shown in Fig 4. The driving torques are produces by the rotors and that the moment of inertia depends on the pitch angle of the beam.

Fig 4. Torques around the vertical axis [13]

The parameters in the torques around vertical axis are $M_{ih}$ is the moment of a aerodynamic force, $M_{ih2}$ is a Moment of friction.

$$M_{ih} = l_{w} S_{v} F_{i}(\alpha) \cos \alpha$$

(17)

The angular velocity $\omega_{i}$ of tail propeller is a nonlinear function of a rotation angle of the DC motor describing by:

$$\omega_{i}(u_{ih}) = 2020 u_{ih}^{2} + 194.69 u_{ih}^{3} - 428.315 u_{ih}^{4} - 262.87 u_{ih}^{2} + 3796.83 u_{ih}$$

(18)

Also, the propulsive force $F_{i}$ moving the joined beam in the Horizontal direction is describing by a nonlinear function of the angular velocity $\omega_{i}$

$$F_{i}(\omega_{i}) = -3 \times 10^{-14} \omega_{i}^{8} + 1.595 \times 10^{-11} \omega_{i}^{6} + 2.511 \times 10^{-7} \omega_{i}^{4} - 1.808 \times 10^{-3} \omega_{i}^{2} + 0.8080 \omega_{i}$$

(19)

The model of the motor-propeller dynamics is obtained by substituting the nonlinear system by a serial connection of a linear dynamics system. This can be expressed as:

$$\frac{du_{ih}}{dt} = \frac{1}{T_{vr}} (-u_{ih} + u_{i})$$

(20)

$u_{ih}$ is the input voltage of the DC motor, $T_{vr}$ is the time constant of the tail rotor and $K_{vr}$ is the static gain DC motor.
The dynamics of the TRMS system are described as follows:

\[
\begin{align*}
M_{h2} &= -\Omega_h K_h \\
\frac{dS_h}{dt} &= \frac{1}{J_h(\alpha_i)} \sum_{i=1}^{4} M_{hi} \\
J_h(\alpha_i) &= D \cos^2 \alpha_i + E \sin^2 \alpha_i + F \\
\frac{dS_h}{dt} &= l_{ij} S_j F_i(\alpha_i) \cos \alpha_i - \Omega_h K_h \\
\frac{dS_h}{dt} &= l_{ij} S_j F_i(\alpha_i) \cos \alpha_i - \Omega_h K_h \\
\frac{d\alpha_h}{dt} &= \Omega_h = S_h + \frac{J_{mr} \omega_m \alpha_i}{D \cos^2 \alpha_i + E \sin^2 \alpha_i + F}
\end{align*}
\]

Where \( S_h \) the angular momentum in the horizontal plane of the beam, \( J_h \) the sum of inertia moments in the vertical plane, \( J_{mr} \) the moment of inertia in DC motor main propeller subsystem, \( K_h \) the friction constant, and \( S_j \) the balance scale.

The dynamics of the TRMS system are described as follows:

\[
\begin{align*}
\frac{dS_v}{dt} &= \frac{1}{J_v} \left\{ l_{m} S_j F_i(\alpha_i) - K_v \Omega_v + g(A - B) \cos \alpha_i - C \sin \alpha_i \right\} \\
\Omega_v &= \frac{\alpha_v}{dt} \\
\Omega_v &= S_v + \frac{J_{mr} \omega_m \alpha_i}{J_v} \\
\frac{dS_h}{dt} &= l_{ij} S_j F_i(\alpha_i) \cos \alpha_i - \Omega_h K_h \\
\frac{d\alpha_h}{dt} &= \Omega_h = S_h + \frac{J_{mr} \omega_m \alpha_i}{D \cos^2 \alpha_i + E \sin^2 \alpha_i + F}
\end{align*}
\]

The model developed in (26) can be rewritten in the state-space form:

\[
\dot{X} = f(X) + g(X, U) \quad \text{and} \quad X = [x_1, \ldots, x_6]^T \text{ is the state vector of the system such as:}
\]

\[
\begin{align*}
X &= [\alpha_i, S_i, u_{i_1}, \alpha_h, S_h, u_{i_0}] \\
U &= [u_v, u_h] \\
Y &= [\alpha_i, \alpha_h]
\end{align*}
\]

From (26), (28) and (29) we obtain the following state representation:

\[
\begin{align*}
\dot{x}_1 &= x_2 + \frac{J_{mr}}{J_v} \omega_m [\alpha_i] \\
\dot{x}_2 &= \frac{1}{J_v} \left\{ l_{m} S_j F_i(\alpha_i) - K_v \Omega_v + g(A - B) \cos x_2 - C \sin x_2 \right\} \\
\dot{x}_3 &= \frac{1}{J_{mr}} \left[ -x_3 + K_{mr} u_i \right]
\end{align*}
\]

III. DECOUPLED MODELS OF THE TRMS SYSTEM

Since the characteristic of TRMS is very complex in the nature, it would be convenient to design a controller for TRMS with the TRMS decoupled into horizontal and vertical subsystems by fixing the horizontal angle \( \alpha_h \) and posing \( u_{i_0} = 0 \), from Eq. (30), it is easy to see that state equations with the state vector \( x_i \) for the vertical subsystem of the TRMS could be defined as:

\[
X_v = [x_{i_1}, x_{i_2}, x_{i_3}] = [\alpha_i, S_i, u_{i_1}]
\]

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{J_v} \left\{ l_{m} S_j F_i(\alpha_i) - K_v [x_{i_2}] + g(A - B) \cos x_2 - C \sin x_2 \right\} \\
\dot{x}_3 &= \frac{1}{J_{mr}} \left[ -x_3 + K_{mr} u_i \right]
\end{align*}
\]

Where \( u_{i_1} \) is a control action of the vertical subsystem

Likewise, we have the horizontal subsystem by posing \( \alpha_h(0) = \alpha_{h_0} \) and \( u_{i_1} = 0 \)

\[
X_h = [x_{i_1}, x_{i_2}, x_{i_3}] = [\alpha_h, S_h, u_{i_0}]
\]

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_1 &= \frac{1}{J_v} \left\{ l_{m} S_j F_i(\alpha_h) \cos \alpha_h - K_h x_1 \right\} \\
\dot{x}_2 &= \frac{1}{J_{mr}} \left[ -x_2 + K_{mr} u_i \right]
\end{align*}
\]
IV. BACKSTEPPING CONTROLLER DESIGN

The backstepping approach as a recursive algorithm for the control-low synthesis, we all the stages of calculation concerning the tracking errors and lyapunov function. From (32) we obtain the following state representation of the vertical subsystem

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = f_2(x_1) - b_1 x_2 + g_2(x_1) \\
\dot{x}_3 = -C_3 x_3 + d_3 u_3
\]

(35)

\[
b_1 = \frac{K_1}{J_1} \\
c_3 = \frac{1}{T_{mr}} - d_3 = \frac{K_{mr}}{T_{mr}}
\]

(36)

\[
f_2(x_1) = \frac{l_z}{J_1} S_1 F_1(v_m(x_3)) \\
g_2(x_1) = \frac{K}{J_1} ((A - B) \cos(x_1) - C \sin(x_1))
\]

(37)

The first error considered in designing the backstepping Lyapunov theory is used while using the Lyapunov function \( z_{1v} \) as a positive definite and its time derivative as a negative semi definite,

\[
V_{1v} = \frac{1}{2} z_{1v}^2
\]

Its derivative is given by

\[
\dot{V}_{1v} = z_{1v} \dot{z}_{1v} = z_{1v} (x_2 - \dot{x}_{1d})
\]

(38)

There is no control input in (38). By letting \( x_2 \) be the virtual control, the desired virtual control is defined as:

\[
(x_2)_{1d} = -\alpha_{1v} z_{1v} + \dot{x}_{1d} \quad \alpha_{1v} > 0
\]

Where \( \alpha_{1v} \) is a positive constant for increasing the convergence speed of the vertical angle tracking loop.

Now, the virtual control is \( x_2 \) where the second error tracking is defined by:

\[
z_{2v} = x_2 - (x_2)_{1d} = x_2 + \alpha_{1v} z_{1v} - \dot{x}_{1d}
\]

The augmented Lyapunov function for the second step is given by

\[
V_{2v} = \frac{1}{2} \dot{z}_{2v}^2 + \frac{1}{2} z_{1v}^2
\]

(39)

The derivative of (53) is given by:

\[
\dot{V}_{2v} = z_{1v} \dot{z}_{1v} + z_{2v} \dot{z}_{2v}
\]

(40)

By putting \( \dot{z}_{2v} \) in (40), the following can be obtained

\[
\dot{V}_{2v} = z_{1v} \dot{z}_{1v} + z_{2v} \left( f_1(x_1) - b_1 x_2 + g_1(x_1) - \alpha_{1v} (x_2 - \dot{x}_{1d}) - \dot{x}_{1d} \right)
\]

(41)

By letting \( f_1(x_1) + g_1(x_1) \) be the virtual control, the desired virtual control in the second step is defined as:

\[
(f_1(x_1) + g_1(x_1))_{1d} = -\alpha_{2v} z_{2v} + b_1 x_2 + \alpha_{1v} (x_2 - \dot{x}_{1d}) - \dot{x}_{1d}
\]

(42)

Now, the virtual control is \( f_1(x_1) + g_1(x_1) \) where the sliding surface is defined in the third step by:

\[
z_{3v} = f(x_1) + g(x_1) - (f_1(x_1) + g_1(x_1))_{1d} = f(x_1) + g(x_1) + \alpha_{2v} z_{2v} - \dot{x}_{1d} - \alpha_{1v} (x_2 - \dot{x}_{1d}) - b_1 x_2
\]

(43)

(44)

The augmented Lyapunov function for the third step is given by
The derivative of (45) is given by:
\[
V_{3h} = \frac{1}{2} \dot{z}_{3h}^2 + \frac{1}{2} \dot{z}_{3h2}^2 + \frac{1}{2} \dot{z}_{3h4}^2
\]  
(45)

The derivative of (45) is given by:
\[
\dot{V}_{3h} = z_{3h} \ddot{z}_{3h} + z_{3h2} \ddot{z}_{3h2} + z_{3h4} \ddot{z}_{3h4}
\]  
(46)

By putting \( \ddot{z}_{3h} \), in (46), the following can be obtained
\[
\dot{V}_{3h} = z_{3h} \ddot{z}_{3h} + z_{3h2} \ddot{z}_{3h2} + z_{3h4} \ddot{z}_{3h4}(\frac{\partial g_i(z)}{\partial x} + \alpha_\alpha \tilde{\xi}_h + \alpha_\beta \{v - i_{h0}\} - \ddot{\tilde{z}_{3h}})
\]  
(47)

As for the backstepping approach, the control input \( u_\nu \) is extracted:
\[
u_\nu = \frac{1}{d} \frac{\partial f_h(x)}{\partial x}
\]  
(48)

Likewise From (34) we obtain the following state representation of the horizontal subsystem
\[
\begin{align*}
x_h &= x_h \\
f_h(x_h) &= f_h(x_h) \\
\dot{x}_h &= -C_h x_h + d_h u_h \\
b_h &= \frac{K_h}{J_h(\alpha_\beta)} \\
c_h &= \frac{1}{T_\nu} \\
f_h(x_h) &= \frac{l_1}{J_h(\alpha_\beta)} S_f f_h(x_h) \cos(\alpha_\beta)
\end{align*}
\]  
(49)

The first error considered in designing the backstepping \( \ddot{z}_{3h} = x_h - x_{4d} = \alpha_h - \alpha_{1d} \)

Lyapunov theory is used while using the Lyapunov function \( \ddot{z}_{3h} \) as a positive definite and its time derivative as a negative semi definite,
\[
V_{1h} = \frac{1}{2} \dot{z}_{3h}^2
\]  
Its derivative is given by
\[
\dot{V}_{1h} = z_{3h} \ddot{z}_{3h}
\]  
(50)

There is no control input in (50). By letting \( x_h \) be the virtual control, the desired virtual control is defined as:
\[
(x_h) = -\alpha_\beta x_{3h} + x_{4d} \quad \alpha_{1d} > 0
\]  
(52)

Where \( \alpha_{1d} \) is a positive constant for increasing the convergence speed of the vertical angle tracking loop.

Now, the virtual control is \( x_h \) where the second error tracking is defined by:
\[
z_{3h} = x_h - (x_h) = x_h - \alpha_{1d} x_{1h} - x_{4d}
\]

The augmented Lyapunov function for the second step is given by
\[
V_{2h} = \frac{1}{2} \dot{z}_{2h}^2 + \frac{1}{2} \dot{z}_{2h2}^2
\]  
(53)

The derivative of (53) is given by:
\[
\dot{V}_{2h} = z_{3h} \ddot{z}_{3h} + z_{3h2} \ddot{z}_{3h2} + z_{3h4} \ddot{z}_{3h4}(\frac{\partial g_i(z)}{\partial x} + \alpha_\alpha \tilde{\xi}_h + \alpha_\beta \{v - i_{h0}\} - \ddot{\tilde{z}_{3h}})
\]  
(54)

By putting \( \ddot{z}_{3h} \) in (54), the following can be obtained
\[
\dot{V}_{2h} = z_{3h} \ddot{z}_{3h} + z_{3h2} \ddot{z}_{3h2} + z_{3h4} \ddot{z}_{3h4}(\frac{\partial g_i(z)}{\partial x} + \alpha_\alpha \tilde{\xi}_h + \alpha_\beta \{v - i_{h0}\} - \ddot{\tilde{z}_{3h}})
\]  
(55)

By letting \( f_h(x_h) \) be the virtual control, the desired virtual control in the second step is defined as:
\[
(x_h) = -\alpha_\beta x_{3h} + x_{4d} \quad \alpha_{1d} > 0
\]

Now, the virtual control is \( f_h(x_h) \) where the sliding surface is defined in the third step by:
\[
z_{3h} = S_h = f_h(x_h) - (f_h(x_h))
\]  
(56)

The augmented Lyapunov function for the third step is given by:
\[
V_{3h} = \frac{1}{2} \dot{z}_{3h}^2 + \frac{1}{2} \dot{z}_{4d}^2 + \frac{1}{2} \dot{z}_{3h4}^2
\]  
(57)

The derivative of (59) is given by:
\[
\dot{V}_{3h} = z_{3h} \ddot{z}_{3h} + z_{3h4} \ddot{z}_{3h4} + z_{3h2} \ddot{z}_{3h2} + z_{3h4} \ddot{z}_{3h4}(\frac{\partial g_i(z)}{\partial x} + \alpha_\alpha \tilde{\xi}_h + \alpha_\beta \{v - i_{h0}\} - \ddot{\tilde{z}_{3h}})
\]  
(58)

As for the backstepping approach, the control input \( u_h \) is extracted:
\[
u_h = \frac{1}{d} \frac{\partial f_h(x_h)}{\partial x}
\]  
(59)

V. SIMULATION RESULTS

To show the performance of the proposed approach, the corresponding algorithm is implemented in simulation to the TRMS model as shown in Fig.6. The results obtained for the vertical and horizontal subsystems are given in Fig.7, and Fig.8. One can see that, the backstepping controller ensures a good tracking. To show the robustness of the TRMS with the backstepping controller, an external disturbance is added to each of the vertical and horizontal angle at \( t=10 \) sec. From the performance of the TRMS in Fig.9, it can be seen that the TRMS with the backstepping controller is robust to the external disturbances.
VI. CONCLUSION

In this paper, we presented stabilizing control laws synthesis by backstepping technique. Firstly, we start by the development of the dynamic model of the TRMS taking into account the different physics phenomena. A highly coupled nonlinear TRMS is decomposed into a set of horizontal and vertical subsystems with the coupling effect considered as the uncertainties. Simulation results also validate that the presented backstepping has a satisfactory tracking performance.

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