REAL TIME IMPLEMENTATION OF ROBUST CONTROLLER BASED TUNING FOR DESIRED CLOSED-LOOP RESPONSE FOR SI/SO SYSTEMS

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Abstract: In this article, the IMC-PID approach is generalized to obtain the PID parameters for general models by approximating the ideal controller with a Maclaurin series in the Laplace variable. It turns out that the PID parameters so obtained provide somewhat better closed-loop responses than those obtained by PID controller tuned by other methods. Further all of the PID parameters depend on the desired closed-loop time constant in a manner consistent with engineering intuition. The effectiveness of the PID controllers tuned by the proposed tuning method will be validated both by simulation studies and real time implementation.

Key words: IMC PID, Maclaurin series, ISE, Controller tuning.

1. INTRODUCTION

Since the proportional, integral and derivative controller finds widespread use in process industries a great deal of effort has been directed at finding the best choices for the controller parameters for various process models. Among the performance criteria used for PID controller parameter tuning, the criterion to keep the response of the process close to the desired closed-loop response has gained widespread acceptance in the chemical process industries, because of its simplicity, robustness and successful practical applications. The IMC-PID tuning method and direct synthesis method are typical tuning methods for achieving a desired loop response. Also current tuning methods yield PID parameters only for a restricted class of process models. There is no general methodology for arbitrary process other than approximating them with a first or second-order models and applying tuning rules for the approximate models. In this article, the IMC-PID approach is generalized to obtain the PID parameters for general models by approximating the ideal controller with a Maclaurin series in the Laplace variable. It turns out that the PID parameters so obtained provide somewhat better closed-loop responses.

2. DEVELOPMENT OF GENERAL TUNING ALGORITHM FOR PID CONTROLLER:

Single Degree of Freedom Controller

Consider a stable (that is no right half plane poles) process model of the form [1].

\[ G(s) = P_m(s)P_a(s) \]  

Where \( P_m(s) \) is the portion of the model inverted by the controller (it must be minimum phases) \( P_a(s) \) is the portion of the model not inverted by the controller (it is usually non minimum phase that is it contains dead times and/or right half plane zeros) and \( P_a(0) = 1 \).

Often, the portion of the model not inverted by the controller is chosen to be all pass that is, of the form

\[ \frac{1}{s^2 + \zeta \omega_n s + \omega_n^2} \]

where \( \omega_n \) and \( \zeta \) are the natural frequency and damping ratio, respectively.

The effectiveness of the PID controllers tuned by the proposed tuning method will be validated both by simulation studies and real time implementation.

Fig.1.- Feedback control system

\[ PA(s) = \prod_{i,j} \left( \frac{-\tau_i s + 1}{\tau_i s + 1} \right) \left( \frac{-\tau_i^2 s^2 - 2\tau_i \zeta j s + 1}{\tau_i^2 s^2 - 2\tau_i \zeta j s + 1} \right) \rightarrow (2) \]

\[ \tau, \tau_i, j \geq 0 ; 0 \leq \zeta \leq 1 \]
Figure 1 shows the feedback control system with IMC-PID controller $G_c$. Let as take $G_0$ and $q_r = 1$. Since this choice gives the best least-squares response. The requirement that $P_A(0) = 1$ is necessary for the controlled variable to track its set point. The aim is to choose the controller $G_c$ of Figure 1 to give the desired closed-loop response, $C/R$ given by

$$
\frac{C}{R} = \frac{P_A(s)}{(\lambda s + 1)^r} \rightarrow (3)
$$

The term $1/(\lambda s + i)^r$ functions as a filter with an adjustable time constant $\lambda$, and an order $r$ chosen so that the controller $G_c$ is realizable.

The ideal controller $G_c$ that yields the desired loop response given by Eq 2 perfectly is given by

$$
G_c(s) = \frac{q}{1 - Gq} = \frac{p_m}{(\lambda s + 1)^r - P_A(s)} \rightarrow (4)
$$

where ‘$q’ is the IMC controller

$$
P_m^{-1}(s) \frac{1}{(\lambda s + 1)^r} \rightarrow (5)
$$

The controller $G_c$ can be approximated to obtain a PID controller by first noting that it can be expressed as

$$
G_c = \frac{f(s)}{s} \rightarrow (6)
$$

whereas $G_c$ has a pole at the origin because $P_A(0)$ is one, $f(s)$ will not have such a pole because the derivative of $(\lambda s + 1)^r - P_A(s)/s$ at the origin is never zero for $r$ greater than Zero.

Expanding $G_c(s)$ in a maclaurin series in $s$ gives

$$
G_c(s) = \frac{1}{2} \left[ f(0) + f'(0)s + \frac{f''(0)}{2} s^2 + \ldots \right] \rightarrow (7)
$$

It should be noted that the resulting controller has the proportional term, integral term and derivative term, in addition to an infinite number of higher-order derivative terms. Since the controller given by Eq.7 is equivalent of the ideal controller given by Eq.4, the desired closed-loop response can be perfectly achieved if all terms in Eq.7 are implemented. In practice, however, it is impossible to implement controller given by Eq. 7 because of the infinite number of high-order derivative terms. Infact, in an actual control situation low and middle frequencies are much more important than high frequencies, and only the first three terms in Eq. 7 are often sufficient to achieve the desired closed-loop performance. The controller given by Eq. 7 can be approximated to the PID controller by using only the first three terms (1/s, 1, s) in Eq. 7 and truncating all other high-order terms ($S^2, S^3, \ldots$). The first three terms of the above expansion can be preted as the standard PID controller given by

$$
G_c(s) = K_c (1 + \frac{1}{T_i s} + T_D s) \rightarrow (8)
$$

where $K_c = f'(0) \rightarrow (9)$

$$
T_i = \frac{f''(0)}{f(0)} \rightarrow (10)
$$

$$
T_D = \frac{f''(0)}{2 f'(0)} \rightarrow (11)
$$

In order to evaluate the PID controller

Parameters given by the above Eqs. we let

$$
D(s) = (\lambda s + 1)^r - P_A(s) / s \rightarrow (12)
$$

Then, by Maclaurin series expansion we get

$$
D(0) = r \lambda - P_A(0) \rightarrow (13)
$$

$$
D(0) = [r(r - 1) \lambda^2 - P_A(0)] / 2 \rightarrow (14)
$$

$$
D(0) = [r(r - 1)(r - 2) \lambda^3 - P_A(0)] / 3 \rightarrow (15)
$$
Using Eq. 12 the function \( f(s) \) and its first and second derivatives, all evaluated at the origin, are given by

\[
f(0) = \frac{1}{K_P D(0)} \quad \text{(16)}
\]

\[
f'(0) = \frac{[P'(0) D(0) + K_P D'(0)]}{[K_P D(0)]^2} \quad \text{(17)}
\]

\[
f''(0) = f'(0) \quad \text{(18)}
\]

\[
\left[ \left( \frac{[P'(0) D(0) + 2P''(0)]}{P'(0) D(0)} \right) \right] \frac{2f'(0)}{f(0)} \quad \text{(19)}
\]

Where

\[
K_P = P_\text{m}(0) = G(0) \quad \text{(20)}
\]

The above formulas can be used to obtain the controller gain, and integral and derivative time constants as analytical functions of the process model parameters and the closed-loop time constant \( \lambda \).

### 3. DERIVATION OF THE PARAMETERS OF APPROXIMATED IMC-PID CONTROLLER:

**One degree of Freedom Controllers:**

\[
G(s) = \frac{K e^{-\theta_k}}{\tau_s + 1} \quad \text{(21)}
\]

\[
G_i = \frac{1}{G_{\text{mm}}} \quad \text{(22)}
\]

\[
G_{\text{mm}} = \frac{K}{\tau_s + 1} = P_\text{m}(s) \quad \text{(23)}
\]

\[
G_{\text{ma}} = e^{-\theta_k} = P_A(s) \quad \text{(24)}
\]

\[
C = e^{-\theta_k} = \frac{P_A(s)}{\lambda s + 1} \rightarrow \text{(25)}
\]

\[
K_c = \frac{T_i}{K(\lambda + \theta)} \rightarrow \text{(26)}
\]

\[
T_i = \tau + \frac{\theta^2}{2(\lambda + \theta)} \rightarrow \text{(27)}
\]

\[
T_D = \frac{\theta^2}{2(\lambda + \theta)} \left( 1 - \frac{\theta}{3T_i} \right) \rightarrow \text{(28)}
\]

### 4. IMC-PID CONTROLLER

The example Process I chosen for simulation study is

\[
G(s) = \frac{e^{-3s}}{10s + 1} \rightarrow \text{(29)}
\]

This \( G(s) \) is divided into

\[
G_{\text{mm}} = \frac{1}{10s + 1} \rightarrow \text{(29)}
\]

\[
G_{\text{ma}} = e^{-3s} \rightarrow \text{(30)}
\]

where \( G_{\text{mm}} \) is the portion of the model inverted by the controller (it must be minimum phase) \( G_{\text{ma}} \) is the portion not inverted by the controller (it is usually non minimum phase). The controller parameters derived for the proposed method of tuning IMC-PID controller as shown in Table I. Referring the Table I the controller parameters for the first-order process with delay are given as below:

\[
K_G = \frac{T_i}{K(\lambda + \theta)} \rightarrow \text{(31)}
\]

\[
T_i = \tau + \frac{\theta^2}{2(\lambda + \theta)} \rightarrow \text{(32)}
\]

\[
T_D = \frac{\theta^2}{6(\lambda + \theta)} \left( 3 - \frac{\theta}{T_i} \right) \rightarrow \text{(33)}
\]

where

- \( k \) = Gain of the process
- \( \tau \) = Time constant of the process
- \( \theta \) = Dead time of the process
- \( \lambda \) = Time constant of the desired closed-loop response

For the example process I the values of controller
parameters are calculated as

\[ K_c = 2.3 \]
\[ T_i = 11 \]
\[ T_d = 0.909 \text{ for } \lambda = 1.5 \]

The unit step response for \( \lambda = 1.5 \) is shown in Figure 3.
\[ \lambda_{\text{adjusted}} = 3.48 \]

\( \lambda \) is adjusted for desired closed-loop response with \( \lambda = 3.48 \).

The PID parameters are

\[ K_c = 0.64 \]
\[ T_i = 4.17 \]
\[ T_d = 0.527 \text{ for } \lambda_{\text{adjusted}} = 3.48 \]

The unit step response for \( \lambda_{\text{adjusted}} = 3.48 \) is shown in Figure 3.

This proposed method of tuning is compared with Rivera method. The responses are shown in Fig.3. It is seen from all the above figures that the proposed method of tuning gives the response closer to desired response than other tuning methods.

**Process II:**

The example process II chosen for simulation study is

\[ G(s) = \frac{e^{-\theta s}}{12s + 1}; \]

\[ \lambda = 1.5 ; \lambda_{\text{adjusted}} = 3.48. \]

The Integral Square Error (ISE) is calculated for the proposed method of tuning IMC-PID controller and is compared with Rivera method of tuning IMC-PID controller. The graph is plotted for various values of \( \theta/\tau \) and is shown in Figure 2. From this figure, it is seen that the proposed tuning rule gives the smallest ISE among all tuning rules over entire range of \( \theta/\tau \). The difference in the values of the ISE becomes more significant as the dead time effect dominates. However it is observed that the magnitude of the ISE obtained is not in good agreement with the expected magnitude of the ISE.

\[ K_c = \frac{\tau_i}{K(\lambda + \theta)} \rightarrow (35) \]
\[ T_i = \tau + \frac{\theta^2}{2(\lambda + \theta)} \rightarrow (36) \]
\[ T_d = \frac{\theta^2}{6(\lambda + \theta)} \left[ 3 - \frac{\theta}{\tau_i} \right] \rightarrow (37) \]

where

- \( k \) = Gain of the process
- \( \tau \) = Time constant of the process
- \( \lambda \) = Time constant of the desired closed-loop response
Table NO. 1 - Various tuning rules to give the desired Closed loop response

<table>
<thead>
<tr>
<th>Process Model</th>
<th>Tuning Method</th>
<th>$K_c$</th>
<th>$T_i$</th>
<th>$T_d$</th>
<th>$T_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G = \frac{Ke^{-\theta}}{\tau s + 1}$</td>
<td>Rivera et al.</td>
<td>$\frac{2\tau + \theta}{K (\lambda + \theta)}$</td>
<td>$\tau + \frac{\theta}{2}$</td>
<td>$\frac{\tau\theta}{2\tau + \theta}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rivera et al. (with Filter)</td>
<td>$\frac{2\tau + \theta}{2K (\lambda + \theta)}$</td>
<td>$\tau + \frac{\theta}{2}$</td>
<td>$\frac{\tau\theta}{2\tau + \theta}$</td>
<td>$\frac{\lambda\theta}{2(\lambda + \theta)}$</td>
</tr>
<tr>
<td></td>
<td>Proposed</td>
<td>$\frac{T_i}{K(\lambda + \theta)}$</td>
<td>$\tau + \frac{\theta^2}{2(\lambda + \theta)}$</td>
<td>$\frac{\theta^2}{6(\lambda + \theta)} \left[ 3 - \frac{\theta}{T_f} \right]$</td>
<td></td>
</tr>
<tr>
<td>$G = \frac{Ke^{-\theta}}{\tau s + 1}$</td>
<td>Smith</td>
<td>$\frac{T_i}{K(\lambda + \theta)}$</td>
<td>$\tau$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rivera et al. Improved IMC-PI</td>
<td>$\frac{2\tau + \theta}{2K (\lambda + \theta)}$</td>
<td>$\tau + \frac{\theta}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proposed</td>
<td>$\frac{T_i}{K(\lambda + \theta)}$</td>
<td>$\tau + \frac{\theta^2}{2(\lambda + \theta)}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G = \frac{Ke^{-\theta}}{(\tau^2 s^2 + 2\zeta \tau s + 1)}$</td>
<td>Smith</td>
<td>$\frac{\tau_1 + \tau_2}{(K(\lambda + \theta)}$</td>
<td>$\tau_1 + \tau_2$</td>
<td>$\frac{\tau_1\tau_2}{\tau_1 + \tau_2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proposed</td>
<td>$\frac{T_i}{K(2\lambda + \theta)}$</td>
<td>$2\zeta \tau - \frac{2\lambda^2 - \theta^2}{2(2\lambda + \theta)}$</td>
<td>$\tau_i - 2\zeta \tau + \frac{\tau^2 - \frac{\theta^2}{6(2\lambda + \theta)}}{T_i}$</td>
<td></td>
</tr>
</tbody>
</table>
For the example process II the values of controller parameters are calculated as
\[ K_c = 2.3, \quad T_i = 11, \quad T_d = 0.909 \text{ for } \lambda = 1.5. \]
The unit step response for \( \lambda = 1.5 \) is shown in Figure 4.

6. IMPLEMENTATION OF THE APPROXIMATED IMC-PID CONTROLLER

The controller parameters are calculated for the modeled process by proposed method and they are
\[ K_c = 4.88, \quad \tau_i = 0.081, \quad \tau_d = 0.194 \text{ with } \lambda = 1.5. \]

5. REAL TIME IMPLEMENTATION:

DESCRIPTION OF THE PROCESS

The circuit diagram of the process taken for study is shown in Figure 5. This process is modeled using process reaction curve method. The open-loop response of the process is shown in Figure 6. From this response, the process model is determined as \( G(s) = e^{-s}/(12s+1) \) and hence this is used in simulations study as process II.
The unit step response with these parameters is shown in Figure 7. The desired closed–loop response is shown in the Figure 8. The proposed method is compared with Rivera method of tuning IMC–PID. These responses are compared with the simulated responses. They are found to be in good agreement with each other. It is also found that the response of the proposed method of tuning is closer to desired response than the other methods was found from the results that the responses of both the software simulated and hardware simulated

REFERENCES:


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