MODELING AND SIMULATION OF A MATRIX CONVERTER / INDUCTIVE LOAD SYSTEM

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Abstract
In this paper, a system composed of a matrix converter and an inductive load is modeled and simulated using Matlab / Simulink software package. The modulation techniques used for the matrix converter are Venturini and optimum Venturini methods. The voltages and currents waveforms of the system are presented and compared to those obtained from previously published results to validate the simulation process.

Keywords: Matrix converter, Inductive load, Matlab / Simulink.

1. Introduction
The matrix converter is a direct AC-AC converter which uses an array of controlled bidirectional switches to create a controllable output voltage system with unrestricted frequency [1-12]. Recently, matrix converters receive a lot of attention because they have several advantages; such as they have simple and compact power circuit, generation of load voltage with arbitrary amplitude and frequency can be obtained, they operate with nearly sinusoidal input and output currents with harmonics only around or above the switching frequency, operation with unity input displacement factor for any load can be obtained and they have bidirectional power flow [1-12].

Few investigations have used Matlab / Simulink for simulation of the matrix converter to obtain its performance when used in some applications [4,13,14]. However either the transfer ratios were calculated by m-file in Matlab [4] or a simplified solution algorithm [13,14] was used. In this paper Matlab / Simulink package is used to simulate the matrix converter, in detail, when Venturini and optimum Venturini modulation techniques [3, 6] are used. The matrix converter is loaded by an inductive static load.

2. Matrix Converter Model
Fig. (1) shows a simplified representation of a matrix converter system. It consists of nine bidirectional power switches which are controlled to connect the input three-phase voltage source to a three-phase load. Controlling of these switches is achieved according to a certain modulation strategy. The modulation strategy is based on a desired output voltage magnitude and frequency and an input displacement factor [1-7]. The matrix converter will be modeled by its output voltages and input currents in terms of switching functions of the switches as follows:

\[
\begin{align*}
 v_a(t) &= [S_{Aa}(t) \ S_{Ba}(t) \ S_{Ca}(t)] \ v(t) \\
 v_b(t) &= [S_{Ab}(t) \ S_{Bb}(t) \ S_{Cb}(t)] \\
 v_c(t) &= [S_{Ac}(t) \ S_{Bc}(t) \ S_{Cc}(t)] \\
 i_A(t) &= [S_{Aa}(t) \ S_{Ab}(t) \ S_{Ac}(t)] \ i(t) \\
 i_B(t) &= [S_{Ba}(t) \ S_{Bb}(t) \ S_{Bc}(t)] \\
 i_C(t) &= [S_{Ca}(t) \ S_{Cb}(t) \ S_{Cc}(t)]
\end{align*}
\]

The switching functions in eqns. (1) and (2) are defined as follows [3, 5]:

\[ S_{Kj} = \begin{cases} 1, & \text{switch } S_{Kj} \text{ closed} \\ 0, & \text{switch } S_{Kj} \text{ open} \end{cases} \quad K = \{A,B,C\} , \quad j = \{a,b,c\} \]

In order to avoid short-circuited input terminals and open-circuited output phases, these switching functions should satisfy the following constraint equation [3, 5]:

\[ S_{Aj} + S_{Bj} + S_{Cj} = 1, \]

The shape of the switching functions depends on the switching pattern used; a typical switching pattern is shown in Fig. (2) [3].
Fig. (1) Simplified representation of a matrix converter system.

Fig. (2) General form of the switching pattern.
By considering that the bidirectional power switches work with high switching frequency, a low-frequency output voltage of variable amplitude and frequency can be generated by modulating the duty cycle of the switches using their respective switching functions. The switching frequency is usually greater than 20 times the output frequency in order to obtain an output voltage with low harmonic content [4].

The low-frequency transfer matrix, also known as modulation matrix [6, 7], is defined by [3, 4]:

\[
\mathbf{M}(t) = \begin{bmatrix}
    m_{Aa}(t) & m_{Ba}(t) & m_{Ca}(t) \\
    m_{Ab}(t) & m_{Bb}(t) & m_{Cb}(t) \\
    m_{Ac}(t) & m_{Bc}(t) & m_{Cc}(t)
\end{bmatrix}
\]

(3)

where \( m_{Kj}(t) = t_{Kj}/T_{seq} \)

(4)

The constraint equations for the matrix converter can be written as [1-4]:

\[
\sum_{K=A,B,C} m_{Ka}(t) = \sum_{K=A,B,C} m_{Kb}(t) = \sum_{K=A,B,C} m_{Kc}(t) = 1 \tag{5}
\]

There are several modulation techniques used to obtain the modulation matrix, \( \mathbf{M}(t) \), such as Venturini method, Venturini optimum method, scalar method and space vector modulation method [3]. In this paper, the Venturini and Venturini optimum methods will be used.

When the voltage gain ratio, \( q \), is less than or equal to 0.5 for unity input displacement factor, the modulation duty cycles can be obtained using the Venturini method from the following compact form [3, 6]:

\[
m_{Kj} = \frac{1}{3} \left[ 1 + \frac{2v_{Kj}}{v_{im}^2} + \frac{4q}{3\sqrt{3}} \right] \left( \sin(\omega_0 t + \beta_K) \sin(3\omega_0 t) \right) \tag{6}
\]

for \( K = A, B, C \), \( j = \{a, b, c\} \), \( \beta_K = 0, 2\pi/3, 4\pi/3 \) for \( K = A, B, C \) respectively. The reference output voltages, \( v_j \), are obtained from [2, 3, 4, 6]:

\[
v_o = q v_{im} \begin{bmatrix}
    \cos(\omega_0 t) - \frac{1}{6} \cos(3\omega_0 t) + \frac{1}{2\sqrt{3}} \cos(3\omega_0 t) \\
    \cos(\omega_0 t + 2\pi/3) - \frac{1}{6} \cos(3\omega_0 t) + \frac{1}{2\sqrt{3}} \cos(3\omega_0 t) \\
    \cos(\omega_0 t + 4\pi/3) - \frac{1}{6} \cos(3\omega_0 t) + \frac{1}{2\sqrt{3}} \cos(3\omega_0 t)
\end{bmatrix} \tag{10}
\]

3. Inductive Load model

The inductive load will be modeled in terms of the load current differential equation given by:

\[
D[I] = [VL] + [LR][I] \tag{11}
\]

where

\[
[I] = \begin{bmatrix}
    i_a \\
    i_b \\
    i_c
\end{bmatrix}, \quad [VL] = \begin{bmatrix}
    v_a / L \\
    v_b / L \\
    v_c / L
\end{bmatrix}, \quad [LR] = \begin{bmatrix}
    -R / L & 0 & 0 \\
    0 & -R / L & 0 \\
    0 & 0 & -R / L
\end{bmatrix},
\]

\( R \) and \( L \) are the load resistance and inductance.

4. Method of Analysis

The performance of the system under consideration will be obtained for given voltage gain ratio, \( q \), output frequency, \( f_o \), and unity input power factor using the following procedure. When the voltage gain ratio, \( q \), is less than or equal to 0.5 for given input voltages, eqn. (7), the reference output voltages are obtained using eqn. (8). The input and output voltages are used in eqn. (6) to obtain the modulation duty cycles of the matrix converter switches.

When the voltage gain ratio, \( q \), is greater than 0.5 and less than or equal to \( \sqrt{3}/2 \), the reference output voltages are obtained from eqn. (10). The input and output voltages are used in eqn. (9) to obtain the modulation duty cycles of the switches.
The modulation duty cycles are used to obtain the conduction periods of the switches during a switching interval, $T_{seq}$, for a given switching frequency, $f_s$. When the switching pattern shown in Fig. (2) is used, the switching functions of the converter switches, $S_k$, can be obtained by comparing the modulation duty cycles with a sawtooth waveform, whose frequency equals the switching frequency and its amplitude is unity. The switching functions and the input voltages are used in eqn. (1) to obtain the three-phase output voltages of the matrix converter. The three-phase output voltages are used in eqn. (11) to obtain the load currents. The corresponding input currents can be obtained using eqn. (2).

5. System Simulation

The system under consideration is simulated using Matlab / Simulink software package. Fig. (3) shows the Simulink block diagram of the system corresponding to the system of Fig. (1). This block diagram consists of three subsystems which are called “Matrix Converter”, “RL-Load” and “MC Input Current”. The blocks labeled “vA”, “vB” and “vC” represent the input voltage waveforms which are given in eqn. (7).

The details of the subsystem “Matrix Converter” are shown in Fig (4). It consists of two subsystems called “$q<=0.5$” and “$0.5<q<=0.866$”. The details of these subsystems are shown in Fig. (5) and (6) respectively. In these figures the block labeled “vst” represents the sawtooth waveform and the block labeled “Enable” is used to enable the subsystem output to be used when its input equals to unity otherwise the output will be disabled. Fig. (5) represents eqns. (1), (6) and eqn. (8). Fig. (6) represents Eqns. (1), (9) and (10). Fig. (7) shows the details of the subsystem “RL-Load” given in Fig. (3). It represents eqn. (11). Fig. (8) shows the details of the subsystem “MC Input Current” of Fig. (3). It represents eqn. (2).
Fig. (4) Subsystem "Matrix Converter".

Fig. (5) Subsystem "q<=0.5".
Fig. (6) Subsystem “0.5<q<=0.866”.

Fig. (7) Subsystem “RL-Load”.

Fig. (8) Subsystem “MC Input Current”.
6. Results

Results are obtained for the system under consideration with a voltage gain ratio $q=0.4$, 100 Hz output frequency, 5 kHz switching frequency, 230 V input voltage (peak value), 50 Hz input frequency, 10 $\Omega$ load resistance and 20 mH load inductance.

Figs. (9) and (10) show the output voltage and current waveforms of the matrix converter respectively. The obtained output current waveforms are compared with those obtained in reference [4], as shown in Fig. (10), in order to validate the simulation process. It is found that the two sets of results are almost identical. It can be noted from Fig. (10) that the output current waveforms are approximately sinusoidal waveforms, therefore they have low harmonic content. Another set of results is obtained for the system under consideration with a voltage gain ratio $q=0.85$, 30 Hz output frequency, 5 kHz switching frequency, 110 V input voltage (peak value) and with the same values used for previous case for the input frequency, load resistance and load inductance. Figs. (11) and (12) show the output voltage and current waveforms of the matrix converter respectively. The obtained output current waveforms are compared with those obtained in reference [4] as shown in Fig. (12), in order to validate the simulation process. It is found that the two sets of results are almost identical. It can be noted from Fig. (12) that the output current waveforms are more distorted compared to those obtained for the first case, Fig. (10).

![Output voltage waveforms at $q=0.4$, $f_o=100$ Hz.](image-url)
Fig. (10) Output current waveforms at $q=0.4$, $f_o=100$ Hz.

Fig. (11) Output voltage waveforms at $q=0.85$, $f_o=30$ Hz.
7. Conclusion
A Matlab / Simulink block diagram has been constructed, in detail, for a matrix converter / an inductive load system. The constructed block diagram for the matrix converter can be used to study and analyze the performance of any application in which the matrix converter is used whether the voltage gain ratio $q<=0.5$ or $0.5<q<=0.866$. The obtained results are almost identical with those obtained from the literature which proves the validity of the simulation using Matlab / Simulink package.

8. References


